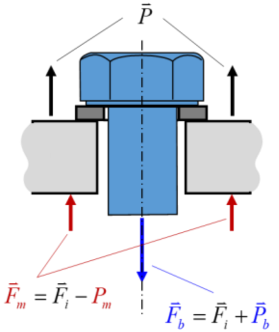
Dynamic and Shear Loading

- **Fatigue Loading**
- **Joints in Shear**

Pre-Loaded Joint Cycled, 0 to P_{max}



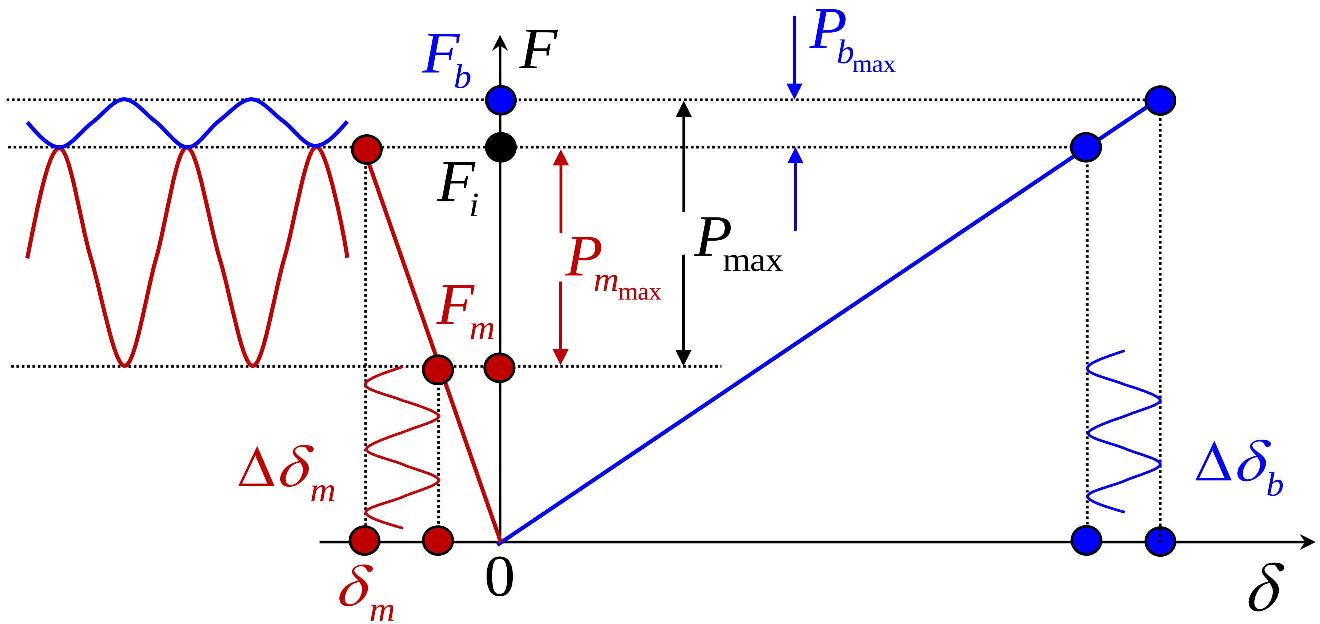
$$\Delta\delta = \Delta\delta_m = \Delta\delta_b$$

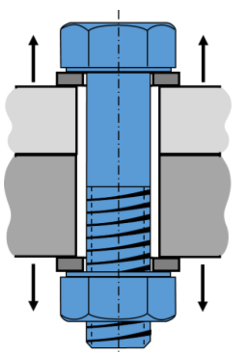
$$P_m = P_b \cdot \frac{k_m}{k_b}$$

$$C = \frac{k_b}{k_m + k_b}$$

$$P_b = P \cdot C$$

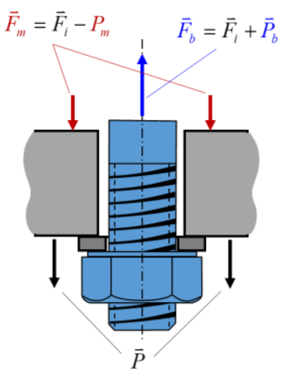
$$P_m = P \cdot (1 - C)$$





$$F_{b_{min}} = C \cdot P_{min} + F_i$$

$$F_{b_{max}} = C \cdot P_{max} + F_i$$



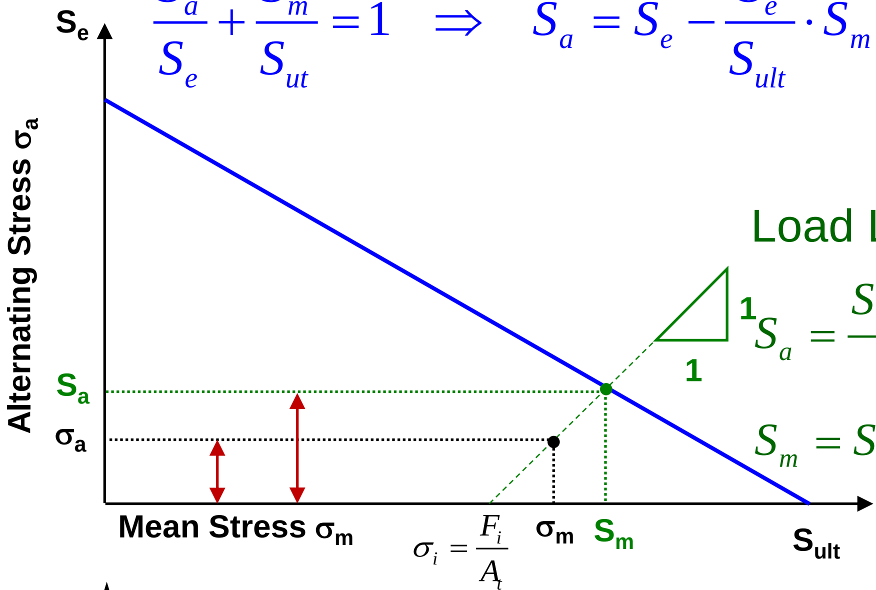
$$\sigma_{b_{alt}} = \frac{F_{b_{max}} - F_{b_{min}}}{2 \cdot A_t} = \frac{(C \cdot P_{max} + F_i) - (C \cdot P_{min} + F_i)}{2 \cdot A_t} = \boxed{\frac{C \cdot (P_{max} - P_{min})}{2 \cdot A_t}}$$

$$\sigma_{b_{mean}} = \frac{F_{b_{max}} + F_{b_{min}}}{2 \cdot A_t} = \frac{(C \cdot P_{max} + F_i) + (C \cdot P_{min} + F_i)}{2 \cdot A_t} = \frac{C \cdot (P_{max} + P_{min})}{2 \cdot A_t} + \frac{F_i}{A_t}$$

Goodman Diagram for Fatigue

Goodman Line

$$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1 \quad \Rightarrow \quad S_a = S_e - \frac{S_e}{S_{ult}} \cdot S_m$$



