FATIGUE

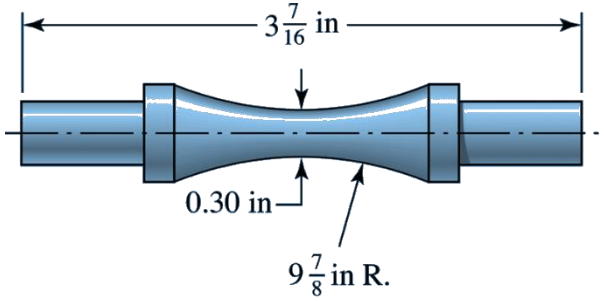
- **Finite Life/Strength**
- **Cumulative Fatigue Damage**

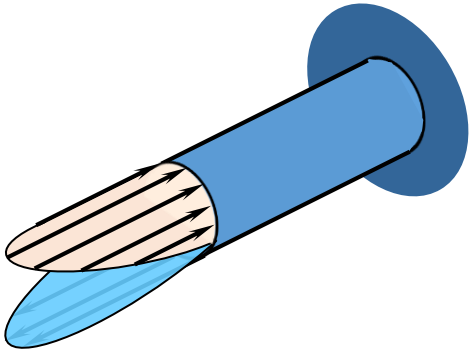
Fatigue and Fracture

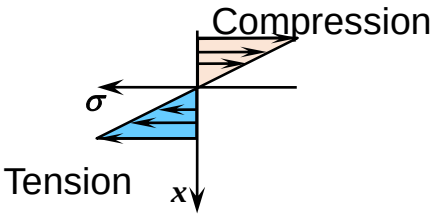
- Introduction to Fatigue
- Fatigue-Life Methods
- **Fatigue Strength and Endurance Limits**
- **Endurance Limit Modifying Factors**
- **Stress Concentrations and Notch Sensitivity**
- **Fluctuating Stresses**
- **Combination of Loading Modes**
- **Varying, Fluctuating Stresses; Cumulative Fatigue Damage**

