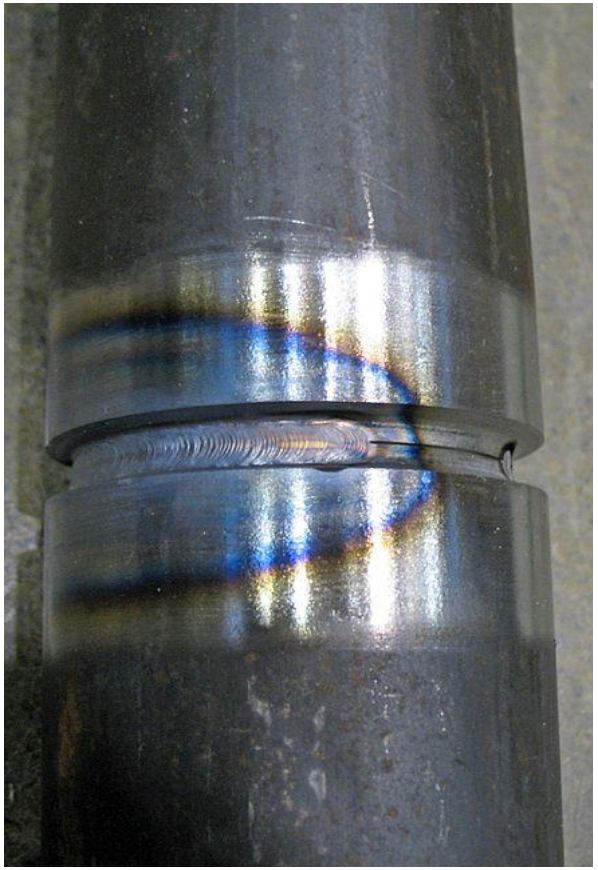
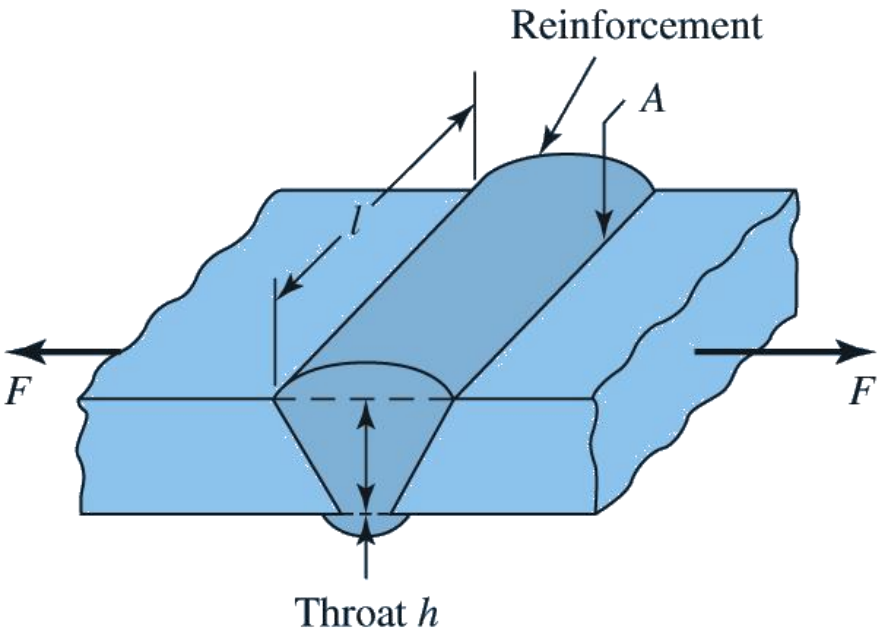
Welding

- **Butt Joints**
  - **Tension**
  - **Shear**
- **Fillet Welds**
  - **Parallel**
  - **Transverse**



BUTT JOINT: Tensile Stress Calculations

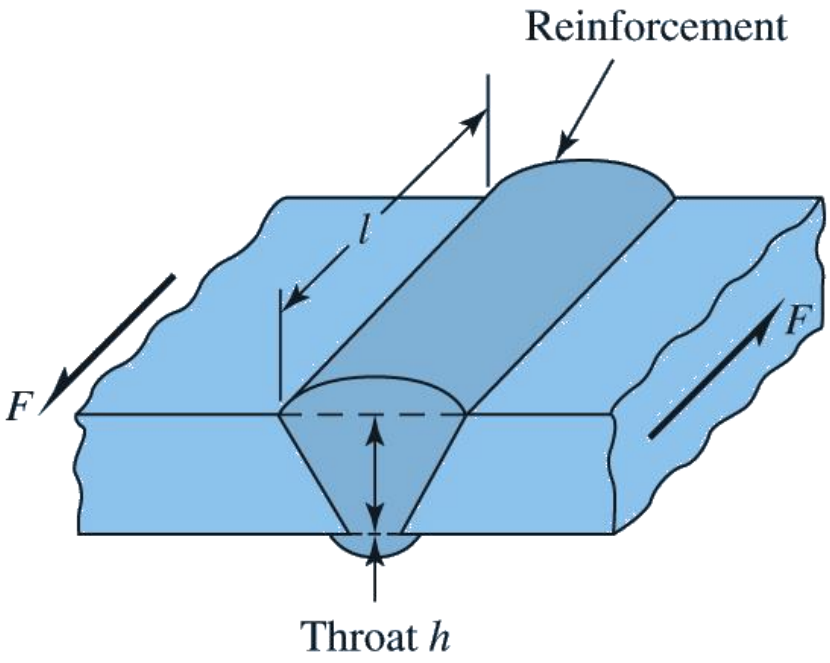


- **Normal Stress**

$$\sigma = \frac{F}{h \cdot l}$$

- **Reinforcement caused Stress Concentration at A**
- **Reinforcement Ground Off to Improve Fatigue Strength**