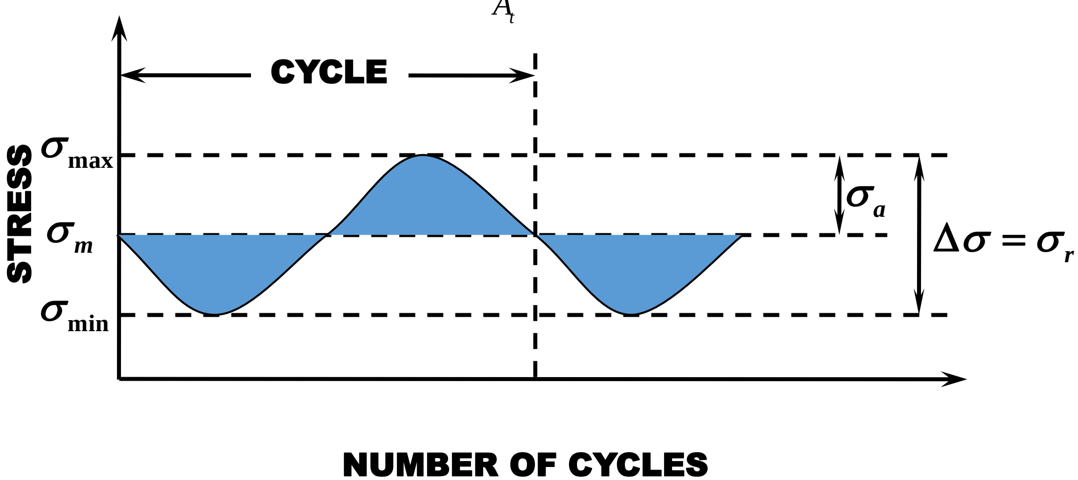
Load Line

$$S_a = \frac{S_e \cdot (S_{ut} - \sigma_i)}{S_{ut} - S_e}$$

$$S_m = S_a + \sigma_i$$

FATIGUE FACTOR OF SAFETY

$$n_f = \frac{S_a}{\sigma_a} = \frac{2 \cdot S_e \cdot (S_{ut} \cdot A_t - F_i)}{C \cdot P \cdot (S_{ut} - S_e)}$$



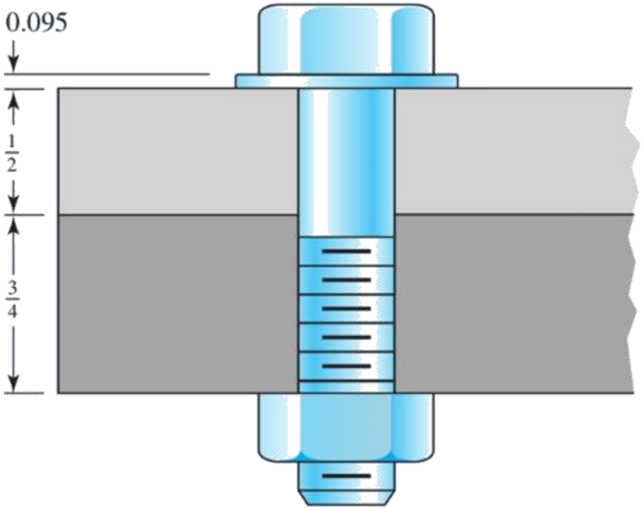
$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{F_{max} + F_{min}}{2 \cdot A_t}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{F_{max} - F_{min}}{2 \cdot A_t}$$

Example

Two plates are clamped by 10 washer-faced ½in-20 UNF x 2 SAE grade 10 bolts each with a standard ½ N steel plain washer. An external load that fluctuates between 0 and 6 kip is applied.

a. Calculate the fatigue factor of safety in the bolt



PROOF FACTOR OF SAFETY

$n_p = 1.095$

LOAD FACTOR OF SAFETY

$n_{Lp} = 7.609$

JOINT SEPARATION FACTOR OF SAFETY

$n_o = 29.03$

PREVIOUS RESULTS

$k_b = 3.978 \times 10^6 \ \frac{lb}{in}$

$k_m = 9.378 \times 10^6 \ \frac{lb}{in}$

$C = 0.2978$

$1 - C = 0.7022$

$F_i = 12.232 \times 10^3 \ lb$

$P \equiv 600 \ lb$

$P_b = 178.7 \ lb$

$P_m = 421.3 \ lb$

$F_b = 12.411 \times 10^3 \ lb$

$F_m = 11.811 \times 10^3 \ lb$

Solution

Table 8–2

Diameters and Area of Unified Screw Threads UNC and UNF*

		Coarse Series—UNC			Fine Series—UNF		
Size Designation	Nominal Major Diameter in	Threads per Inch N	Tensile-Stress Area A_t in^2	Minor-Diameter Area A_r in^2	Threads per Inch N	Tensile-Stress Area A_t in^2	Minor-Diameter Area A_r in^2
$\frac{1}{2}$	0.5000	13	0.141 9	0.125 7	20	0.159 9	0.148 6

$$F_i = 12.232 \times 10^3 \ lb$$

$$P = \ 600 \ lb$$

$$A_t = 0.1599 \, in^2$$

$$C = 0.2978$$

$$1 - C = 0.7022$$

Solution

Table 8–2

Diameters and Area of Unified Screw Threads UNC and UNF*

Size Designation	Nominal Major Diameter in	Coarse Series—UNC: Threads per Inch N	Coarse Series—UNC: Tensile-Stress Area A_t in^2	Coarse Series—UNC: Minor-Diameter Area A_r in^2	Fine Series—UNF: Threads per Inch N	Fine Series—UNF: Tensile-Stress Area A_t in^2	Fine Series—UNF: Minor-Diameter Area A_r in^2
$\frac{1}{2}$	0.5000	13	0.141 9	0.125 7	20	0.159 9	0.148 6

$$F_i = 12.232\times10^3\ lb$$

$$P = 600\ lb$$

$$A_t = 0.1599\,in^2$$

$$C = 0.2978$$

$$1-C = 0.7022$$

$$\sigma_{b_{alt}} = \frac{C\cdot(P_{\max}-P_{\min})}{2\cdot A_t} = \frac{0.2978\cdot(600\ lb - 0\ lb)}{2\cdot(0.1599\ in^2)} = 558.7\ \tfrac{lb}{in^2}$$

$$\sigma_{b_{mean}} = \frac{C\cdot(P_{\max}+P_{\min})}{2\cdot A_t} + \frac{F_i}{A_t} = \frac{0.2978\cdot(600\ lb + 0\ lb)}{2\cdot(0.1599\ in^2)} + \frac{12.232\times10^3\ lb}{0.1599\ in^2} = 77.06\times10^3\ \tfrac{lb}{in^2}$$

$$\sigma_{b_i} = \frac{F_i}{A_t} = \frac{12.232\times10^3\ lb}{0.1599\ in^2} = 76.50\times10^3\ \tfrac{lb}{in^2}$$

Solution

Table 8–17

Fully Corrected Endurance Strengths for Bolts and Screws with Rolled Threads*

Grade or Class	Size Range	Endurance Strength
SAE 5	$\frac{1}{4}$–1 in	18.6 kpsi
	$1\frac{1}{8}$–$1\frac{1}{2}$ in	16.3 kpsi
SAE 7	$\frac{1}{4}$–$1\frac{1}{2}$ in	20.6 kpsi