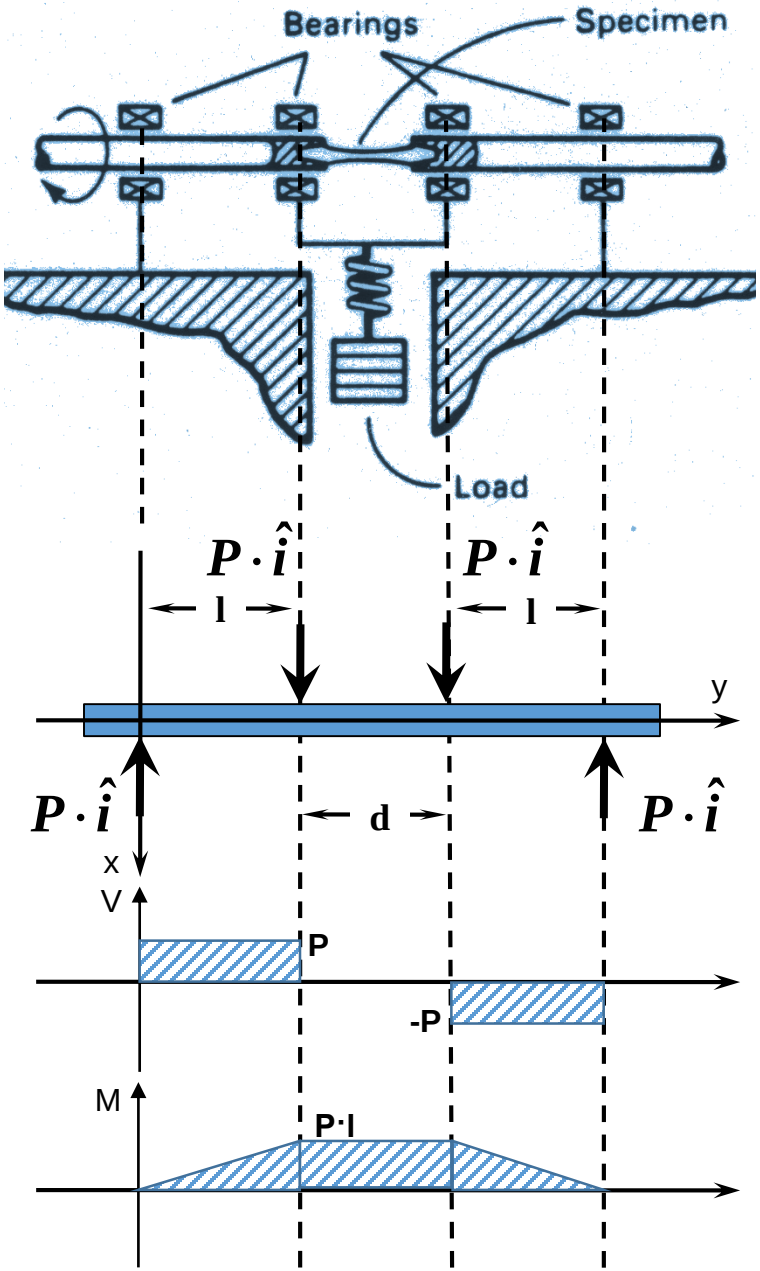
Stress-Life Method: R.R Moore Machine

- Repeated or varying loads
- Specified Magnitude



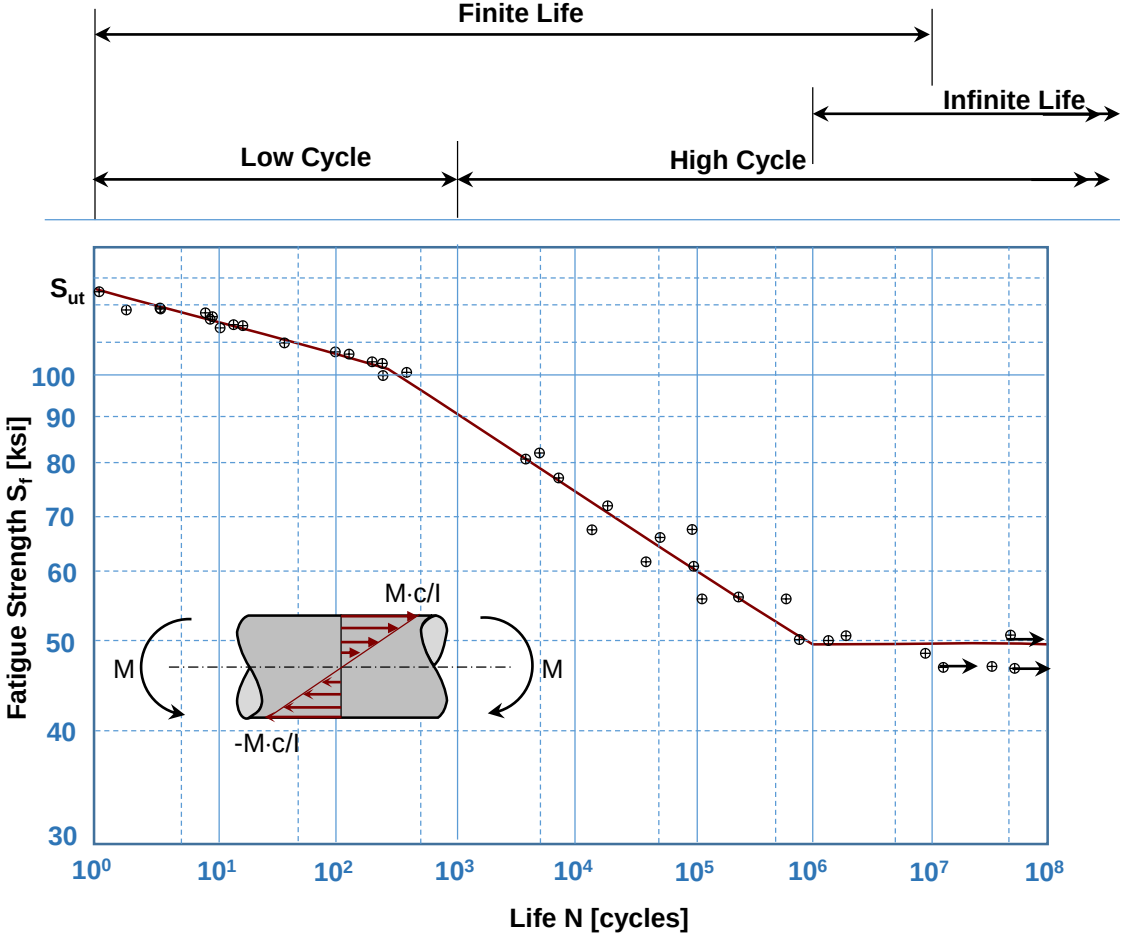






Stress-Life Method: S-N Diagram – Log-Log Axes

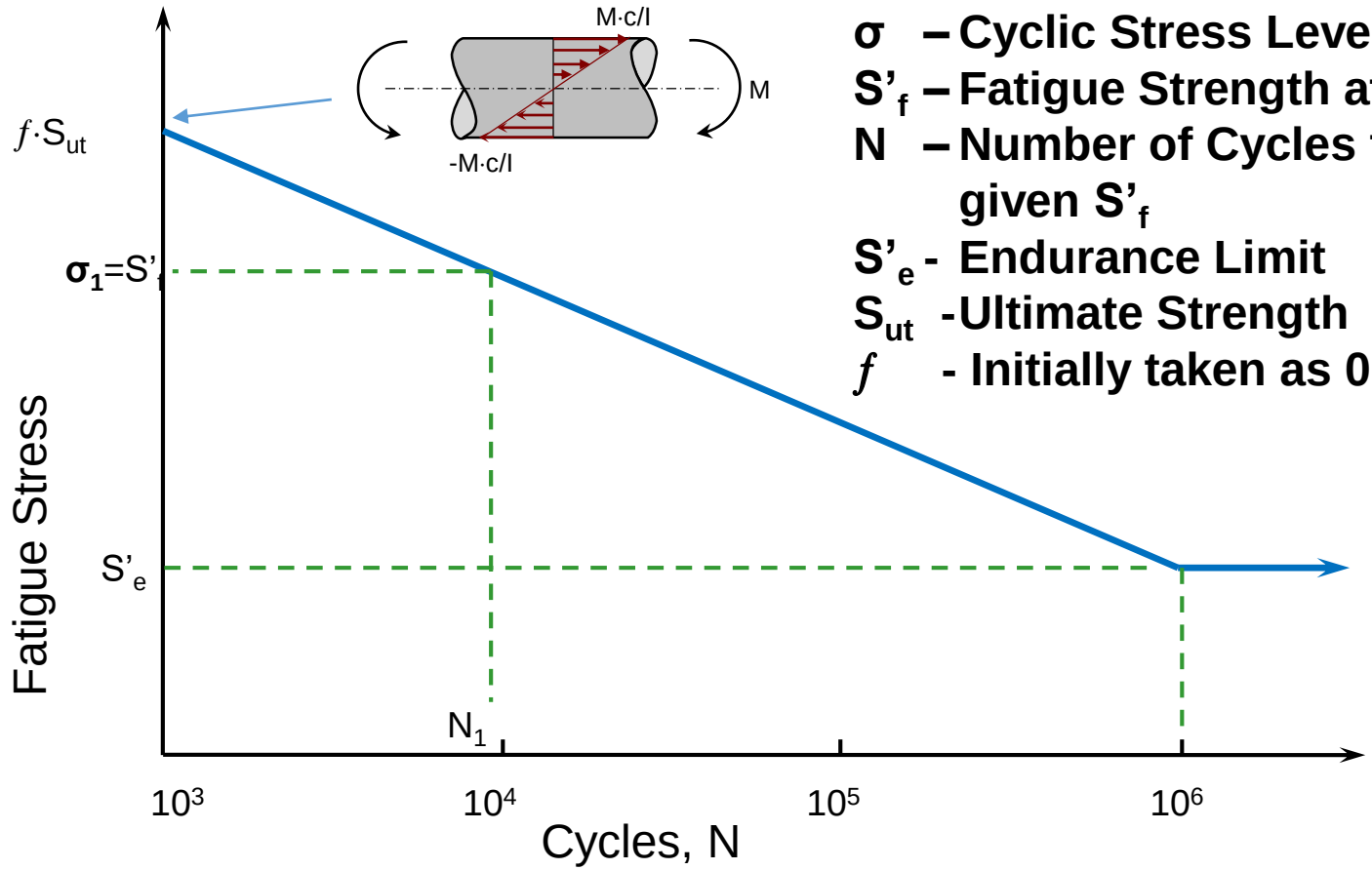
- **Plotted on Log-Log Scale**
- **Three Regions Observed**
  - **Low Cycle**
    - **Significant Plastic Deformation**
    - **Difficult to Predict**
  - **Finite Life, S_f**
  - **Endurance Limit, S_e**
    - **Mostly Ferrous Metals**
    - **Infinite Life**



Fatigue and Fracture

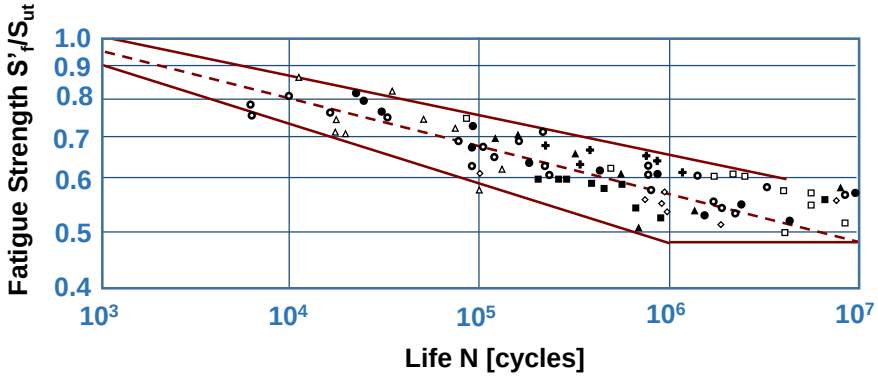
- Introduction to Fatigue
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Stress-Life Method: Calculating Fatigue Life