BUTT JOINT: Shear Stress Calculations



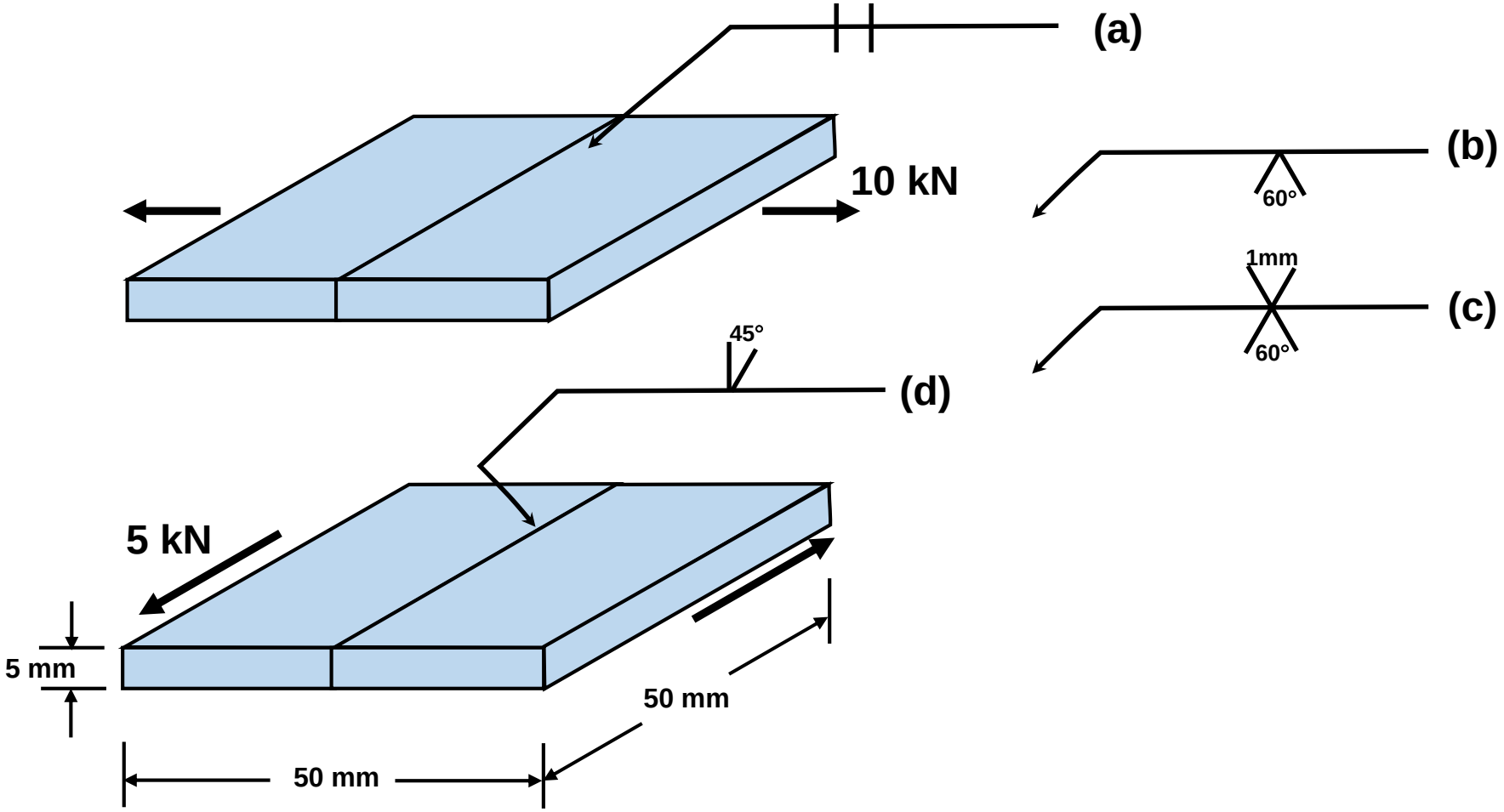
- **AVERAGE Shear Stress**

$$\tau = \frac{F}{h \cdot l}$$

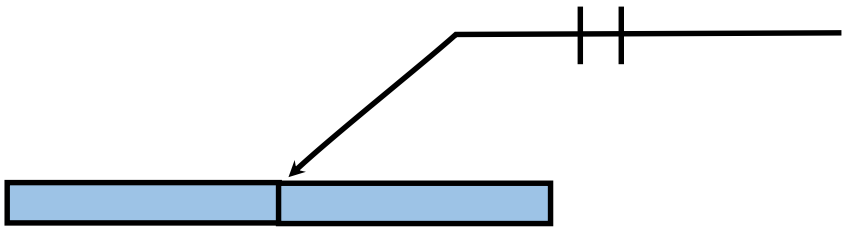
- **Reinforcement caused Stress Concentration at A**
- **Reinforcement Ground Off to Improve Fatigue Strength**

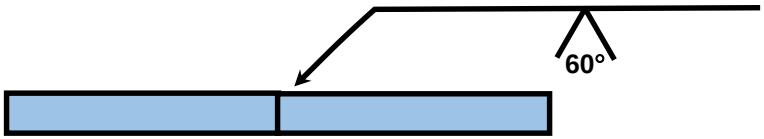
Example

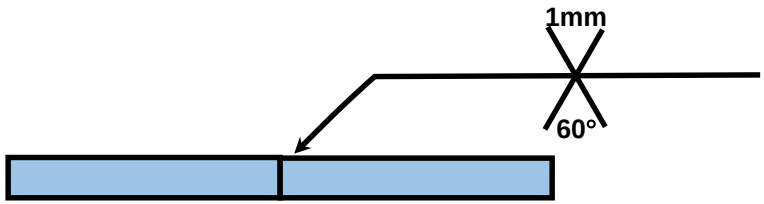
For the two load scenarios shown, calculate the normal and shearing stress in each weld shown.