Table 8–9

SAE Specifications for Steel Bolts

SAE Grade No.	Size Range Inclusive, in	Minimum Proof Strength,* kpsi	Minimum Tensile Strength,* kpsi	Minimum Yield Strength,* kpsi	Material	Head Marking
5	$\frac{1}{4}$–1	85	120	92	Medium carbon, Q&T	
	$1\frac{1}{8}$–$1\frac{1}{2}$	74	105	81		

$$\sigma_{b_{alt}} = 561.9 \frac{lb}{in^2}$$

$$\sigma_{b_{mean}} = 77.06 \times 10^3 \frac{lb}{in^2}$$

$$\sigma_{b_i} = 76.50 \times 10^3 \frac{lb}{in^2}$$

$$F_i = 12.232 \times 10^3 \ lb$$

$$P = 600 \ lb$$

$$A_t = 0.1599 \, in^2$$

$$C = 0.2978$$

$$1 - C = 0.7022$$

Solution

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	$1\frac{1}{8}$–$1\frac{1}{2}$	74	105	81		

$$\sigma_{b_{alt}} = 561.9\,\frac{lb}{in^2}$$

$$\sigma_{b_{mean}} = 77.06\times10^3\,\frac{lb}{in^2}$$

$$\sigma_{b_i} = 76.50\times10^3\,\frac{lb}{in^2}$$

$$F_i = 12.232\times10^3\ lb$$

$$P = 600\ lb$$

$$A_t = 0.1599\,in^2$$

$$C = 0.2978$$

$$1-C = 0.7022$$

Load Line

$$S_a = \frac{S_e\cdot\left(S_{ut}-\sigma_i\right)}{S_{ut}-S_e} = \frac{\left(18.6\times10^3\ {}^{lb}/_{in^2}\right)\cdot\left[\left(120.0\times10^3\ {}^{lb}/_{in^2}\right)-\left(76.50\times10^3\ {}^{lb}/_{in^2}\right)\right]}{\left(120.0\times10^3\ {}^{lb}/_{in^2}\right)-\left(18.6\times10^3\ {}^{lb}/_{in^2}\right)}$$

$$= 7.98\times10^3\ {}^{lb}/_{in^2}$$

$$S_m = S_a + \sigma_i = \left(7.98\times10^3\ {}^{lb}/_{in^2}\right)+\left(76.50\times10^3\ {}^{lb}/_{in^2}\right)$$

$$= 84.48\times10^3\ {}^{lb}/_{in^2}$$

Solution

Table 8–17

Fully Corrected Endurance Strengths for Bolts and Screws with Rolled Threads*

Grade or Class	Size Range	Endurance Strength
SAE 5	$\frac{1}{4}$–1 in	18.6 kpsi
	$1\frac{1}{8}$–$1\frac{1}{2}$ in	16.3 kpsi
SAE 7	$\frac{1}{4}$–$1\frac{1}{2}$ in	20.6 kpsi

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SAE Specifications for Steel Bolts

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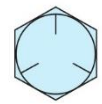
5	$\frac{1}{4}$–1	85	120	92	Medium carbon, Q&T	
	$1\frac{1}{8}$–$1\frac{1}{2}$	74	105	81		

$$\sigma_{b_{alt}} = 561.9\,\frac{lb}{in^2}$$

$$\sigma_{b_{mean}} = 77.06\times10^3\,\frac{lb}{in^2}$$

$$\sigma_{b_i} = 76.50\times10^3\,\frac{lb}{in^2}$$

$$F_i = 12.232\times10^3\ lb$$

$$P = 600\ lb$$

$$A_t = 0.1599\,in^2$$

$$C = 0.2978$$

$$1-C = 0.7022$$

$$S_a = 7.98\times10^3\ {}^{lb}\!/_{in^2}$$

$$S_m = 84.48\times10^3\ {}^{lb}\!/_{in^2}$$

FATIGUE FACTOR OF SAFETY

$$n_f = \frac{S_a}{\sigma_a} = \frac{7.98\times10^3\ {}^{lb}\!/_{in^2}}{561.9\,\frac{lb}{in^2}} = \boxed{14.2}$$

$$= \frac{2\cdot S_e\cdot\left(S_{ut}\cdot A_t - F_i\right)}{C\cdot P\cdot\left(S_{ut}-S_e\right)}$$

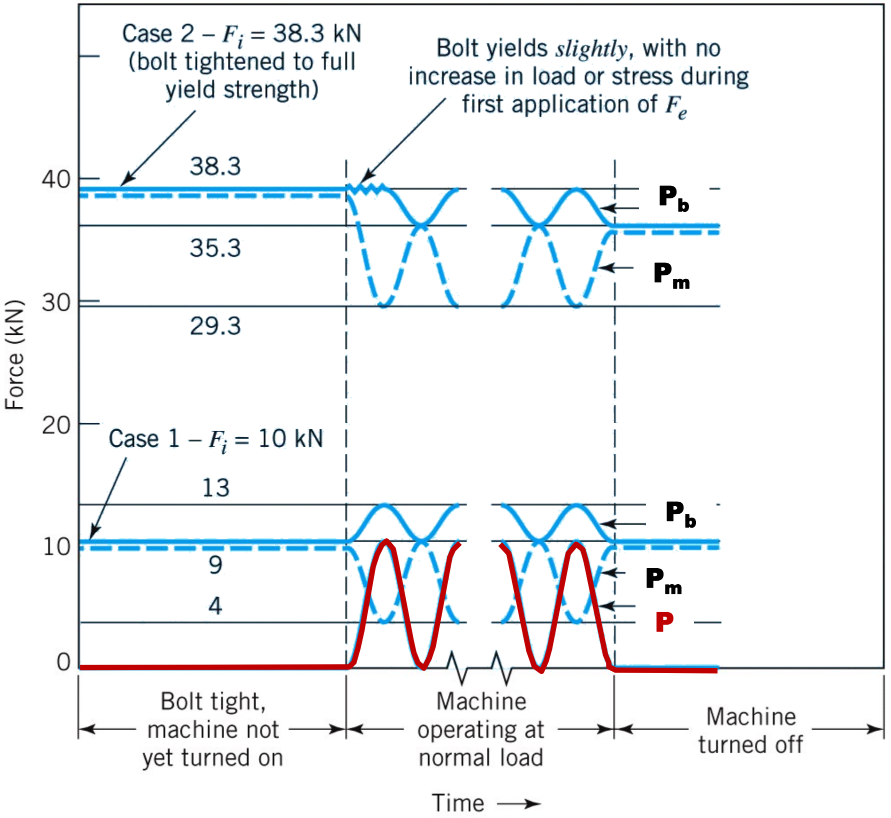
$$= \frac{2\cdot\left(18.6\times10^3\ {}^{lb}\!/_{in^2}\right)\cdot\left[\left(120.0\times10^3\ {}^{lb}\!/_{in^2}\right)\cdot\left(0.1599\,in^2\right)-\left(12.23\times10^3\,lb\right)\right]}{(0.2978)\cdot(600\,lb)\cdot\left[\left(120.0\times10^3\ {}^{lb}\!/_{in^2}\right)-\left(18.6\times10^3\ {}^{lb}\!/_{in^2}\right)\right]}$$