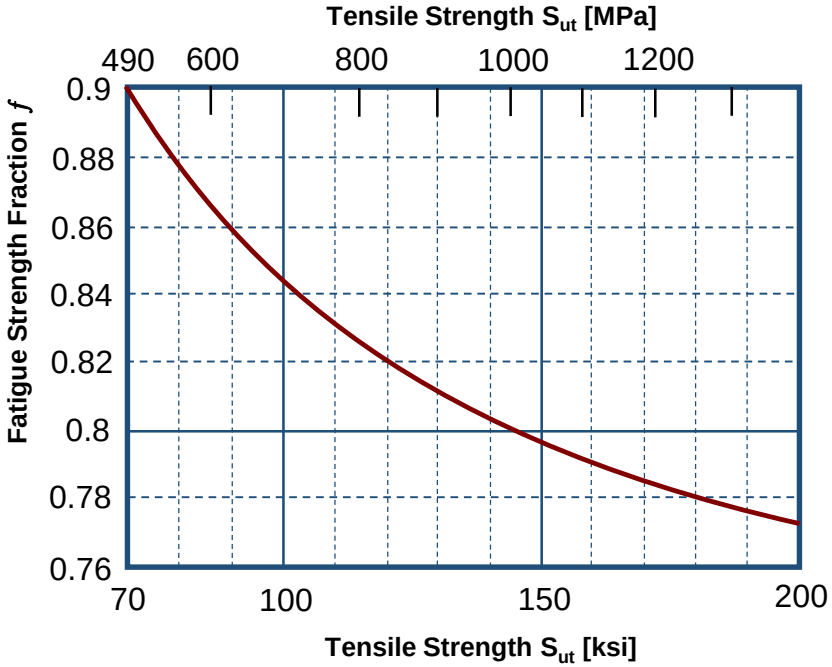


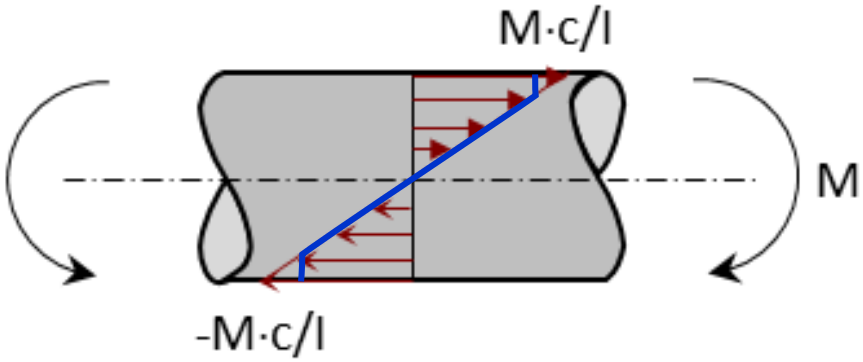
σ – Cyclic Stress Level
S'_f – Fatigue Strength at N
N – Number of Cycles to Failure given S'_f
S'_e - Endurance Limit
S_{ut} -Ultimate Strength
f - Initially taken as 0.9

Fatigue Strength Factors, $S_f(10^3)$

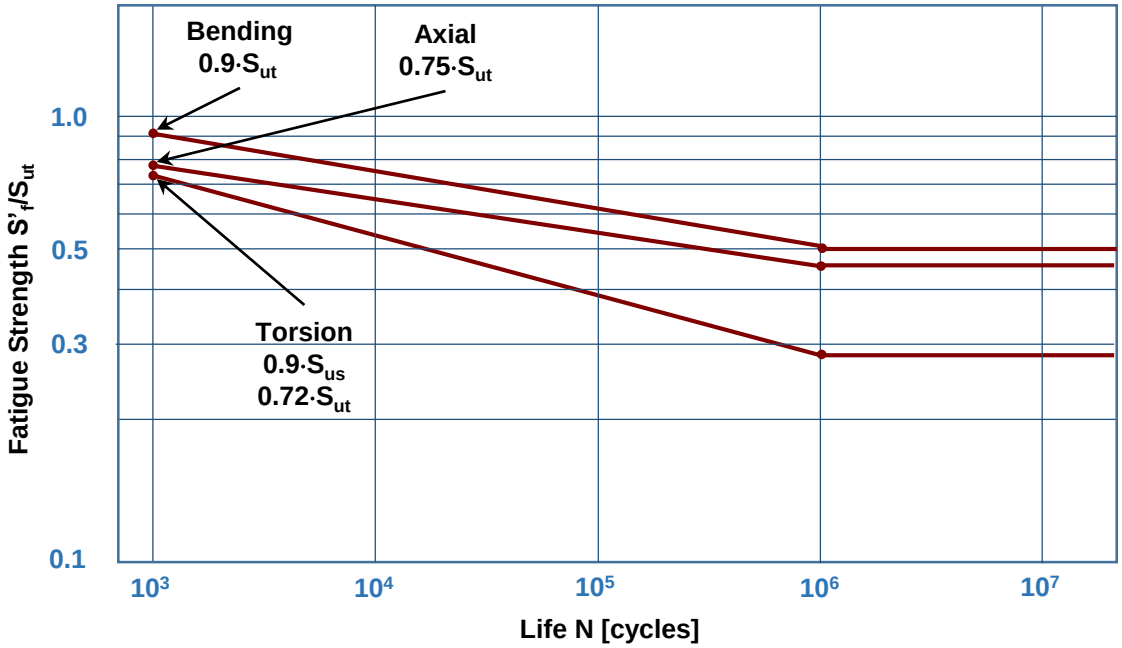


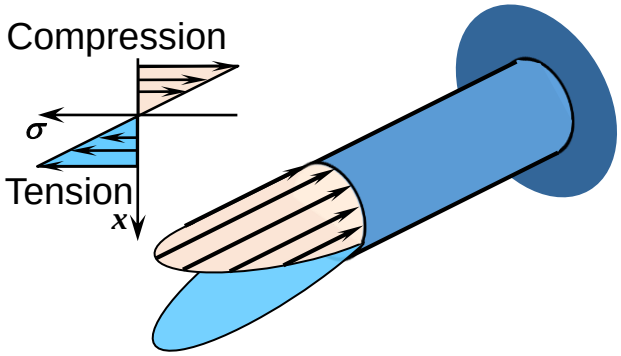
- Actual strength not as high as predicted
- Loading that will cause failure at 10^3 cycles will typically cause **plastic deformation**

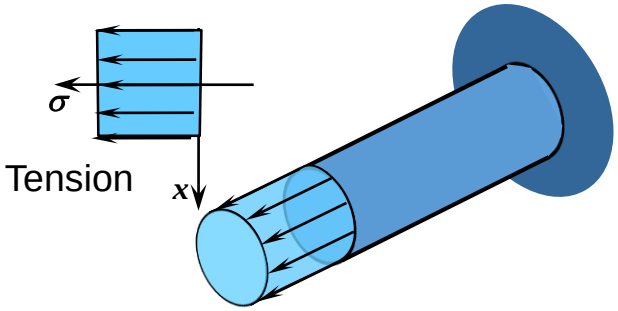


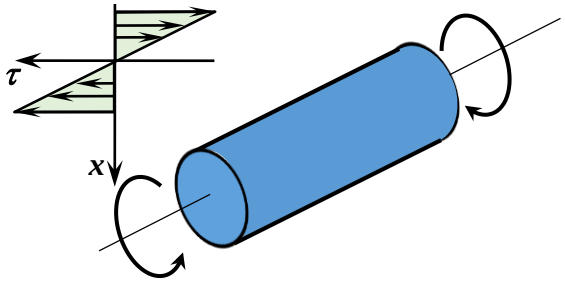


Influence of Loading Configuration on 10^3 Cycle Strength (S_f')