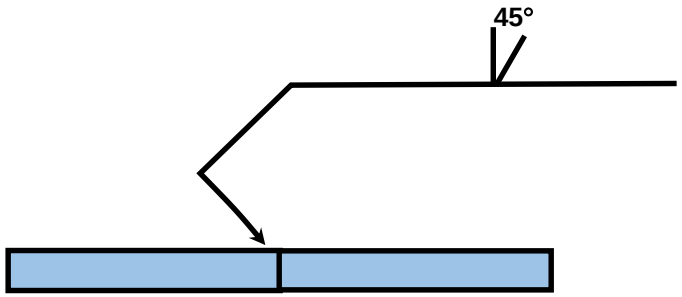


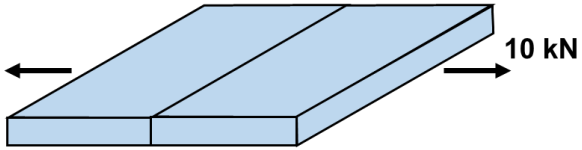
Solution

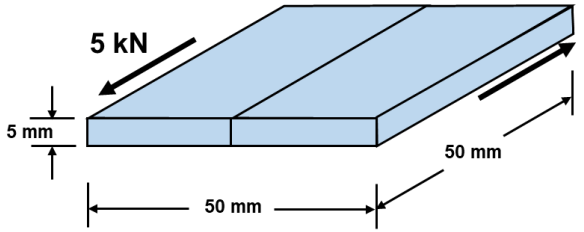




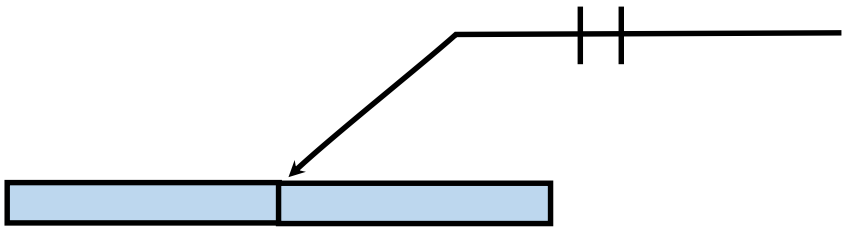


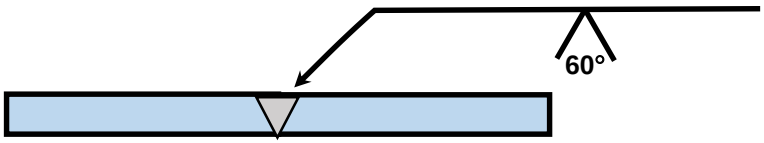


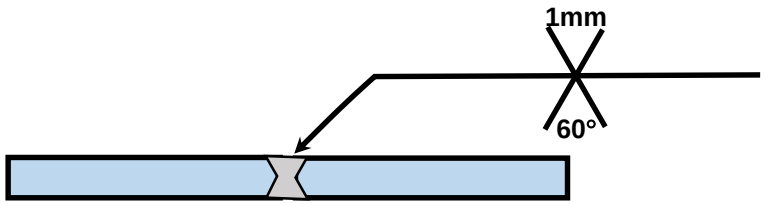


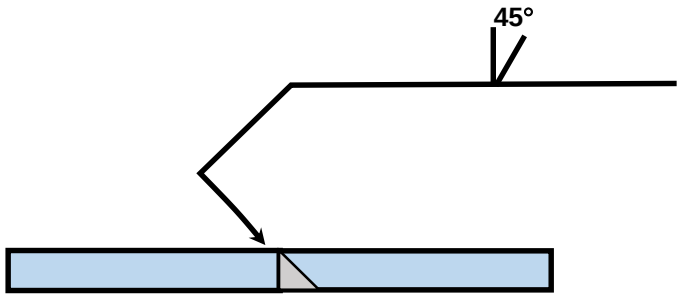


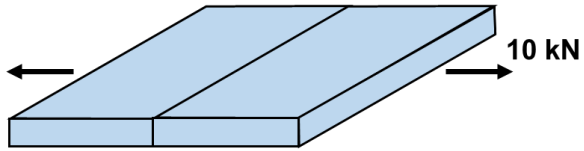
Solution

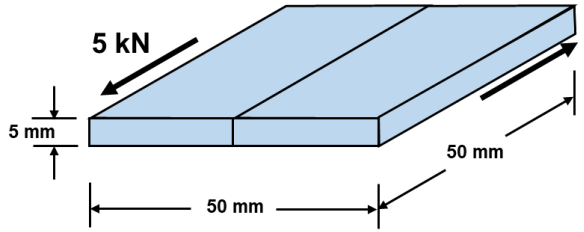








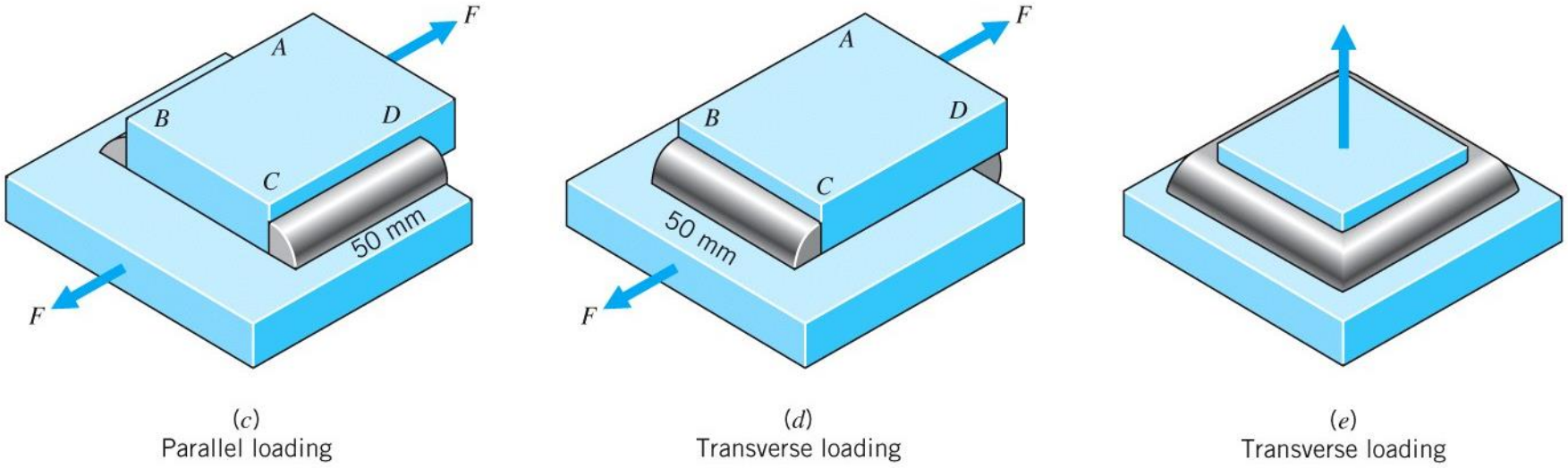




$$\sigma = \frac{F}{l \cdot h} = \frac{10\,kN}{(50\,mm)\cdot(5\,mm)} = 40\,MPa$$

$$\tau = \frac{F}{l \cdot h} = \frac{5\,kN}{(50\,mm)\cdot(5\,mm)} = 20\,MPa$$

FILLET WELD: Classification



Parallel Loading

- **Both plates exert a shear load on the weld**

Transverse Loading

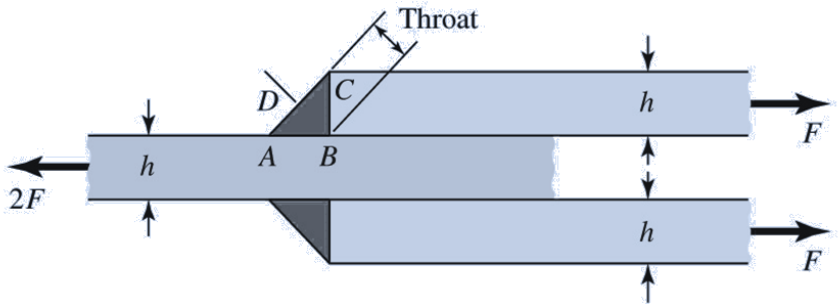
- **One of the plates exerts a shear load on the weld**
- **The other plate exerts a tensile or compressive load on the weld**

Calculating the Maximum Stress in FILLET WELD

Normal and Shear Stress as a Function of θ

$$\sigma = \frac{F_N}{h \cdot l} \quad \Rightarrow \quad F_N = F \cdot \cos\theta \quad \Rightarrow \quad \sigma = \frac{F \cdot \cos\theta}{h \cdot l} \quad ①$$

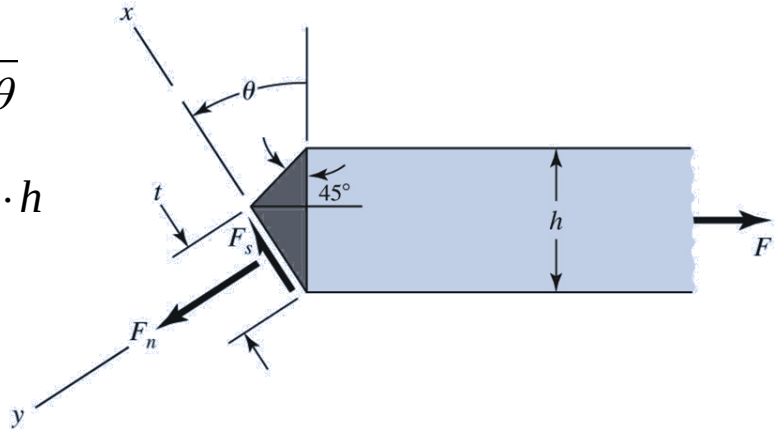
$$\tau = \frac{F_S}{h \cdot l} \quad \Rightarrow \quad F_S = F \cdot \sin\theta \quad \Rightarrow \quad \tau = \frac{F \cdot \sin\theta}{h \cdot l} \quad ②$$



Throat Thickness as a Function of θ

$$\frac{t}{\sin 45°} = \frac{h}{\sin(180° - 45° - \theta)} = \frac{h}{\sin(135° - \theta)} = \frac{\sqrt{2} \cdot h}{\cos\theta + \sin\theta}$$

$$t = \frac{h}{\cos\theta + \sin\theta} \quad ③ \quad \xrightarrow{\theta = 45°} \quad t = \frac{h}{0.7071 + 0.7071} = 1.414 \cdot h$$



Substituting ③ into ① and ②

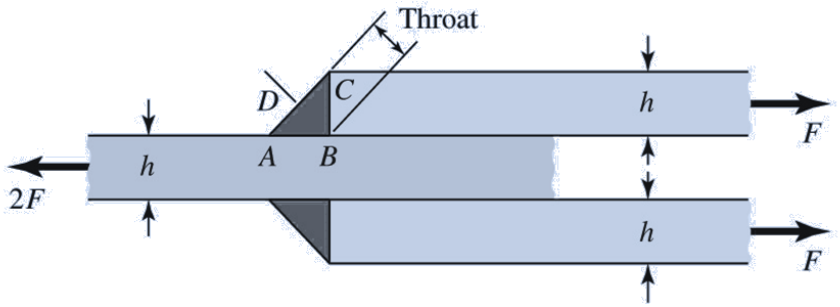
$$\sigma = \frac{F \cdot \sin\theta \cdot (\cos\theta + \sin\theta)}{h \cdot l} = \frac{F}{h \cdot l} \cdot (\sin\theta \cdot \cos\theta + \sin^2\theta) \quad ④$$

$$\tau = \frac{F \cdot \cos\theta \cdot (\cos\theta + \sin\theta)}{h \cdot l} = \frac{F}{h \cdot l} \cdot (\cos^2\theta + \sin\theta \cdot \cos\theta) \quad ⑤$$

Calculating the Maximum Stress in FILLET WELD

$$\sigma = \frac{F \cdot \sin\theta \cdot (\cos\theta + \sin\theta)}{h \cdot l} = \frac{F}{h \cdot l} \cdot (\sin\theta \cdot \cos\theta + \sin^2\theta)$$

$$\tau = \frac{F \cdot \cos\theta \cdot (\cos\theta + \sin\theta)}{h \cdot l} = \frac{F}{h \cdot l} \cdot (\cos^2\theta + \sin\theta \cdot \cos\theta) \quad ①$$