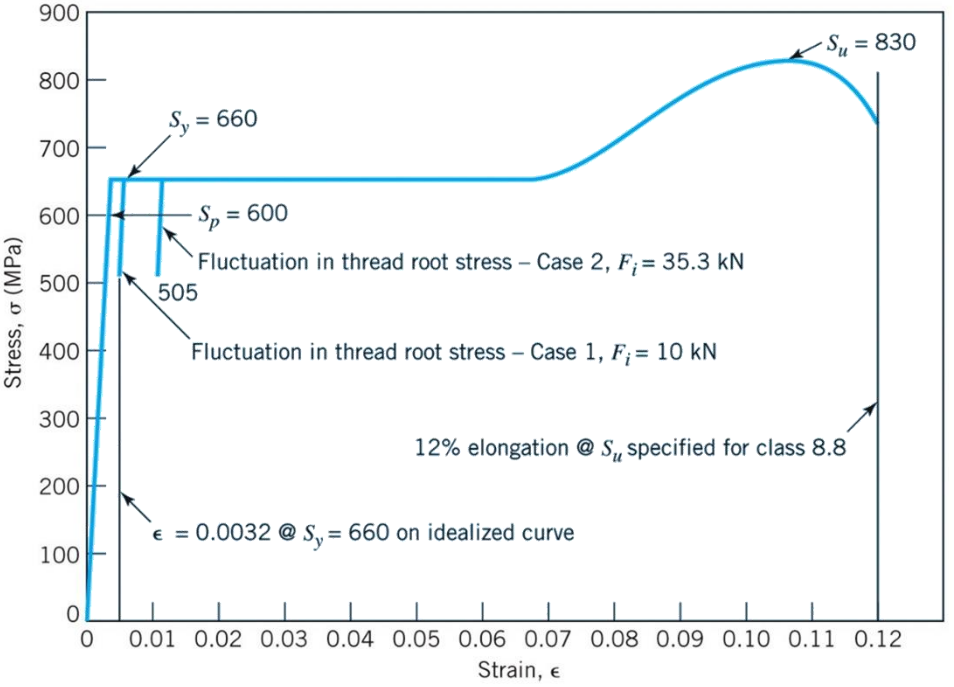
$$= \boxed{14.2}$$

Pre-Loaded to Yield in Fatigue

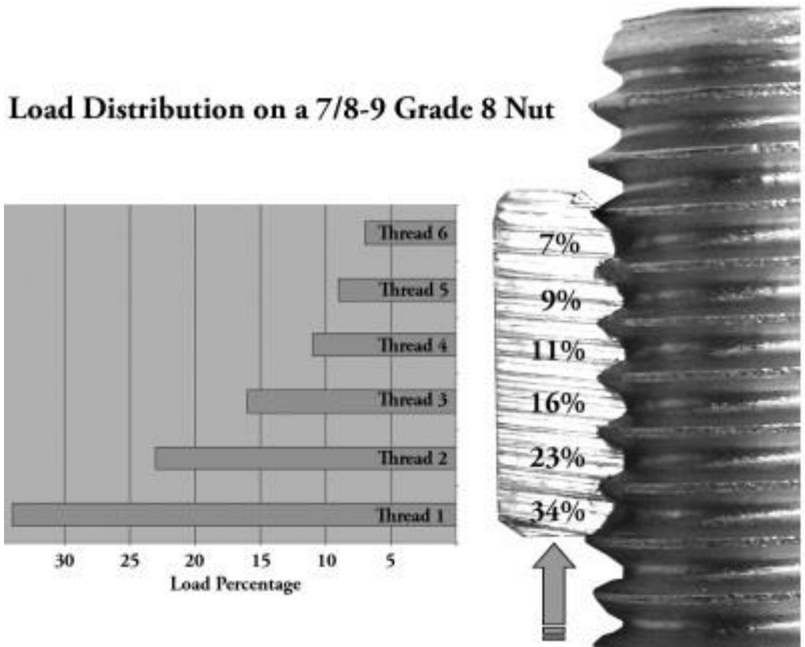
Consider the fatigue strength of an <u>M10x1.5 SAE Class 8.8</u> bolt, A_t=58.27mm^2, K_f=3.0

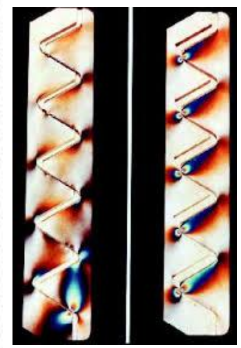
- **k_m=2·k_b**
- **P=F_e fluctuates between 0 and 9kN**
- **Preload**
  - **Case 1: F_i=10kN, → Thread Root Stress = (10kN/58.27mm^2)·3.0 = 515 MPa**
    - **F_b=13kN, → Thread Root Stress = (13kN/58.27mm^2)·3.0 = 670 Mpa**
  - **Case 2: F_i=Full Yield = S_y·A_t = 660MPa·58.0mm^2 =38.3kN**

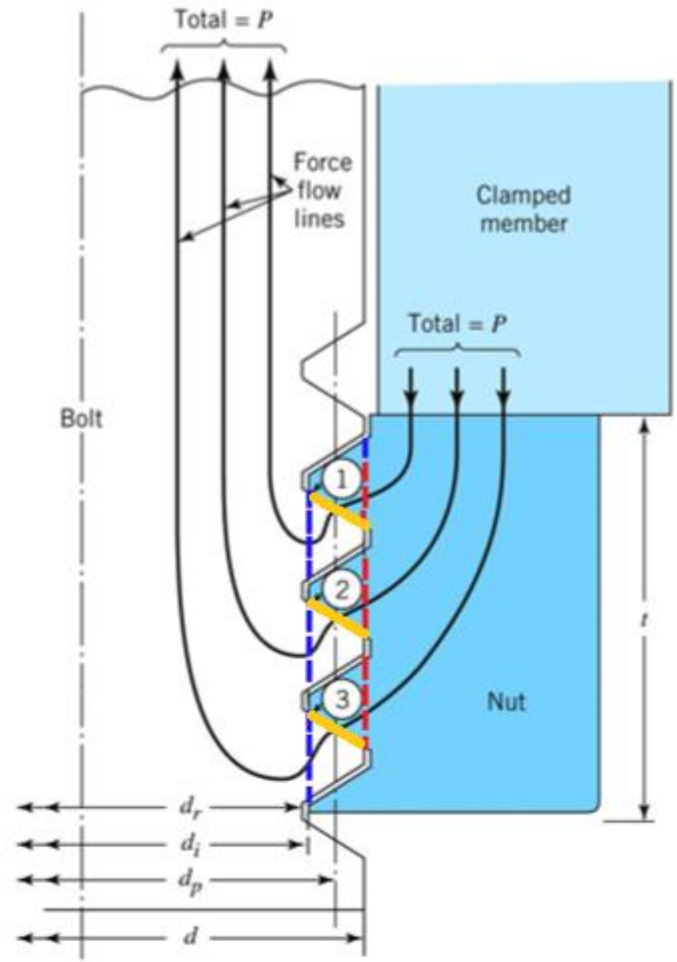




Actual Load Shearing in Threads







$$\sigma_i = \frac{P}{A} = \frac{4 \cdot P}{\pi \cdot \left(d^2 - d_i^2\right)} \cdot L_{\%}$$

$$A \equiv \text{Projected Area} = \frac{\pi \cdot \left(d^2 - d_i^2\right)}{4}$$

- d_i is the minor diameter of the internal threads
- For threaded fasteners, $d_i \approx d_r$

$L_{\%} \equiv$ Percent of load on thread

Obtaining More Uniformly Distributed Thread Loading

1. **Make the NUT from a softer material than the Bolt**
   a. **Highly loaded first thread will deflect, thereby transferring more of the load to the other threads**
   b. **This may require increasing the number of threads in contact**