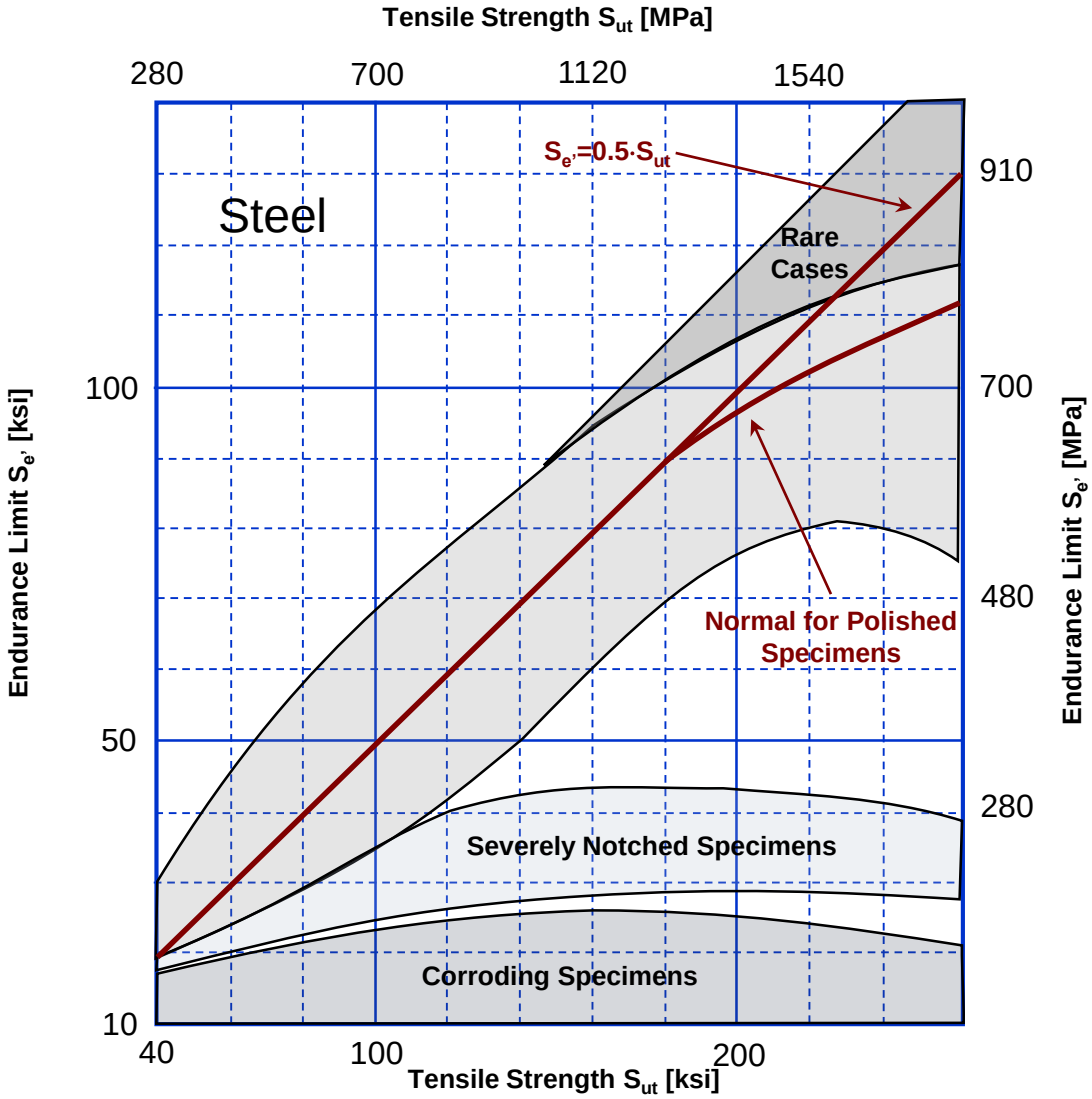








Endurance Limit for Std. Specimen as a function of Ultimate Strength



- **Steel**
  - English Units

$$\bar{S}'_e = 0.5 \cdot S_{UT} \quad when \quad S_{UT} \le 200ksi$$

$$\bar{S}'_e = 100ksi \quad when \quad S_{UT} > 200ksi$$

  - Metric Units

$$\bar{S}'_e = 0.5 \cdot S_{UT} \quad when \quad S_{UT} \le 1400MPa$$

$$\bar{S}'_e = 700MPa \quad when \quad S_{UT} > 1400MPa$$

- **Cast Iron**

$$\bar{S}'_e = 0.4 \cdot S_{UT}$$

- **Aluminum and Magnesium ($5x10^8$)**

$$\bar{S}'_{f\ 5\times10^8} \approx 0.3\ to\ 0.4 \cdot S_{UT}$$

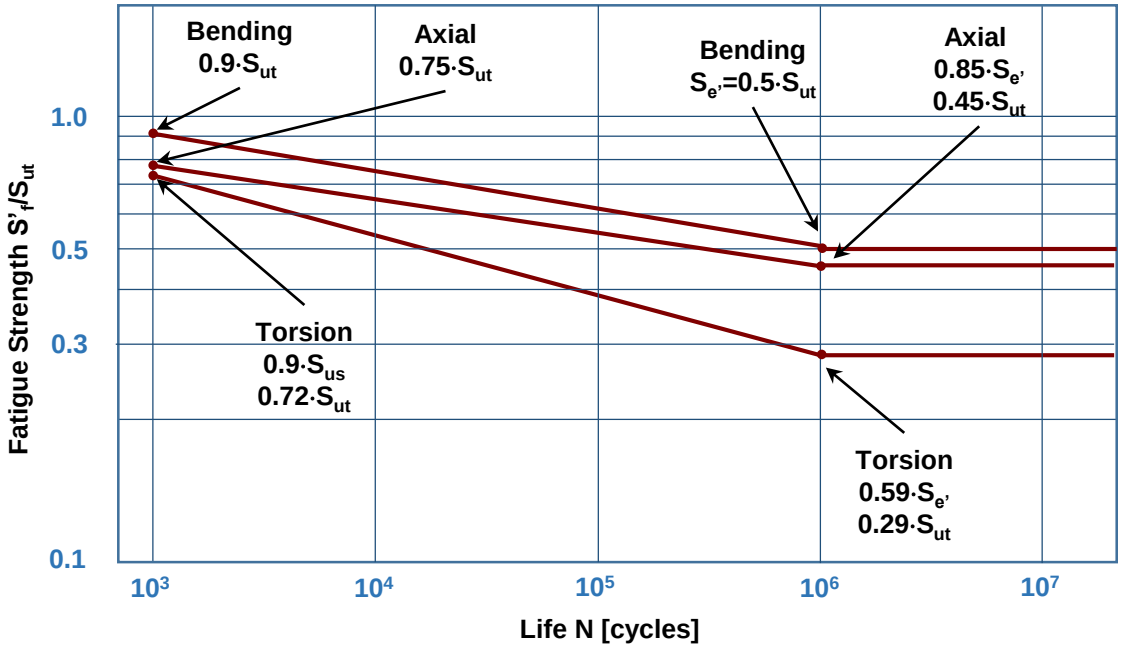
- **Copper Allows (10^8) (Brass, bronzes, etc.)**

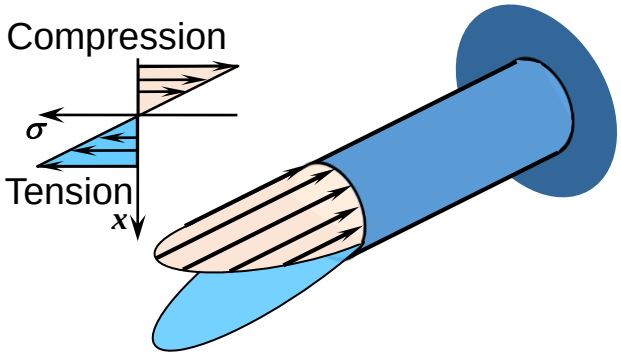
$$\bar{S}'_{f\ 10^8} \approx 0.25\ to\ 0.5 \cdot S_{UT}$$