The von Mises Stress is written

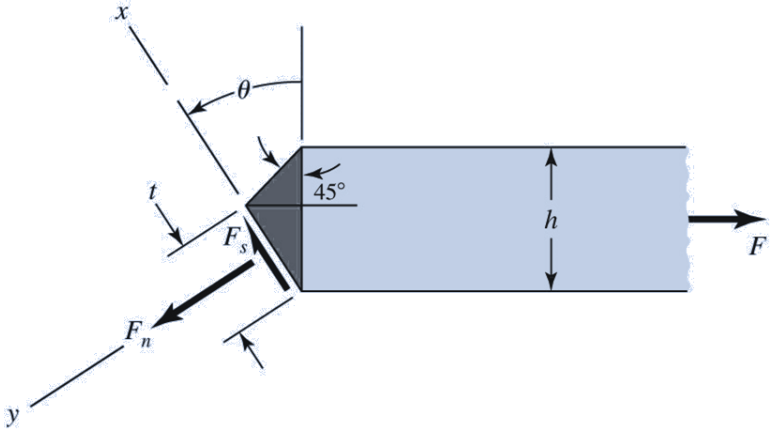
$$\sigma' = (\sigma^2 + 3 \cdot \tau^2)^{1/2} = \frac{F}{h \cdot l} \cdot \left[(\cos^2\theta + \sin\theta \cdot \cos\theta)^2 + 3 \cdot (\sin\theta \cdot \cos\theta + \sin^2\theta)^2 \right]$$

Maximum von Mises Stress σ'

$$\theta = 62.5° \rightarrow \sigma' = \frac{2.16 \cdot F}{h \cdot l} \rightarrow \sigma = \frac{0.623 \cdot F}{h \cdot l}, \tau = \frac{1.196 \cdot F}{h \cdot l}$$

Maximum Shear Stress (Differentiating ① and setting to 0)

$$\theta = 67.5° \rightarrow \sigma = \frac{0.5 \cdot F}{h \cdot l}, \tau_{max} = \frac{1.207 \cdot F}{h \cdot l}$$

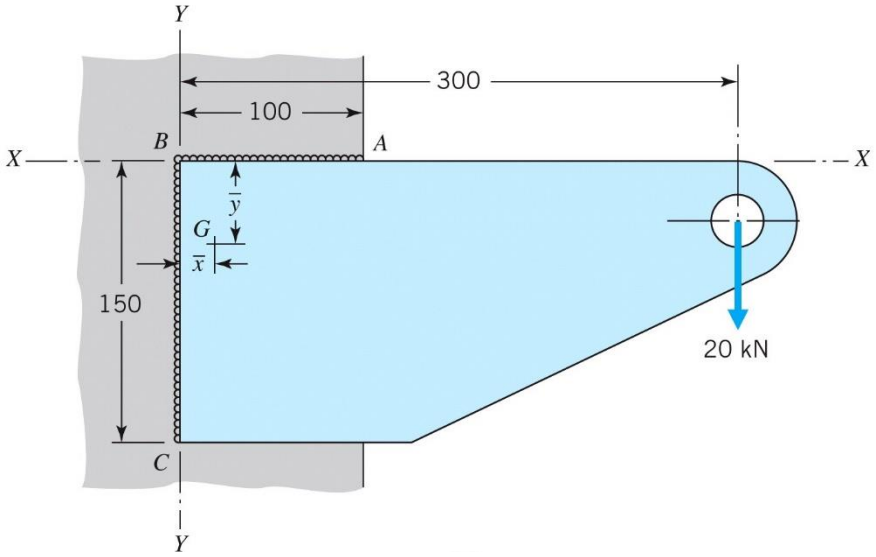


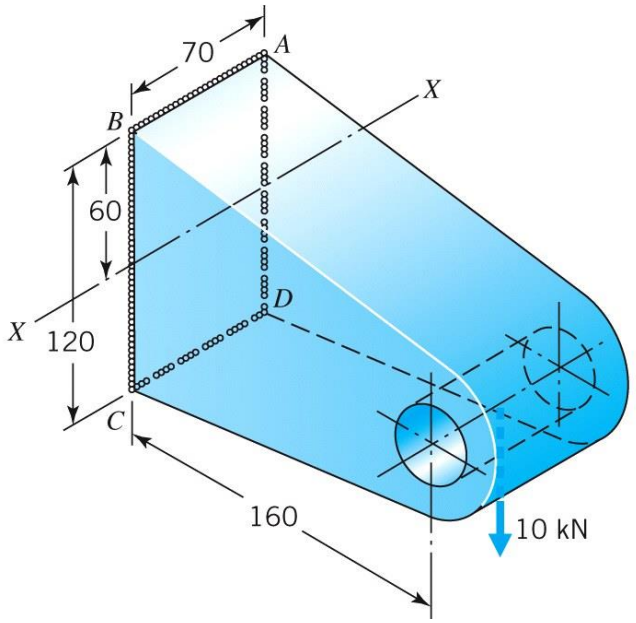
Assuming entire force in Shear across the minimum Throat Area

$$\tau = \frac{F}{0.7071 \cdot h \cdot l} = \boxed{\frac{1.414 \cdot F}{h \cdot l}}$$

Most Conservative, will be used in analysis

FILLET WELD: Load Orientation





Welds in the Plane of the Load

- **Welds subjected to <u>Torsion</u>**
- **Welds also subjected to direct loading**

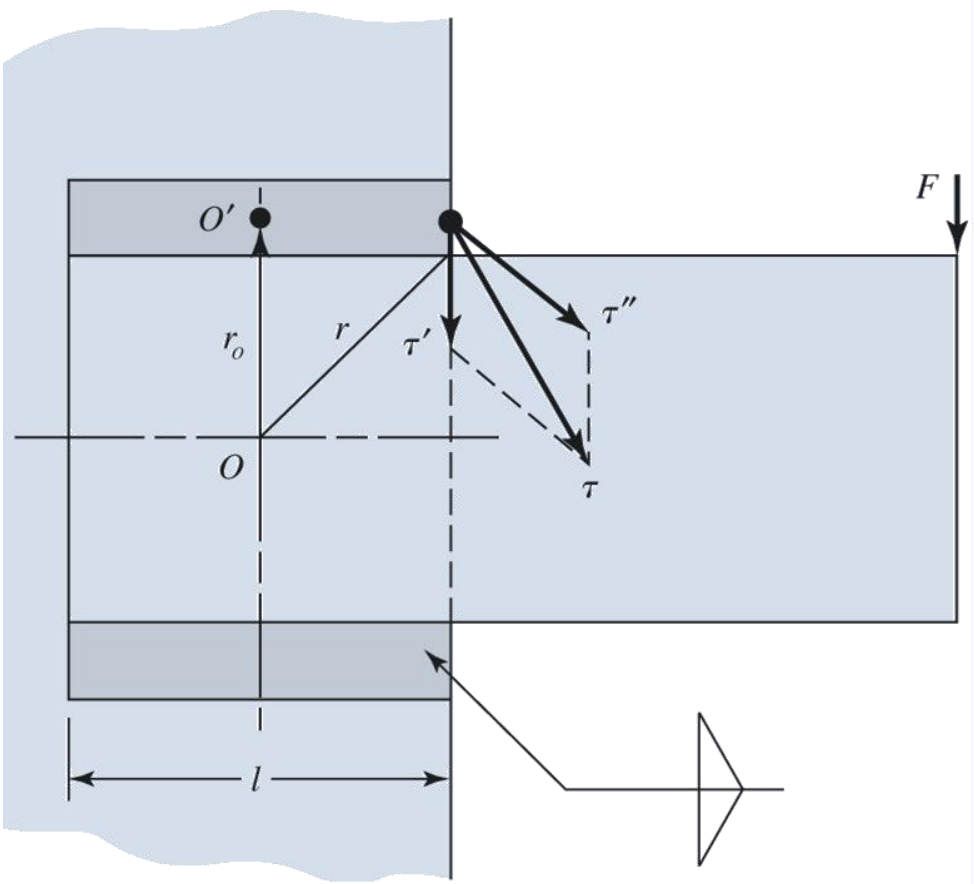
Welds out of the Plane of the Load

- **Welds subjected to <u>Bending</u>**
- **Welds also subjected to direct loading**