2. **Manufacturing the nut threads with a SLIGHTLY grater pitch than that of the bolt threads so that the two pitches are theoretically equal AFTER the load is applied.**
   a. **The thread clearance and precision of manufacture must be such that the nut and bolt can be readily assembled**

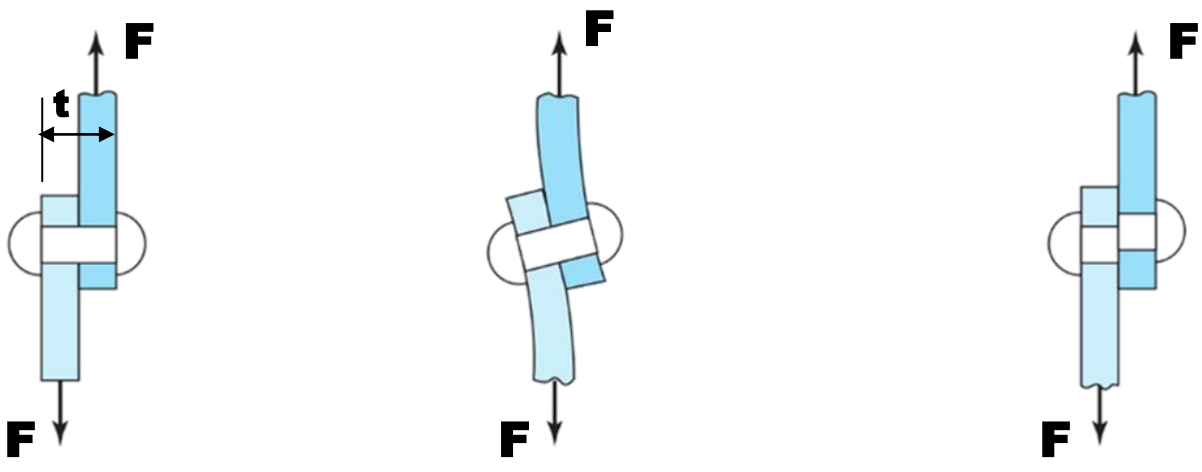
3. **Modify the nut design so that the nut loading puts the region of the top threads in tension**
   a. **This will cause the elastic changes in pitch to approximately match the bolt pitch**
   b. **Such nuts are expensive and have been used onlyh in critical applications involving fatigue loading**

Failure of Bolted and Riveted Joints Loaded in Shear

Failure of Rivets and Bolts Treated the same

Failure by Bending

Failure by Pure Shear



(a) (b) (c)

$$M = F \cdot t/2$$

$$\sigma = \frac{M \cdot c}{I}$$

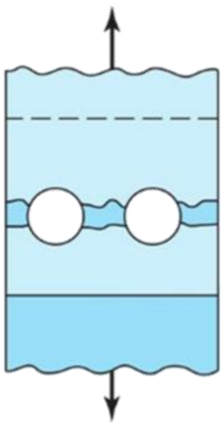
I/c is the Section Modulus of the Weaker Member

$$\tau = \frac{F}{A}$$

A is the Nominal Area of the Fastener

Failure of Bolted and Riveted Joints Loaded in Shear

Rupture

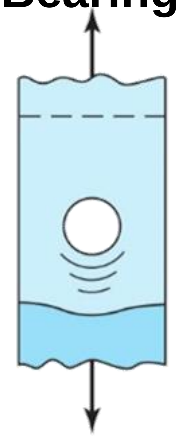


(d)

$$\sigma = K \cdot \frac{F}{A}$$

A is the Net Area of the Plate

Crushing/ Bearing



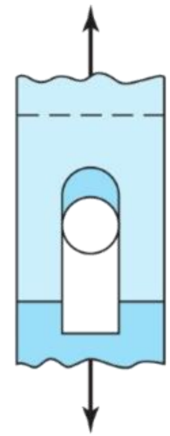
(e)

$$\sigma = -\frac{F}{A}$$

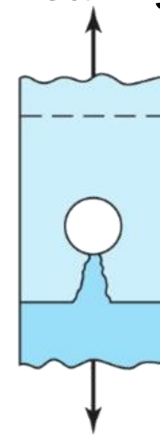
$$A = t \cdot d$$

t ≡Thickness of Thinnest Plate
d≡Fastener Dia.

Edge Shearing or Tearing



(f)

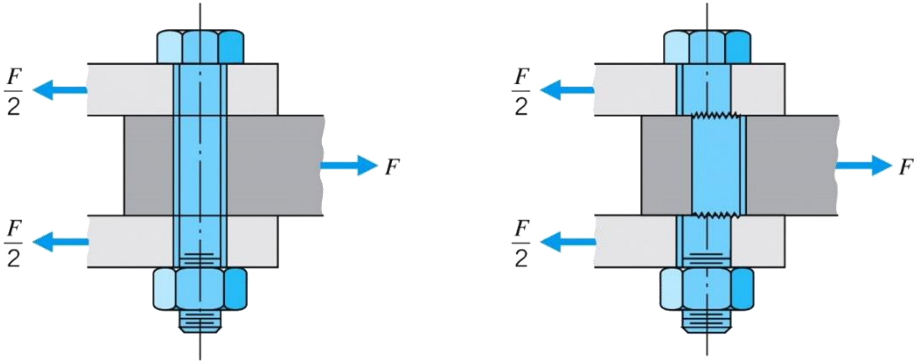


(g)

Avoided by Spacing Fasteners 1.5 Diameters away from Edge

Example of Fastener in Shear

A ½ in-13 UNC Grade 5 steel bolt is loaded in double shear. The clamped plates are made of steel and have clean dry surfaces. The bolt is to be tightened with a torque wrench to its full proof load. What force is the joint capable of withstanding.



(a) Normal load, carried by friction forces

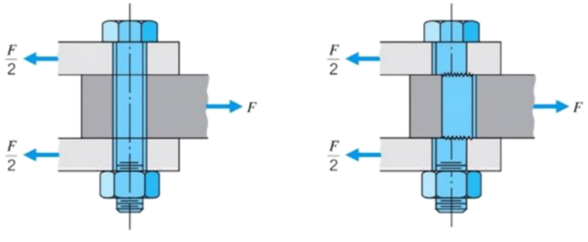
(b) Overload, causing shear failure

For a ½ in-13UNC Grade 5 Steel Bolt

- **A_t = 0.1419 in^2**
- **S_P = 85 ksi**
- **S_y= 92 ksi**
- **S_{us} = 74 ksi**
- **Conservative estimate after a few weeks of service**
  - **±30% Torque Wrench variation**
  - **10% tension loss during service**

- **The force required to slip each plate**
  - **μ = 0.4**

Solution



(a) Normal load, carried by friction forces

(b) Overload, causing shear failure

For a ½ in-13UNC Grade 5 Steel Bolt

- **A_t = 0.1419 in^2**
- **S_P = 85 ksi**
- **Initial Tension $F_i = S_P \cdot A_t$ = (85 000psi)·(0.1419 in^2) = 12 060 lb**
- **Conservative estimate after a few weeks of service**
  - **±30% Torque Wrench variation**
  - **10% tension loss during service**
  - **F_i = (0.9)·(0.7)·12 060 lb = 7 600 lb**