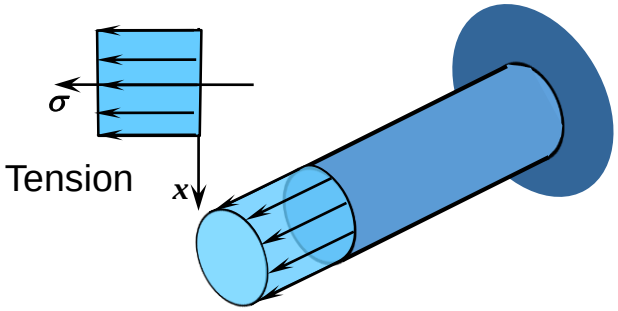
- **Titanium**

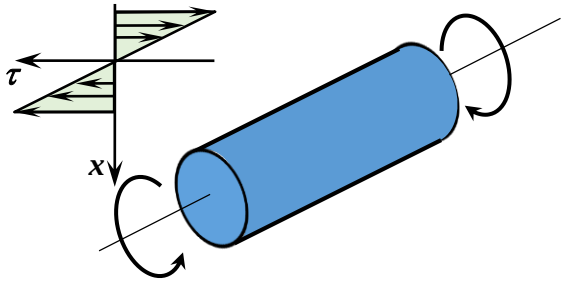
$$\bar{S}'_e \approx 0.45\ to\ 0.65 \cdot S_{UT}$$

Influence of Loading Configuration on 10^6 Cycle Strength (S_e')









Example 1

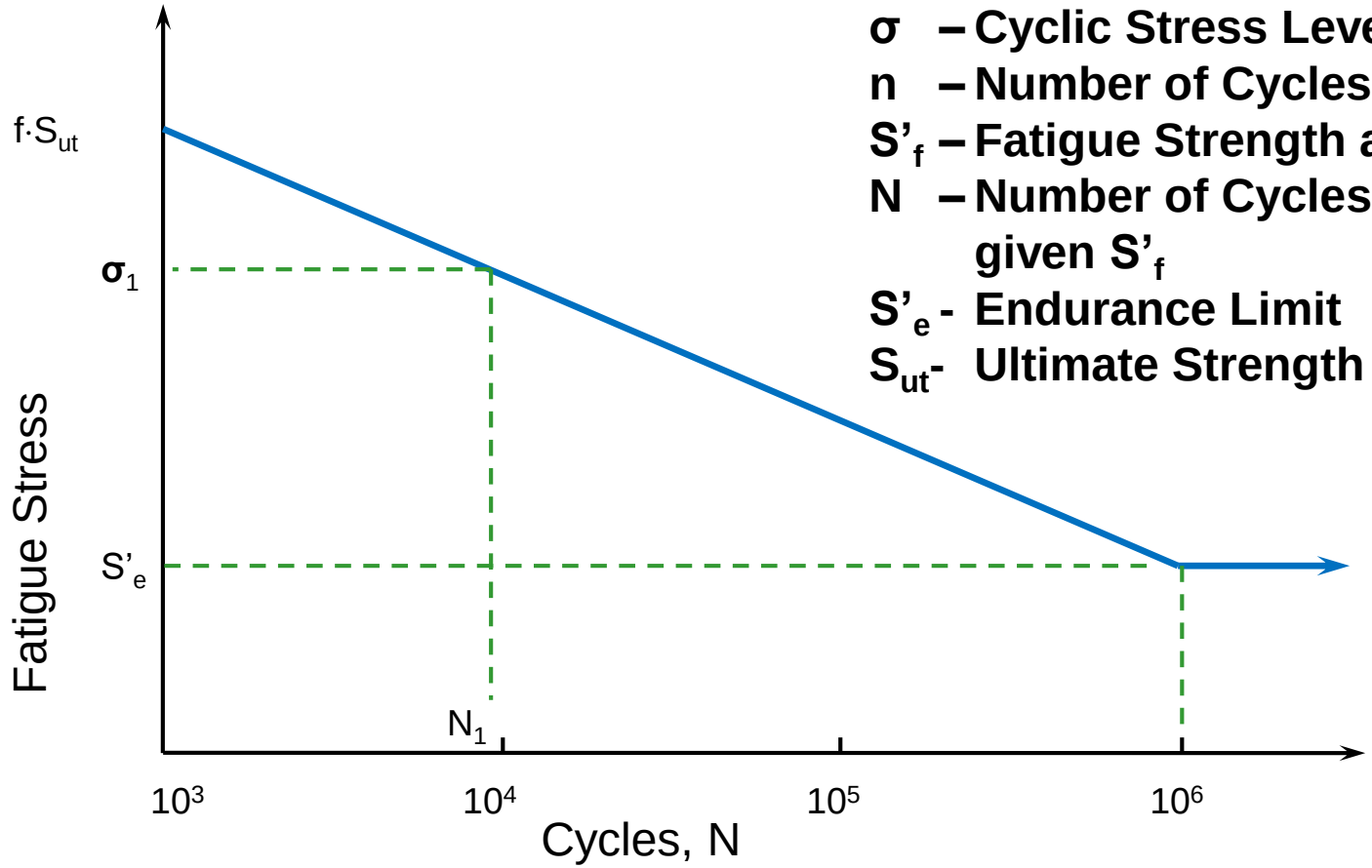
- Given an Alloy Steel with a Tensile Strength of 1580 MPa, determine the 10^3 fatigue strength and the endurance limits for the following loading cases: Standard Rotating Beam Specimen, Axial Loading (no eccentricity), and Pure Torsion.

Typical Material Fatigue Values

(Reported by F.B. Stulen, H.N. Cummings, and W.C. Schulte, Preventing Fatigue Failures, Part 5, *Machine Design*, vol. 33, P. 161, 22 June 1961)

Material UNS No. (Alloys are heat treated, hot worked, specimens smooth, subjected to long life rotating beam test)	**Tensile Strength, S_{ut}**		**Endurance Limit, S'_E**		**Standard Deviation**	
	MPa	ksi	MPa	ksi	ksi	%
G43400 Steel	965 1310 1580 1790	140 190 230 260	489 586 620 668	71 85 90 97	3.5 6.7 5.3 6.3	4.9 7.8 5.9 6.5
G43500 Steel	2070	300	689	100	4.4	4.4
R50001-series Titanium Alloy	1000	145	579	84	5.4	6.4
A97076 Aluminum Alloy	524	76	186	27	1.6	6.0
C63000 Aluminum Bronze	806	117	331	48	4.5	9.4
C17200 Beryllium Copper	1210	175	248	36	2.7	7.5

Constructing the Log-Log Plot



σ – Cyclic Stress Level
n – Number of Cycles
S'f – Fatigue Strength at N
N – Number of Cycles to Failure given S'f
S'e - Endurance Limit
Sut- Ultimate Strength

Equation for the S-N Curve

S-N Curve Equation

$$\log S'_f = -m \cdot \log N + b$$

where

$$m = \frac{1}{3} \cdot \log\left(\frac{f \cdot S_{ut}}{S'_e} \right)$$

$$b = \log\left(\frac{\left(f \cdot S_{ut} \right)^2}{S'_e} \right)$$

S-N Curve Calculations

In the range $10^3 \le y \le 10^6$

Given N, S'_f is

$$S'_f = \frac{10^b}{N^m}$$

Given S'_f, N is

$$N = \frac{10^{b/m}}{S'^{1/m}_f}$$

Example 2

For the standard rotating beam specimen with an ultimate tensile strength of 990 MPa, determine the following.

a. What is the intercept of the S-N Curve. What life does the intercept correspond to?

b. What is the slope of the S-N Curve?

c. What fatigue strength corresponds to a life of $70(10^3)$ cycles?

d. What is the life of the specimen corresponding to a fatigue strength of 700 MPa?