Design Strategy

- Examine primary shear stresses due to external forces
- Examine secondary shear stresses due to torsion and bending moments
- Estimate the strength(s) of parent metal(s)
- Estimate the strength of deposited weld metal
- Estimate permissible load(s) for parent metal(s)
- Estimate permissible load for deposited weld metal

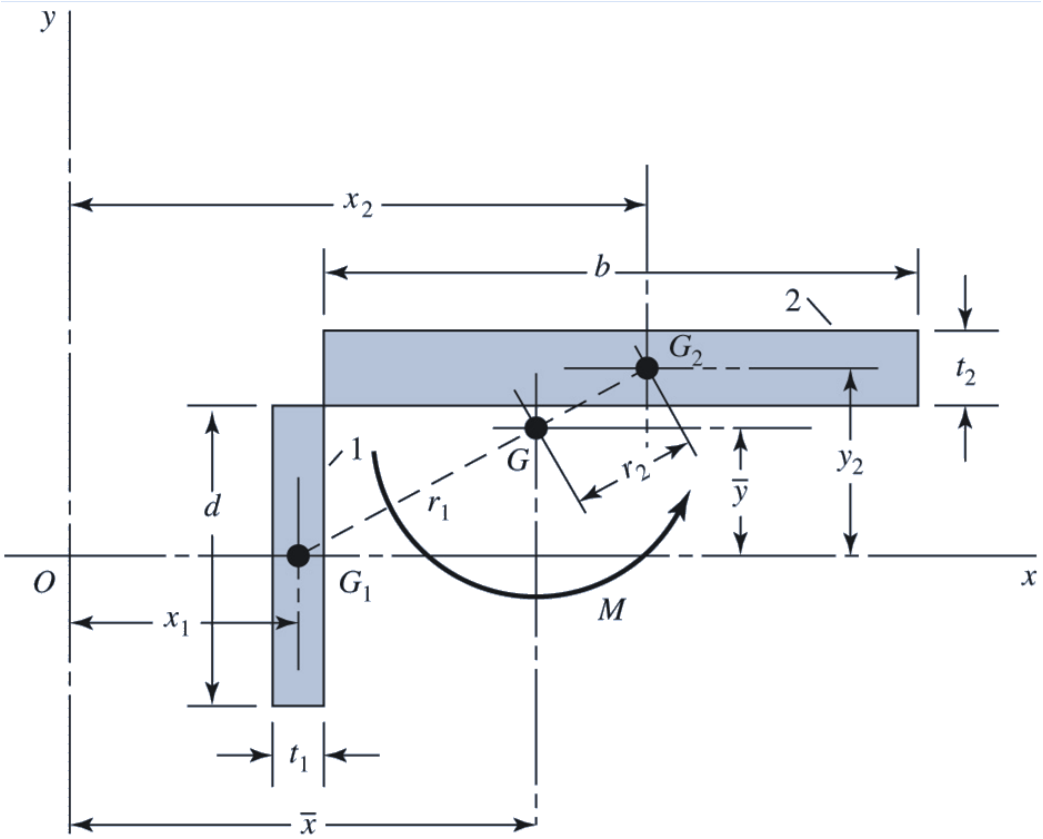
FILLET WELDS Loaded in Torsion



$$\tau' = \frac{V}{A}$$

$$\tau'' = \frac{T \cdot r}{J}$$

Calculations Needed to Find A and J



Weld Area

$$A = A_1 + A_2 = t_1 \cdot d + t_2 \cdot b$$

Area Moments of Inertia of Area 1

$$I_{x1} = \frac{t_1 \cdot d^3}{12} \qquad I_{y1} = \frac{d \cdot t_1^3}{12}$$

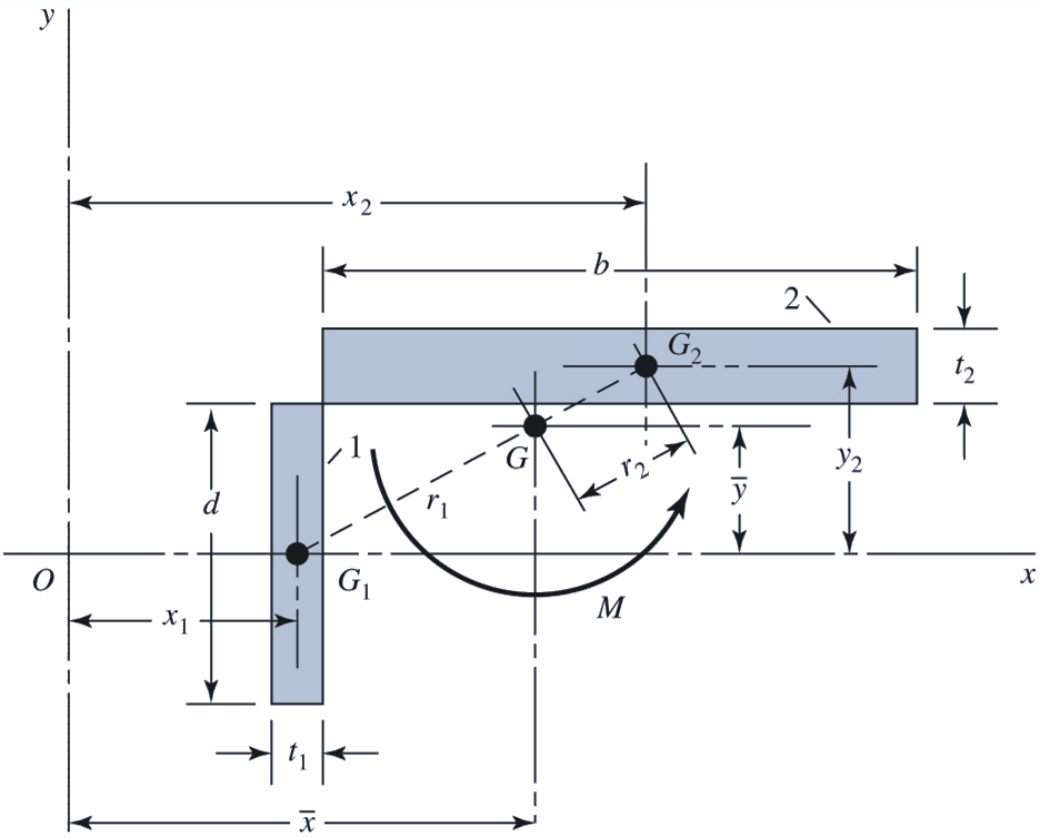
$$J_{G1} = I_{x1} + I_{y1} = \frac{t_1 \cdot d^3}{12} + \frac{d \cdot t_1^3}{12}$$

Area Moments of Inertia of Area 2

$$I_{x2} = \frac{b \cdot t_2^3}{12} \qquad I_{y2} = \frac{t_2 \cdot b^3}{12}$$

$$J_{G2} = I_{x2} + I_{y2} = \frac{b \cdot t_2^3}{12} + \frac{t_2 \cdot b^3}{12}$$

Simplifying the Calculation of J



Locating the Centroid of the Weld Group

$$\bar{x} = \frac{A_1 \cdot x_1 + A_2 \cdot x_2}{A_1 + A_2} \qquad \bar{y} = \frac{A_1 \cdot y_1 + A_2 \cdot y_2}{A_1 + A_2}$$

r_1 and r_2 Needed for Parallel Axis Calculation

$$r_1 = \left[\left(\bar{x} - x_1 \right)^2 + \left(\bar{y} - y_1 \right)^2 \right]^{1/2}$$

$$r_2 = \left[\left(y_2 - \bar{y} \right)^2 + \left(x_2 - \bar{x} \right) \right]^{1/2}$$

Section Polar Moment of Inertia

$$J = \left(J_{G1} + A_1 \cdot r_1^2 \right) + \left(J_{G2} + A_2 \cdot r_2^2 \right)$$

$$J = \left(\frac{t_1 \cdot d^3}{12} + \cancel{\frac{d \cdot t_1^3}{12}} + t_1 \cdot d \cdot r_1^2 \right)$$