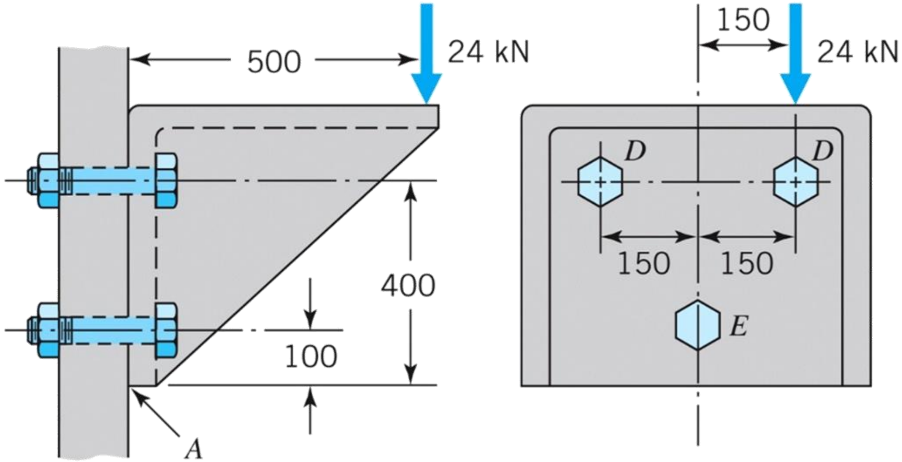
- **The force required to slip each plate**
  - **μ = 0.4**
  - **Force to overcome friction, f = 7 600lb·0.4 = 3040 lb**
  - **Force to slip the two plates F = 2·3040lb ≈ 6080 lb**

- **Total load that can be transmitted through bolts**
  - **$F = 2 \cdot S_{sy} \cdot A$**
  - **Area of the bolt at the SHEAR PLANE, A = π·(0.5in)^2/4 = 0.196in^2**
  - **Estimate of Shear Yield Strength (from Distortional energy)**
    $S_{sy} = 0.58 \cdot S_y$ = (0.58)·(92 ksi) = 53 ksi
  - **For yielding the two shear planes F = 2·(0.196in^2)·(53 ksi) = 21000 lb**
  - **Total failure load calculated by replacing S_{sy} with Sus = 74 ksi, F = 29000 lb**

Example of Eccentric Loading, Neglect Friction, Bolts Shear

A vertically loaded bracket is attached to a fixed member by three identical bolts. Although the 24-kN load is normally applied in the center, the bolts are to be selected on the basis that the load eccentricity shown could occur. Because of safety considerations, SAE class 9.8 steel bolts and a minimum safety factor of 6 (based on proof strength)are to be used. Determine the appropriate bolt size.

S_p=650 MPa



Assumptions:

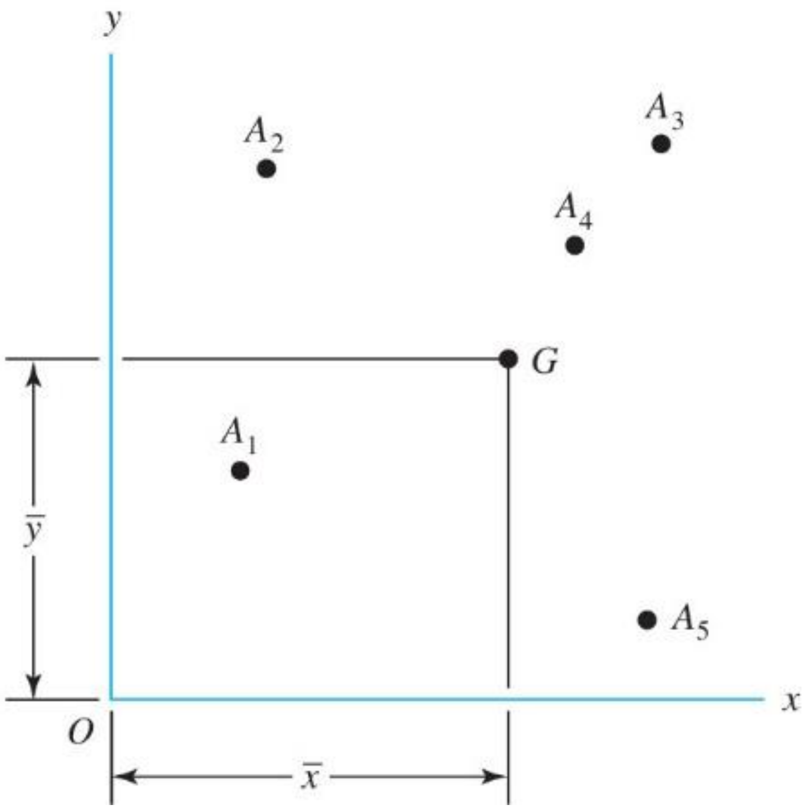
1. **Shear forces caused by eccentric vertical load are carried completely by bolts**
2. **Vertical shear load is distributed equally among the three bolts**
3. **Tangential shear force carried by each bolt is proportional to its distance from the center of gravity of the group of bolts**

Distribution of Tangential Shear Forces, No Friction

Distribution of Tangential Shear Load Proportional to distance from Centroid.

$$\bar{x} = \frac{\sum_1^n A_i \cdot x_i}{\sum_1^n A_i}$$

$$\bar{y} = \frac{\sum_1^n A_i \cdot y_i}{\sum_1^n A_i}$$

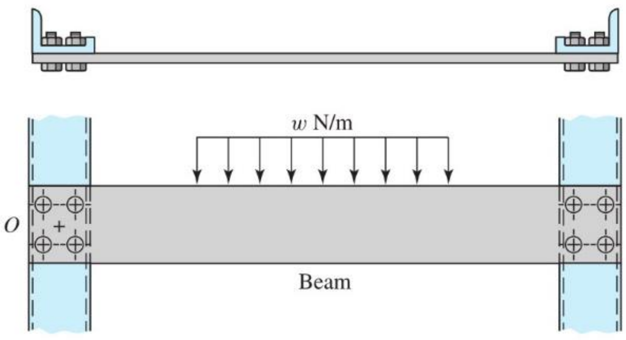


Secondary Shear Distribution

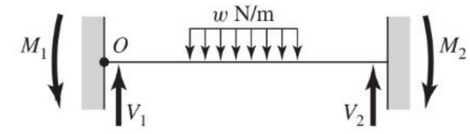
$$M_1 = F_A'' \cdot r_A + F_B'' \cdot r_B + F_C'' \cdot r_C + \cdots$$

$$\frac{F_A''}{r_A} = \frac{F_B''}{r_B} = \frac{F_C''}{r_C} = \cdots$$

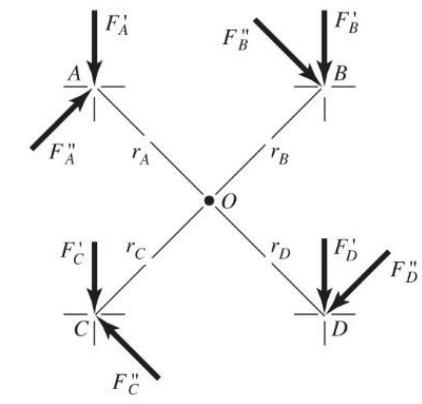
$$F_i'' = \frac{M_1 \cdot r_i}{r_A^2 + r_B^2 + r_C^2 + \cdots}$$



(a)



(b)