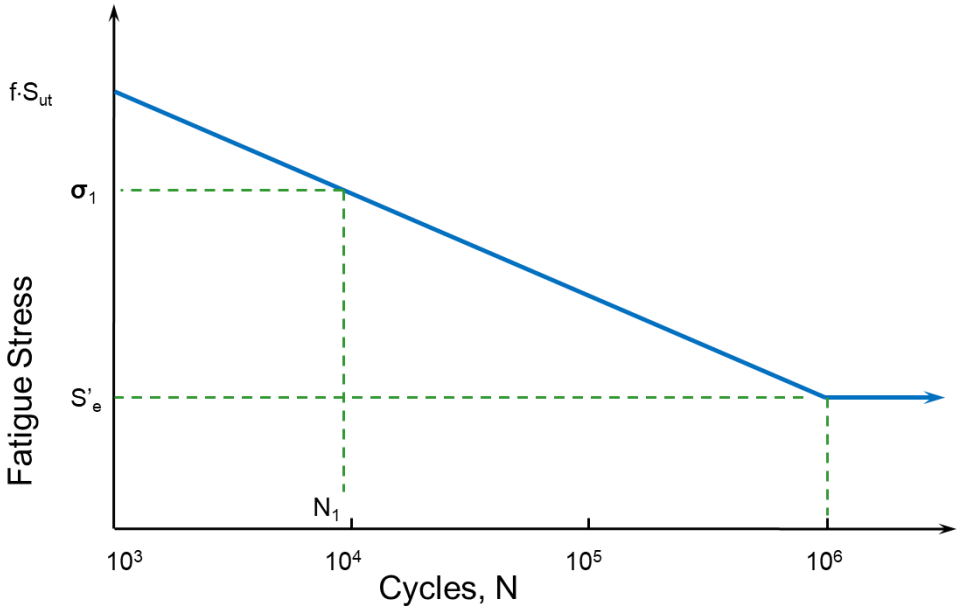
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Endurance Limit Modifying Factors

- Unrealistic to expect Structural or Mechanical Component to Match Laboratory Specimen

- $S_e = k_a k_b k_c k_d k_e k_f S'_e$
  - k_a-surface factor
  - k_b-size factor
  - k_c-load factor
  - k_d-temperature factor
  - k_e-reliability factor
  - k_f-miscellaneous effects factor

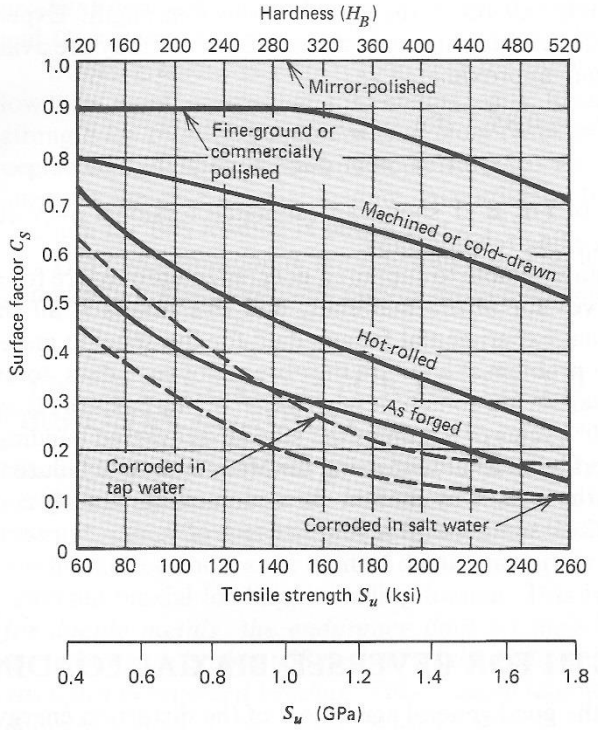


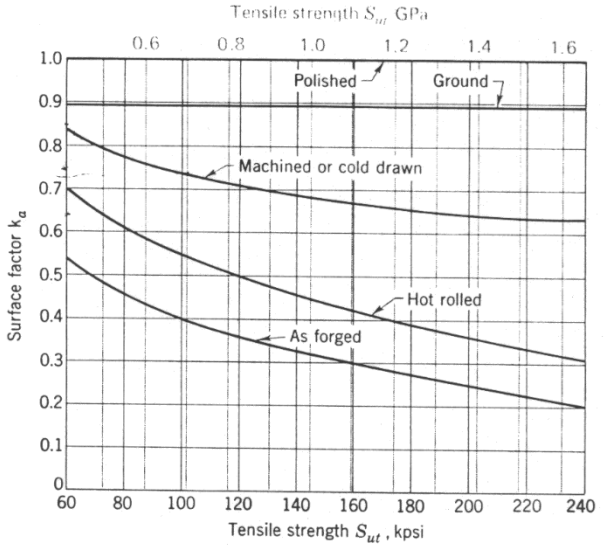
k_a – Surface factor

$$k_a = a \cdot S_{ut}^{b}$$

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

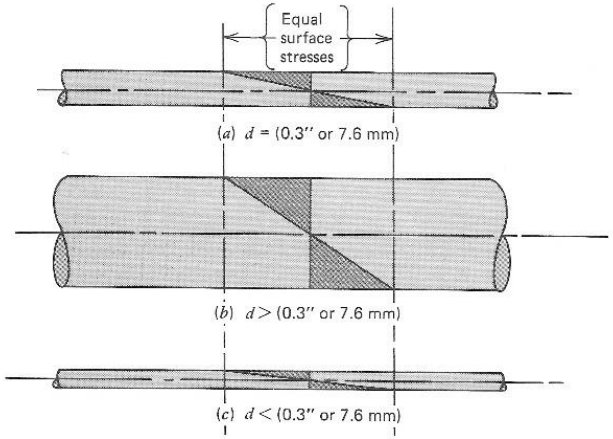
k_a for Ordinary Gray Cast Iron ≅ 1





k_b – Size Effect, Circular X-Sections

- Rotating beam test specimen is 0.3 in in diameter



- For rotating bending and torsion, k_b

$$k_b = \begin{cases} \left(d/0.3\right)^{-0.107} = 0.879 \cdot d^{-0.107} & 0.11in \le d \le 2in \\ 0.91 \cdot d^{-0.157} & 2in \le d \le 10in \\ \\ \left(d/7.62\right)^{-0.107} = 1.24 \cdot d^{-0.107} & 2.79mm \le d \le 51mm \\ 1.51 \cdot d^{-0.157} & 51mm \le d \le 254mm \end{cases}$$