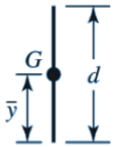
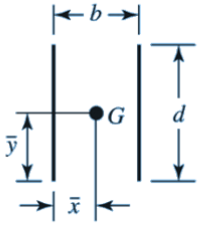
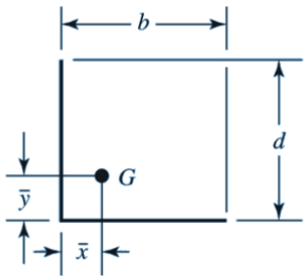
$$+ \left(\cancel{\frac{b \cdot t_2^3}{12}} + \frac{t_2 \cdot b^3}{12} + t_2 \cdot b \cdot r_2^2 \right)$$

$$J = t_1 \cdot \left(\frac{d^3}{12} + d \cdot r_1^2 \right) + t_2 \cdot \left(\frac{b^3}{12} + b \cdot r_2^2 \right)$$

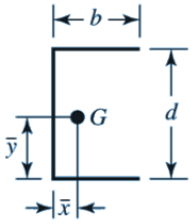
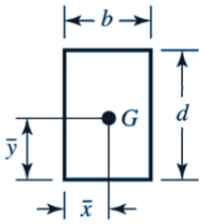
Weld is Uniform, Unit Weld Thickness

$$\boxed{J = 0.7071 \cdot h \cdot J_u = t \cdot J_u}$$

Common Torsional Properties of Fillet Welds

Weld	Throat Area	Location of G	Unit Second Polar Moment of Area
1. 	$A = 0.707\,hd$	$\bar{x} = 0$ $\bar{y} = d/2$	$J_u = d^3/12$
2. 	$A = 1.414\,hd$	$\bar{x} = b/2$ $\bar{y} = d/2$	$J_u = \dfrac{d(3b^2 + d^2)}{6}$
3. 	$A = 0.707h(b + d)$	$\bar{x} = \dfrac{b^2}{2(b + d)}$ $\bar{y} = \dfrac{d^2}{2(b + d)}$	$J_u = \dfrac{(b + d)^4 - 6b^2d^2}{12(b + d)}$

Common Torsional Properties of Fillet Welds

Weld	Throat Area	Location of G	Unit Second Polar Moment of Area
4. 	$A = 0.707h(2b + d)$	$\bar{x} = \dfrac{b^2}{2b + d}$ $\bar{y} = d/2$	$J_u = \dfrac{8b^3 + 6bd^2 + d^3}{12} - \dfrac{b^4}{2b + d}$
5. 	$A = 1.414h(b + d)$	$\bar{x} = b/2$ $\bar{y} = d/2$	$J_u = \dfrac{(b + d)^3}{6}$
6. 	$A = 1.414\,\pi hr$		$J_u = 2\pi r^3$

*G is centroid of weld group; h is weld size; plane of torque couple is in the plane of the paper; all welds are of unit width.

Allowable Steady Loads and Minimum Fillet Weld Sizes

Schedule A: Allowable Load for Various Sizes of Fillet Welds

	Strength Level of Weld Metal (EXX)						
	60*	70*	80	90*	100	110*	120
	Allowable shear stress on throat, MPa of fillet weld or partial penetration groove weld						
$\tau =$	124	145	165	186	207	228	248
	Allowable Unit Force on Fillet Weld, N/mm						
†$f =$	87.67h	102.52h	116.66h	131.5h	146.35h	161.2h	175.34h

Leg Size h, mm	Allowable Unit Force for Various Sizes of Fillet Welds N/mm						
25	2192	2563	2916	3288	3659	4030	4383
22	1929	2255	2566	2893	3220	3546	3857
20	1753	2050	2333	2630	2927	3224	3506
16	1403	1640	1866	2104	2342	2579	2805
12	1052	1230	1400	1578	1756	1934	2104
11	964	1127	1283	1447	1610	1773	1927
10	877	1025	1167	1315	1463	1612	1753
8	701	820	933	1052	1171	1290	1403
6	526	615	700	789	878	967	1052
5	438	513	583	658	732	806	877
3	263	308	350	395	439	484	526
2	175	205	233	263	293	322	351

*Fillet welds actually tested by the joint AISC-AWS Task Committee.
†$f = 0.707h\ \tau_{all}$.

Schedule B: Minimum Fillet Weld Size, h

Material Thickness of Thicker Part Joined, mm		Weld Size, mm
*To 6 incl.		3
Over 6	To 12	5
Over 12	To 20	6
†Over 20	To 38	8
Over 38	To 58	10
Over 58	To 150	12
Over 150		16

Not to exceed the thickness of the thinner part.
*Minimum size for bridge application does not go below 5 mm.
†For minimum fillet weld size, schedule does not go above 8 mm fillet weld for every 20 mm material.

Source: From Omer W. Blodgett (ed.), *Stress Allowables Affect Weldment Design,* D412, The James F. Lincoln Arc Welding Foundation, Cleveland, May 1991, p. 3. Reprinted by permission of Lincoln Electric Company.

Allowable Steady Loads and Minimum Fillet Weld Sizes

Schedule A: Allowable Load for Various Sizes of Fillet Welds

	Strength Level of Weld Metal (EXX)						
	60*	70*	80	90*	100	110*	120
	Allowable shear stress on throat, ksi (1000 psi) of fillet weld or partial penetration groove weld						
$\tau =$	18.0	21.0	24.0	27.0	30.0	33.0	36.0
	Allowable Unit Force on Fillet Weld, kip/linear in						
$^{\dagger}f =$	$12.73h$	$14.85h$	$16.97h$	$19.09h$	$21.21h$	$23.33h$	$25.45h$

Leg Size h, in	Allowable Unit Force for Various Sizes of Fillet Welds kip/linear in						
1	12.73	14.85	16.97	19.09	21.21	23.33	25.45
7/8	11.14	12.99	14.85	16.70	18.57	20.41	22.27
3/4	9.55	11.14	12.73	14.32	15.92	17.50	19.09
5/8	7.96	9.28	10.61	11.93	13.27	14.58	15.91
1/2	6.37	7.42	8.48	9.54	10.61	11.67	12.73
7/16	5.57	6.50	7.42	8.35	9.28	10.21	11.14
3/8	4.77	5.57	6.36	7.16	7.95	8.75	9.54
5/16	3.98	4.64	5.30	5.97	6.63	7.29	7.95
1/4	3.18	3.71	4.24	4.77	5.30	5.83	6.36
3/16	2.39	2.78	3.18	3.58	3.98	4.38	4.77
1/8	1.59	1.86	2.12	2.39	2.65	2.92	3.18
1/16	0.795	0.930	1.06	1.19	1.33	1.46	1.59

*Fillet welds actually tested by the joint AISC-AWS Task Committee.
$^{\dagger}f = 0.707h\,\tau_{all}$.

Schedule B: Minimum Fillet Weld Size, h

Material Thickness of Thicker Part Joined, in		Weld Size, in
*To $\frac{1}{4}$ incl.		$\frac{1}{8}$
Over $\frac{1}{4}$	To $\frac{1}{2}$	$\frac{3}{16}$
Over $\frac{1}{2}$	To $\frac{3}{4}$	$\frac{1}{4}$
†Over $\frac{3}{4}$	To $1\frac{1}{2}$	$\frac{5}{16}$
Over $1\frac{1}{2}$	To $2\frac{1}{4}$	$\frac{3}{8}$
Over $2\frac{1}{4}$	To 6	$\frac{1}{2}$
Over 6		$\frac{5}{8}$

Not to exceed the thickness of the thinner part.
*Minimum size for bridge application does not go below $\frac{3}{16}$ in.
†For minimum fillet weld size, schedule does not go above $\frac{5}{16}$ in fillet weld for every $\frac{3}{4}$ in material.

Source: From Omer W. Blodgett (ed.), *Stress Allowables Affect Weldment Design,* D412, The James F. Lincoln Arc Welding Foundation, Cleveland, May 1991, p. 3. Reprinted by permission of Lincoln Electric Company.