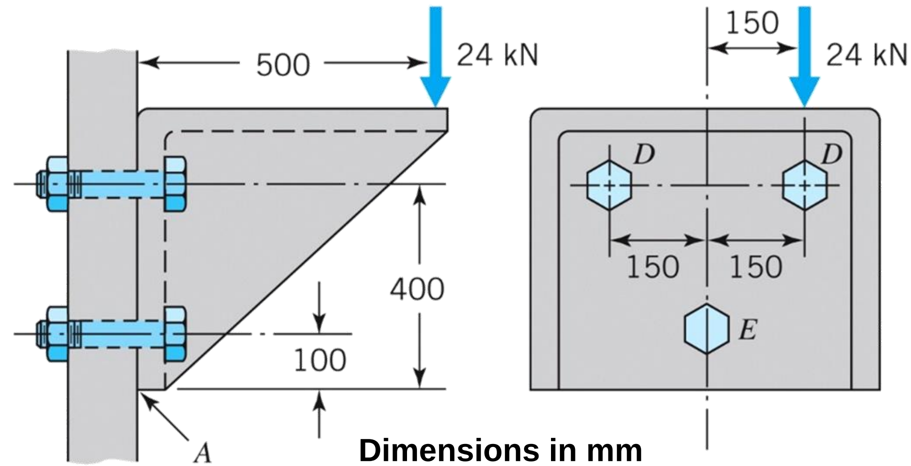


(c)

Example of Eccentric Loading, Assuming Shear Carried by Friction

A vertically loaded bracket is attached to a fixed member by three identical bolts. Although the 24-kN load is normally applied in the center, the bolts are to be selected on the basis that the load eccentricity shown could occur. Because of safety considerations, SAE class 9.8 steel bolts and a minimum safety factor of 6 (based on proof strength)are to be used. Determine the appropriate bolt size.

S_p = 650 Mpa
μ = 0.4
Tension relaxation of 0.55

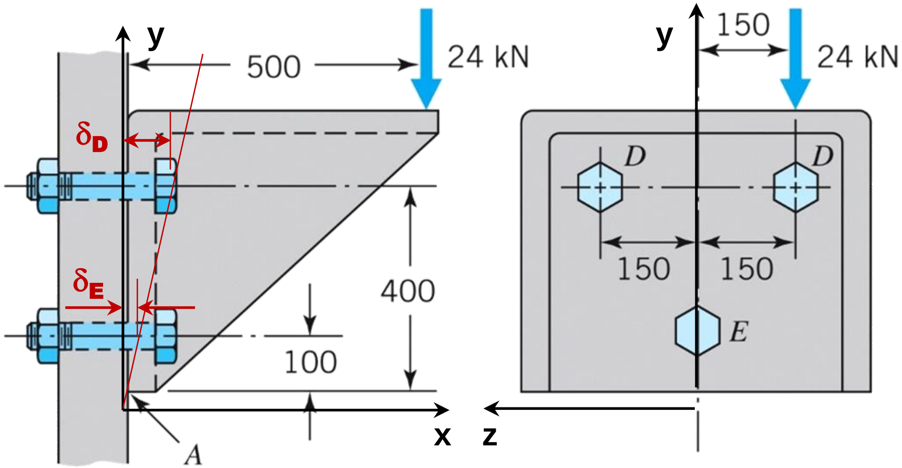


Dimensions in mm

Assumptions:

1. The clamped members are rigid and do not deflect with the load
   a. Eccentricity of the applied load has no effect on the bolt loading
2. The load tends to rotate the bracket about point A
3. The shear loads are carried by friction

Solution-Bolt Size Based on Plate Slip



Load Multiplied by 6 to account for Factor of Safety

Design Load $= 24kN \cdot 6 = 144kN$

Tension in Bolt

Summing Moment in the z-direction at A:

$$144kN \cdot 0.5m = 72kN \cdot m = 0.1m \cdot F_E + 0.4m \cdot F_D + 0.4m \cdot F_D$$

The kinematic assumption, bolts at D stretch 4 times as much as at E

$$\delta_D = 4 \cdot \delta_E \quad \Rightarrow \quad F_D / k = 4 \cdot \left(F_E / k \right) \quad \Rightarrow \quad F_D = 4 \cdot F_E$$

$$\Rightarrow \quad 0.25 \cdot F_D = F_E$$

$$72kN \cdot m = 0.1m \cdot 0.25 \cdot F_D + 0.4m \cdot F_D + 0.4m \cdot F_D = 0.825m \cdot F_D$$

$$F_D = 87.3kN$$

$$F_E = 21.8kN$$

Table 8–11

Metric Mechanical-Property Classes for Steel Bolts, Screws, and Studs

Property Class	Size Range, Inclusive	Minimum Proof Strength,* MPa	Minimum Tensile Strength,* MPa	Minimum Yield Strength,* MPa	Material	Head Marking
9.8	M1.6–M16	650	900	720	Medium carbon, Q&T	9.8

*Minimum strengths are strengths exceeded by 99 percent of fasteners.

Required Tensile Area

$$A_t = \frac{F_D}{S_P} = \frac{87.3kN}{650MPa} = 134\,mm^2$$

Table 8–1

Diameters and Areas of Coarse-Pitch and Fine-Pitch Metric Threads.*

Nominal Major Diameter d mm	Coarse-Pitch Series: Pitch p mm	Coarse-Pitch Series: Tensile-Stress Area A_t, mm²	Coarse-Pitch Series: Minor-Diameter Area A_r, mm²	Fine-Pitch Series: Pitch p mm	Fine-Pitch Series: Tensile-Stress Area A_t, mm²	Fine-Pitch Series: Minor-Diameter Area A_r, mm²
12	1.75	84.3	76.3	1.25	92.1	86.0
14	2	115	104	1.5	125	116
16	2	157	144	1.5	167	157
20	2.5	245	225	1.5	272	259