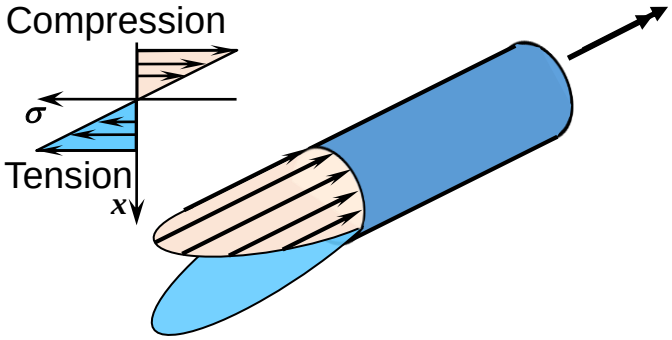
- For Axial loading, k_b=1

k_b – Size Effect, Non-Circular X-Sections

- k_b for NOT round and/or NOT rotating parts
- Equivalent round rotating diameter is (d_e) obtained
- Equating volume of material stressed AT and ABOVE 95% of the maximum rotating-beam stress
- Lengths cancel → Equate Areas
- **ROTATING ROUND** SECTION 95% STRESS AREA

$$A_{0.95\sigma} = \frac{\pi}{4} \cdot \left[d^2 - \left(0.95 \cdot d \right)^2 \right] = 0.0766 \cdot d^2$$

- $d=d_e$

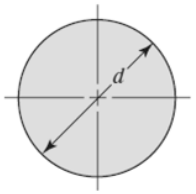


k_b – Size Effect, Non-Circular X-Sections

- For Non-Rotating Round

$$A_{0.95\sigma} = 0.01046 \cdot d^2$$

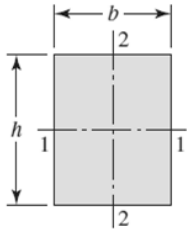
$$d_e = 0.370 \cdot d$$



$$A_{0.95\sigma} = 0.01046 d^2$$
$$d_e = 0.370 d$$

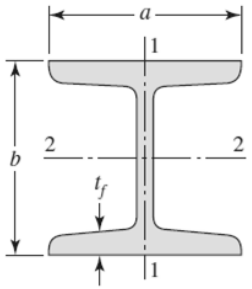
- Non-Rotating Rectangular Section

$$d_e = 0.808 \cdot (b \cdot h)^{1/2}$$

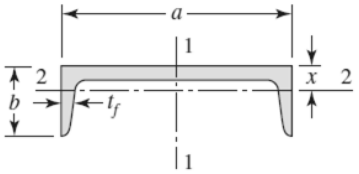


$$A_{0.95\sigma} = 0.05hb$$
$$d_e = 0.808\sqrt{hb}$$

- Other Non-Rotating Sections in Table

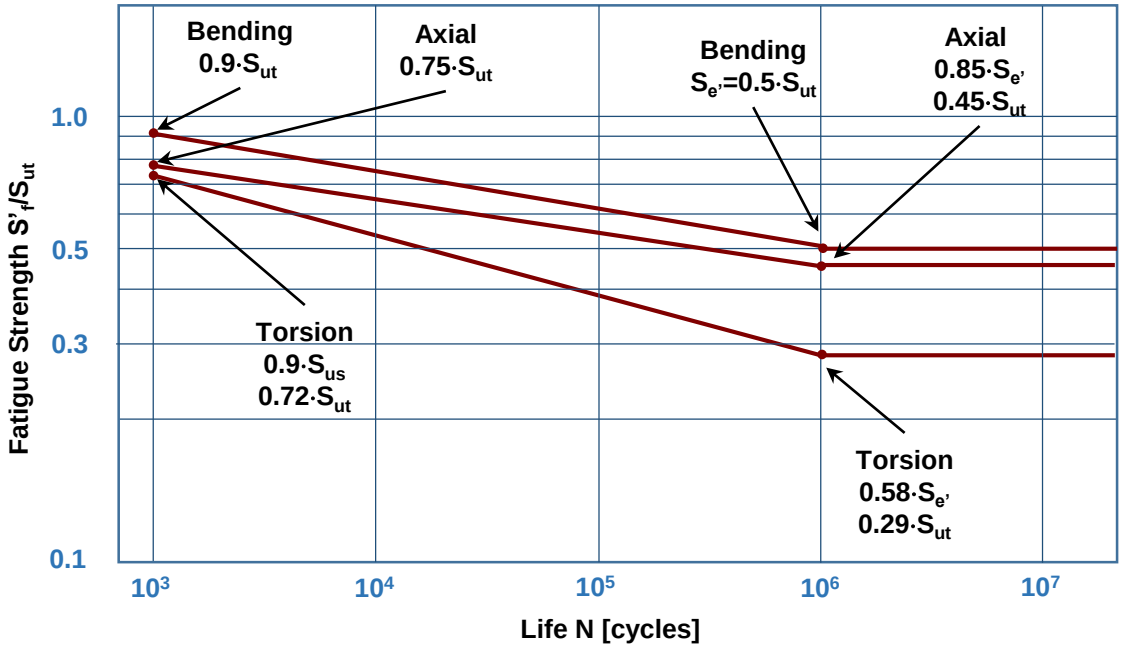


$$A_{0.95\sigma} = \begin{cases} 0.10at_f & \text{axis 1-1} \\ 0.05ba \quad t_f > 0.025a & \text{axis 2-2} \end{cases}$$



$$A_{0.95\sigma} = \begin{cases} 0.05ab & \text{axis 1-1} \\ 0.052xa + 0.1t_f(b-x) & \text{axis 2-2} \end{cases}$$

k_c- Loading Factor



- Bending k_c=1
- Axial k_c=0.85
- Torsion k_c=0.59
  - Distortional Energy Theory
    - S_{us}=0.8·S_u for Steel
    - S_{us}=0.7·S_u for Other Ductile Metals
  - Only for pure torsion
  - Combined loading
    - k_c=1
    - Use Equivalent Von Mises

k_d - Temperature

For Carbon and Alloy Steels, $70°F \le T_F \le 1000°F$

$$k_d = \frac{S_T}{S_{RT}} \equiv \frac{\text{Tensile Strength at Operating Temperature}}{\text{Tensile Strength at Room Temperatue}}$$

$$k_d = 0.975 + 0.432\left(10^{-3}\right)\cdot T_F - 0.115\left(10^{-5}\right)\cdot T_F^2 + 0.104\left(10^{-8}\right)\cdot T_F^3 - 0.595\left(10^{-12}\right)\cdot T_F^4$$

Temperature, °C	S_T/S_{RT}	Temperature, °F	S_T/S_{RT}
20	1.000	70	1.000
50	1.010	100	1.008
100	1.020	200	1.020
150	1.025	300	1.024
200	1.020	400	1.018
250	1.000	500	0.995
300	0.975	600	0.963
350	0.943	700	0.927
400	0.900	800	0.872
450	0.843	900	0.797
500	0.768	1000	0.698
550	0.672	1100	0.567
600	0.549		

k_e - Reliability

$$k_e = 1 - 0.08 \cdot z_a$$

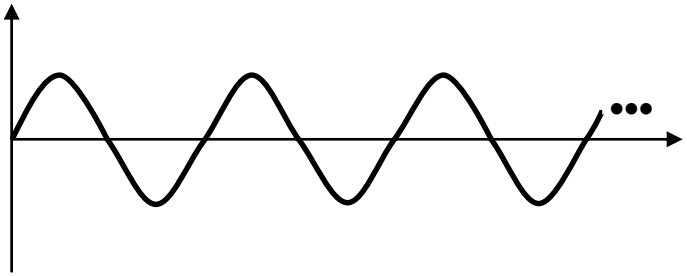
Reliability, R	Standardized Variable, z_R	k_c
0.50	0	1.000
0.90	1.288	0.897
0.95	1.645	0.868
0.99	2.326	0.814
0.999	3.091	0.753
0.999 9	3.719	0.702
0.999 99	4.265	0.659
0.999 999	4.753	0.620
0.999 999 9	5.199	0.584
0.999 999 99	5.612	0.551
0.999 999 999	5.997	0.520