Minimum Weld-Metal Properties

AWS Electrode Number*	Tensile Strength kpsi (MPa)	Yield Strength, kpsi (MPa)	Percent Elongation
E60xx	62 (427)	50 (345)	17–25
E70xx	70 (482)	57 (393)	22
E80xx	80 (551)	67 (462)	19
E90xx	90 (620)	77 (531)	14–17
E100xx	100 (689)	87 (600)	13–16
E120xx	120 (827)	107 (737)	14

*The American Welding Society (AWS) specification code numbering system for electrodes. This system uses an E prefixed to a four- or five-digit numbering system in which the first two or three digits designate the approximate tensile strength. The last digit includes variables in the welding technique, such as current supply. The next-to-last digit indicates the welding position, as, for example, flat, or vertical, or overhead. The complete set of specifications may be obtained from the AWS upon request.

Stresses Permitted by the AISC Code for Weld Metals

Type of Loading	Type of Weld	Permissible Stress	n^*
Tension	Butt	$0.60S_y$	1.67
Bearing	Butt	$0.90S_y$	1.11
Bending	Butt	$0.60–0.66S_y$	1.52–1.67
Simple compression	Butt	$0.60S_y$	1.67
Shear	Butt or fillet	$0.30S_{ut}^{\dagger}$	

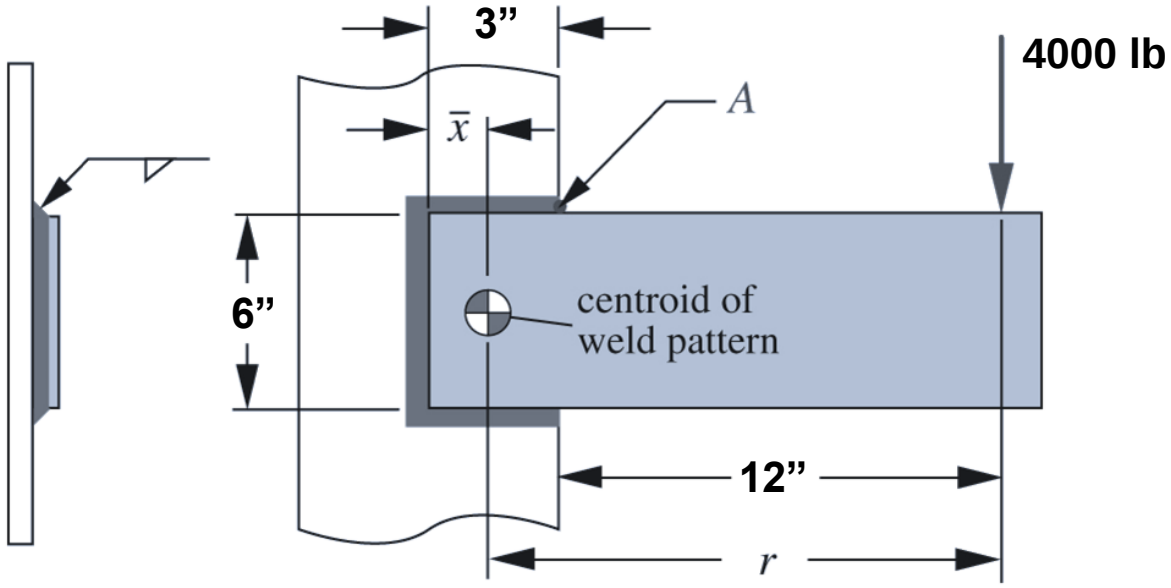
*The factor of safety n has been computed by using the distortion-energy theory.

†Shear stress on base metal should not exceed $0.40S_y$ of base metal.

Example

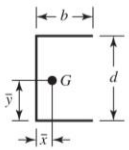
The weldment assembly shown has fillet welds on three sides of the joint. Determine the weld size needed to withstand a static load of P=4000 lb. Check for yielding in the beam also.

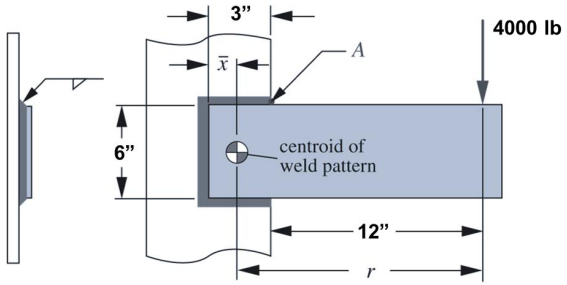
The material is ASTM A36 (S_y=36.3x10^3 lb/in^2, S_{ut}=70.0x10^3 lb/in^2, E= 29.0x10^6 lb/in$^{2)}$ structural steel and an E70xx welding electrode (S_{ut}=70 000 lb/in^2) is used. The plates are ½" thick. Dimension a=12", b=3", and d=6".



Solution

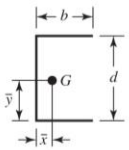
Shigley 10th Table 9-1

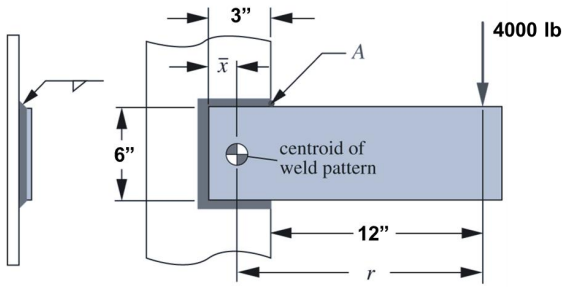
Weld	Throat Area	Location of G	Unit Second Polar Moment of Area
4. 	$A = 0.707h(2b + d)$	$\bar{x} = \frac{b^2}{2b + d}$ $\bar{y} = d/2$	$J_u = \frac{8b^3 + 6bd^2 + d^3}{12} - \frac{b^4}{2b + d}$



Solution

Shigley 10th Table 9-1

Weld	Throat Area	Location of G	Unit Second Polar Moment of Area
4. 	$A = 0.707h(2b + d)$	$\bar{x} = \frac{b^2}{2b + d}$ $\bar{y} = d/2$	$J_u = \frac{8b^3 + 6bd^2 + d^3}{12} - \frac{b^4}{2b + d}$



$$\bar{x} = \frac{b^2}{2 \cdot b + d} = \frac{(3in)^2}{2 \cdot (3in) + (6in)}$$

$$= \boxed{0.75in}$$

$$\bar{y} = \frac{d}{3} = \frac{6in}{2}$$

$$= \boxed{3.0in}$$

$$A_t = 0.7071 \cdot h \cdot (2 \cdot b + d)$$

$$= t \cdot (2 \cdot b + d) = t \cdot (2 \cdot 3in + 6in)$$

$$= \boxed{12in \cdot t}$$

$$J_u = \frac{8 \cdot b^3 + 6 \cdot b \cdot d^2 + d^3}{12} - \frac{b^4}{2 \cdot b + d}$$

$$= \frac{8 \cdot (3in)^3 + 6 \cdot (3in) \cdot (6in)^2 + (6in)^3}{12} - \frac{(3in)^4}{2 \cdot (3in) + (6in)}$$