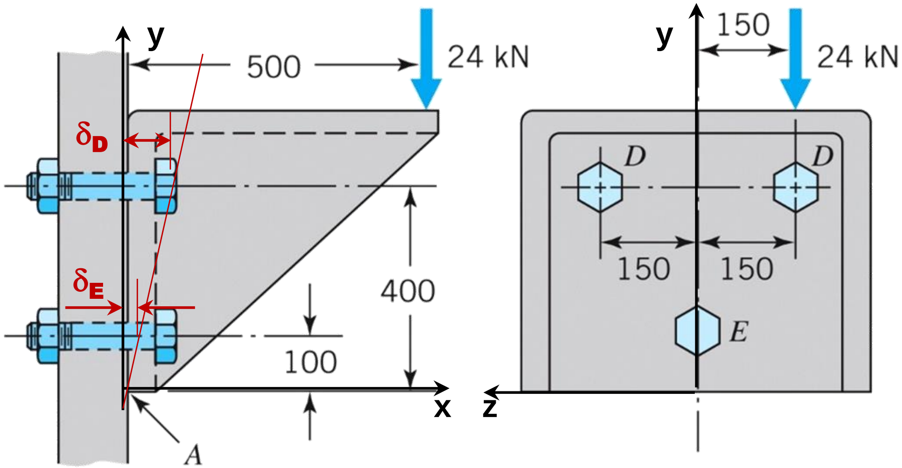
Required thread size M16 x 2

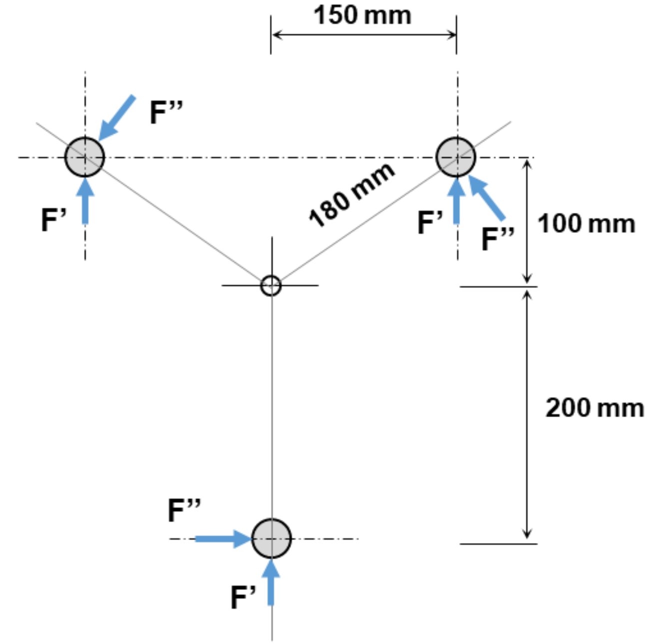
The available friction force

$$f = (\#\text{ of Bolts}) \cdot (\text{Relaxation}) \cdot (\text{Proof Load}) \cdot (\text{Friction Coef})$$
$$= 3 \cdot 0.55 \cdot S_p \cdot A_t \cdot \mu$$
$$= 3 \cdot 0.55 \cdot \left(650 \times 10^6\ {}^{lb}/_{in^2}\right) \cdot \left(167 \times 10^{-6}\ m^2\right) \cdot (0.4)$$
$$= 67\,500\ N = 67.5\,kN$$

This load is compared to the 24 kN load and shows the margin to avoid the plate from slipping.

Solution-Shear Loading





Primary Shear Loading F' - Load Equally Shared

$$F' = \frac{\text{Shear Force}}{\text{Number of Bolts}} = \frac{V}{n} = \frac{144\,kN}{3} = 48\,kN$$

Secondary Shear Loading F''

Finding the bolt pattern centroid (equal areas):

$$\bar{z} = \frac{\sum_1^n A_i \cdot z_i}{\sum_1^n A_i} = \frac{A_t \cdot (150\,mm) + A_t \cdot (-150\,mm) A_t \cdot (0\,mm)}{3 \cdot A_t}$$

$$= 0\,mm$$

$$\bar{y} = \frac{\sum_1^n A_i \cdot y_i}{\sum_1^n A_i} = \frac{A_t \cdot (100\,mm) + A_t \cdot (400\,mm) A_t \cdot (400\,mm)}{3 \cdot A_t}$$

$$= 300\,mm$$

$$F_i'' = \frac{M_1 \cdot r_i}{r_A^2 + r_B^2 + r_C^2 + \cdots}$$

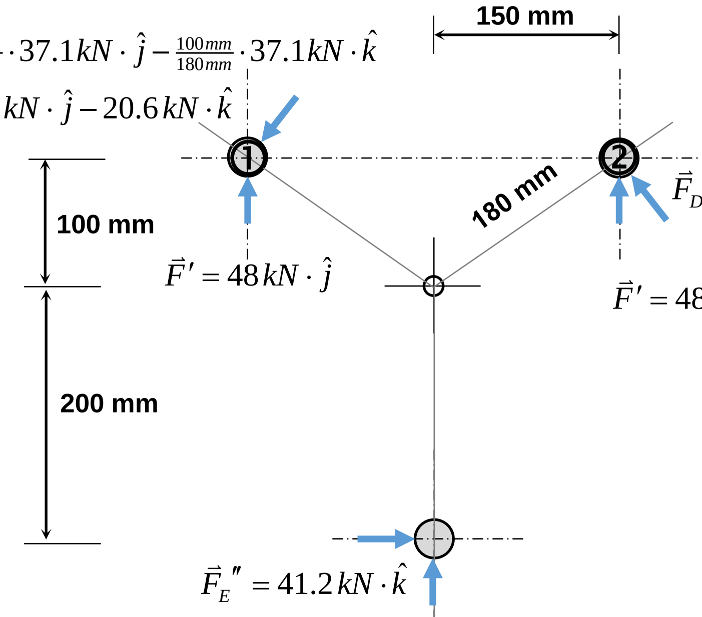
$$M = 0.150\,m \cdot 144 \times 10^3\,N = 21.6 \times 10^3\,N-m$$

$$F_E'' = \frac{(21.6 \times 10^3\,N-m) \cdot (0.200\,m)}{(0.2\,m)^2 + 2 \cdot (0.18\,m)^2} = 41.2 \times 10^3\,N$$

$$F_D'' = \frac{(21.6 \times 10^3\,N-m) \cdot (0.180\,m)}{(0.2\,m)^2 + 2 \cdot (0.18\,m)^2} = 37.1 \times 10^3\,N$$

Solution-Shear Loading

$$\vec{F}_{D1}'' = -\tfrac{100mm}{180mm}\cdot 37.1kN\cdot\hat{j} - \tfrac{100mm}{180mm}\cdot 37.1kN\cdot\hat{k}$$
$$= -30.9\,kN\cdot\hat{j} - 20.6\,kN\cdot\hat{k}$$



$$\vec{F}' = 48\,kN\cdot\hat{j}$$

$$\vec{F}_{D2}'' = \tfrac{100mm}{180mm}\cdot 37.1kN\cdot\hat{j} - \tfrac{100mm}{180mm}\cdot 37.1kN\cdot\hat{k}$$
$$= 30.9\,kN\cdot\hat{j} - 20.6\,kN\cdot\hat{k}$$

$$\vec{F}' = 48\,kN\cdot\hat{j}$$

$$\vec{F}_E'' = 41.2\,kN\cdot\hat{k}$$

$$\vec{F}' = 48\,kN\cdot\hat{j}$$

Max. Secondary Shear Loading is at D2

$$V_{D2} = \sqrt{\left(48\,kN + 30.9\,kN\right)^2 + \left(20.6\,kN\right)^2}$$
$$= 81.5\,kN$$

$N_{D2} = 87.3kN$ (Previous Result)

From the Distortional Energy Theory (VonMises), an equivalent stress can be computed

$$\sigma_{D2} = \sqrt{\sigma^2 + 3\cdot\tau^2} = \sqrt{\left(\frac{87.3\,kN}{A_t}\right)^2 + 3\cdot\left(\frac{81.5\,kN}{A_t}\right)^2}$$
$$= \frac{166.0\,kN}{A_t}$$

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*Minimum strengths are strengths exceeded by 99 percent of fasteners.

Comparing to the Proof Strength

$$S_p = 650\times10^6\,\tfrac{N}{m^2} = \frac{166.0\times10^3\,N}{A}$$
$$A_t = 255.4\times10^{-6}\,m^2$$

The minimum diameter of the shank is

$$A = \frac{\pi\cdot d^2}{4}$$
$$\Rightarrow d = \sqrt{\frac{4\cdot a}{\pi}} = \sqrt{\frac{4\cdot 255.4\times10^{-6}\,m^2}{\pi}}$$
$$= 18.0\times10^{-3}\,m = 18.0\,mm$$

This is larger than the previously calculated nominal diameter, thus it is more critical