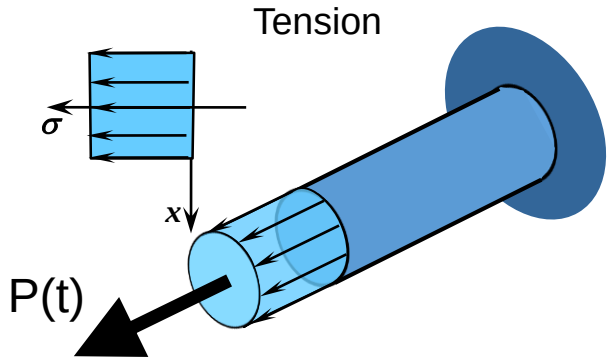
k_f – Miscellaneous – Effects Factors

- Residual Stresses
  - Compressive surface residual stresses improves endurance limit
- Corrosion
  - Corrosion and stressing occur at the same time
  - Try to minimize factors the affect fatigue: mean or static stress, alternating stress, electrolyte concentration, dissolved oxygen in electrolyte, material properties and composition, temperature, cyclic frequency, fluid flow rate around specimen, and local crevices
- Electrolytic Plating
  - Chromium plating, nickel plating, and cadmium plating REDUCE the endurance limit
  - Zinc plating does not affect the fatigue strength
  - Anodic oxidation of light alloys reduces bending endurance limit, but not torsional
- Metal Spraying
- Cyclic Frequency
  - When corrosion or high temperatures are encourntered, cyclic frequency impact crack growth rate
- Frettage Corrosion

Example 1

Using the empirical relationships given in this section, estimate the S-N curve and a family of constant-life fatigue curves (10^3, 10^4, 10^5, 10^6) pertaining to the axial loading of the precision steel parts having S_{ut}= 150 ksi, S_y=120 ksi, and commercially polished surfaces. All cross-section dimensions are to be under 2 in.





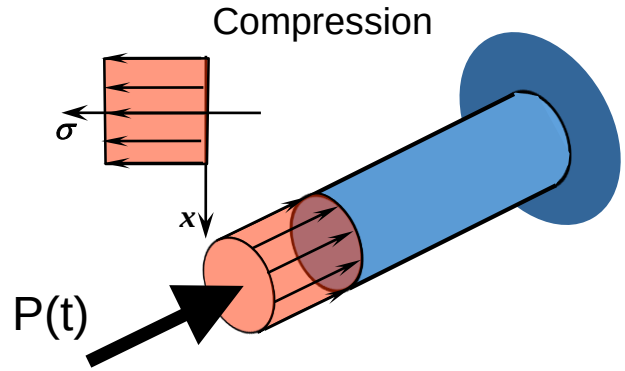


Table A–24

Mechanical Properties of Three Non-Steel Metals

(a) Typical Properties of Gray Cast Iron

[The American Society for Testing and Materials (ASTM) numbering system for gray cast iron is such that the numbers correspond to the *minimum tensile strength* in kpsi. Thus an ASTM No. 20 cast iron has a minimum tensile strength of 20 kpsi. Note particularly that the tabulations are *typical* of several heats.]

ASTM Number	Tensile Strength S_{ut}, kpsi	Compressive Strength S_{uc}, kpsi	Shear Modulus of Rupture S_{su}, kpsi	Modulus of Elasticity, Mpsi		Endurance Limit* S_e', kpsi	Brinell Hardness H_B	Fatigue Stress-Concentration Factor K_f
				Tension†	Torsion			
20	22	83	26	9.6–14	3.9–5.6	10	156	1.00
25	26	97	32	11.5–14.8	4.6–6.0	11.5	174	1.05
30	31	109	40	13–16.4	5.2–6.6	14	201	1.10
35	36.5	124	48.5	14.5–17.2	5.8–6.9	16	212	1.15
40	42.5	140	57	16–20	6.4–7.8	18.5	235	1.25
50	52.5	164	73	18.8–22.8	7.2–8.0	21.5	262	1.35
60	62.5	187.5	88.5	20.4–23.5	7.8–8.5	24.5	302	1.50

*Polished or machined specimens.

†The modulus of elasticity of cast iron in compression corresponds closely to the upper value in the range given for tension and is a more constant value than that for tension.

Table A–24

Mechanical Properties of Three Non-Steel Metals (*Continued*)

(b) Mechanical Properties of Some Aluminum Alloys

[These *are typical* properties for sizes of about $\frac{1}{2}$ in; similar properties can be obtained by using proper purchase specifications. The values given for fatigue strength correspond to $50(10^7)$ cycles of completely reversed stress. Alluminum alloys do not have an endurance limit. Yield strengths were obtained by the 0.2 percent offset method.]

Aluminum Association Number	Temper	Strength			Elongation in 2 in, %	Brinell Hardness H_B
		Yield, S_y, MPa (kpsi)	Tensile, S_u, MPa (kpsi)	Fatigue, S_f, MPa (kpsi)		
Wrought:						
2017	O	70 (10)	179 (26)	90 (13)	22	45
2024	O	76 (11)	186 (27)	90 (13)	22	47
	T3	345 (50)	482 (70)	138 (20)	16	120
3003	H12	117 (17)	131 (19)	55 (8)	20	35
	H16	165 (24)	179 (26)	65 (9.5)	14	47
3004	H34	186 (27)	234 (34)	103 (15)	12	63
	H38	234 (34)	276 (40)	110 (16)	6	77
5052	H32	186 (27)	234 (34)	117 (17)	18	62
	H36	234 (34)	269 (39)	124 (18)	10	74
Cast:						
319.0*	T6	165 (24)	248 (36)	69 (10)	2.0	80
333.0†	T5	172 (25)	234 (34)	83 (12)	1.0	100
	T6	207 (30)	289 (42)	103 (15)	1.5	105
335.0*	T6	172 (25)	241 (35)	62 (9)	3.0	80
	T7	248 (36)	262 (38)	62 (9)	0.5	85

Table A–24

Mechanical Properties of Three Non-Steel Metals (*Continued*)

(b) Mechanical Properties of Some Aluminum Alloys

[These *are typical* properties for sizes of about $\frac{1}{2}$ in; similar properties can be obtained by using proper purchase specifications. The values given for fatigue strength correspond to $50(10^7)$ cycles of completely reversed stress. Alluminum alloys do not have an endurance limit. Yield strengths were obtained by the 0.2 percent offset method.]

(c) Mechanical Properties of Some Titanium Alloys

Titanium Alloy	Condition	Yield, S_y (0.2% offset) MPa (kpsi)	Strength Tensile, S_{ut} MPa (kpsi)	Elongation in 2 in, %	Hardness (Brinell or Rockwell)
Ti-35A†	Annealed	210 (30)	275 (40)	30	135 HB
Ti-50A†	Annealed	310 (45)	380 (55)	25	215 HB
Ti-0.2 Pd	Annealed	280 (40)	340 (50)	28	200 HB
Ti-5 Al-2.5 Sn	Annealed	760 (110)	790 (115)	16	36 HRC
Ti-8 Al-1 Mo-1 V	Annealed	900 (130)	965 (140)	15	39 HRC
Ti-6 Al-6 V-2 Sn	Annealed	970 (140)	1030 (150)	14	38 HRC
Ti-6Al-4V	Annealed	830 (120)	900 (130)	14	36 HRC
Ti-13 V-11 Cr-3 Al	Sol. + aging	1207 (175)	1276 (185)	8	40 HRC

†Commercially pure alpha titanium.