$$= \boxed{83.25in^3}$$

Radius from Applied Load to Centroid

$$r = 12in + 3in - \bar{x} = 12in + 3in - 0.75in$$

$$= \boxed{14.25in}$$

Direct Shear

$$f' = \frac{V}{A_T} = \frac{V}{2 \cdot b + d} = \frac{4000lb}{12in \cdot t}$$

$$= \boxed{\frac{333.3 \frac{lb}{in}}{t}}$$

Torsional Shear Load

$$T = P \cdot r = 4000lb \cdot 14.25in$$

$$= \boxed{57\,000lb \cdot in}$$

Solution

Direct Shear

$$f' = \frac{V}{A_T} = \frac{333.3\,\frac{lb}{in}}{t}$$

Torsional Shear Load

$$T = 57\,000\,lb \cdot in$$

Weld Geometry

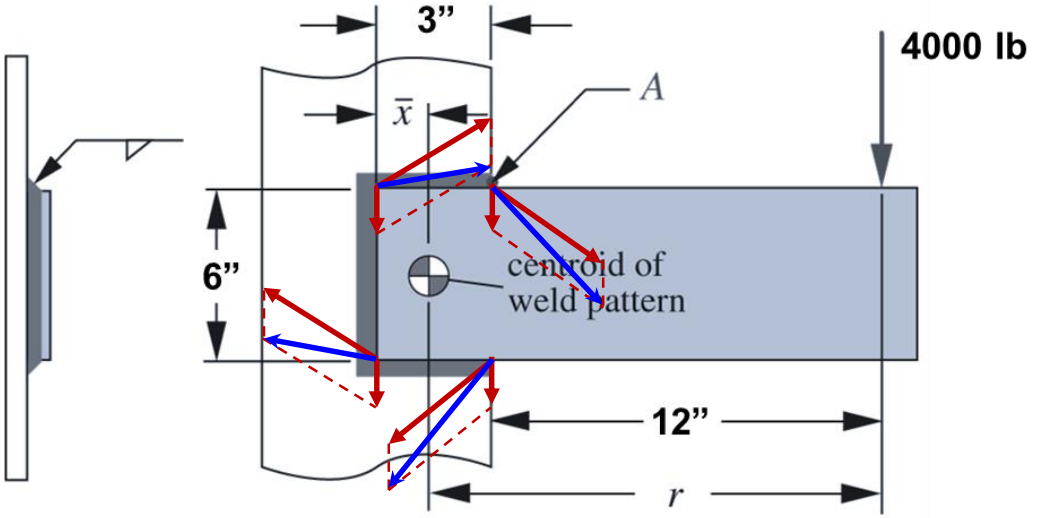
$$\bar{x} = 0.75\,in$$

$$\bar{y} = 3.0\,in$$

$$A_t = 12\,in \cdot t$$

$$r = 14.25\,in$$

$$J_u = 83.25\,in^3$$



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*The factor of safety n has been computed by using the distortion-energy theory.

†Shear stress on base metal should not exceed $0.40S_y$ of base metal.

Solution

Direct Shear

$$f' = \frac{V}{A_T} = \frac{333.3\,\frac{lb}{in}}{t}$$

Torsional Shear Load

$$T = 57\,000\,lb \cdot in$$

Weld Geometry

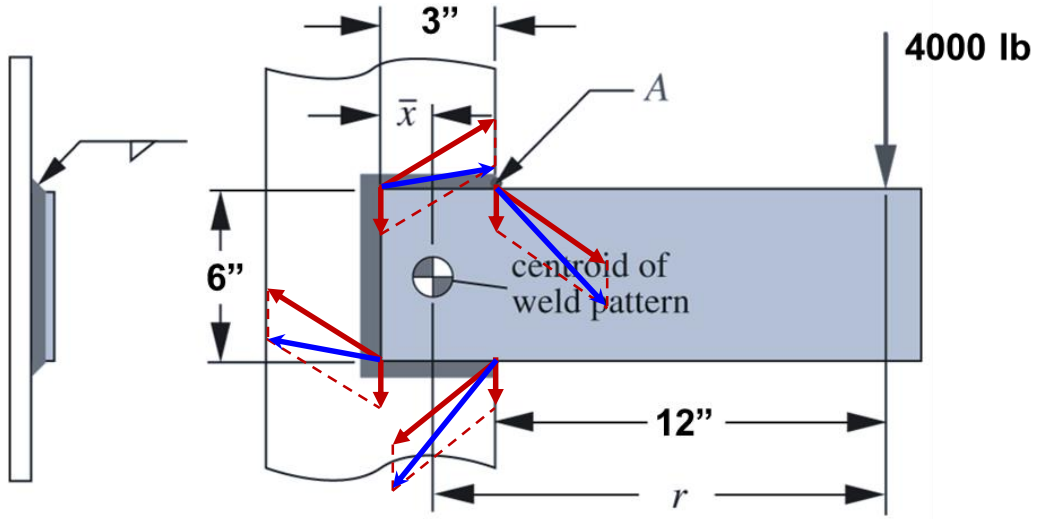
$$\bar{x} = 0.75\,in$$

$$\bar{y} = 3.0\,in$$

$$A_t = 12\,in \cdot t$$

$$r = 14.25\,in$$

$$J_u = 83.25\,in^3$$



Point A is the location of highest resultant shear

The torsional load f''s horizontal f_h'' and vertical f_v'' components are calculated separately

$$f_h'' = \frac{T \cdot r_{AH}}{J_u \cdot t} = \frac{57\,000\,lb \cdot in \cdot \frac{6\,in}{2}}{83.25\,in^3 \cdot t} = \frac{2054.1\,\frac{lb}{in}}{t}$$

$$f_v'' = \frac{T \cdot r_{AV}}{J_u \cdot t} = \frac{57\,000\,lb \cdot in \cdot (14.25\,in - 12\,in)}{83.25\,in^3 \cdot t} = \frac{1540.5\,\frac{lb}{in}}{t}$$

$$F_R = \sqrt{\left(\frac{2054.1\,\frac{lb}{in}}{t}\right)^2 + \left(\frac{1540.5\,\frac{lb}{in}}{t} + \frac{333.3\,\frac{lb}{in}}{t}\right)^2} = \frac{2780\,\frac{lb}{in}}{t}$$

Given the electroid

$$\tau_{allow} = 0.30 \cdot E_{xx} = 0.30 \cdot 70\,000\,\frac{lb}{in^2}$$

$$= 21000\,\frac{lb}{in^2}$$

$$\tau_{allow} = F_R$$

$$21000\,\frac{lb}{in^2} = \frac{2780\,\frac{lb}{in}}{t} \quad \Rightarrow \quad t = \frac{2780\,\frac{lb}{in}}{21000\,\frac{lb}{in^2}} = 0.132\,in$$

$$\Rightarrow \quad h = 1.414 \cdot (0.132\,in) = 0.187\,in \Rightarrow \boxed{\tfrac{3}{16}\,in}$$

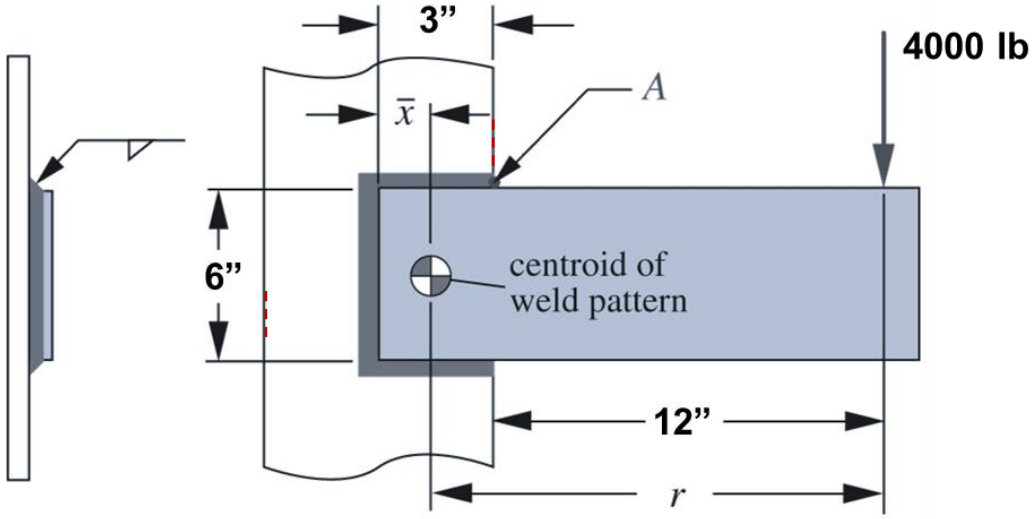
Solution

Yielding in the Beam

Bending Maximum Tensile Stress at A

$$\sigma_{Ax} = \frac{M \cdot c}{I} = \frac{(4000\,lb)\cdot(12\,in)\cdot(3\,in)}{\left[\frac{(0.5\,in)\cdot(6\,in)^3}{12}\right]}$$

$$= \boxed{16.0\times10^3\ \frac{lb}{in^2}}$$



At A the shear stress is 0.

The yield stress for A36 steel is

$$S_y = 36.3\times10^3\ \frac{lb}{in^2}$$

The factor of safety for the parent material

$$n = \frac{36.3\times10^3\ \frac{lb}{in^2}}{16.0\times10^3\ \frac{lb}{in^2}} = \boxed{2.3}$$