

CAMs

☐ **Roller Followers**

☐ **Flat Faced Followers**

Translating Follower CAM Sizing

Prime Circle

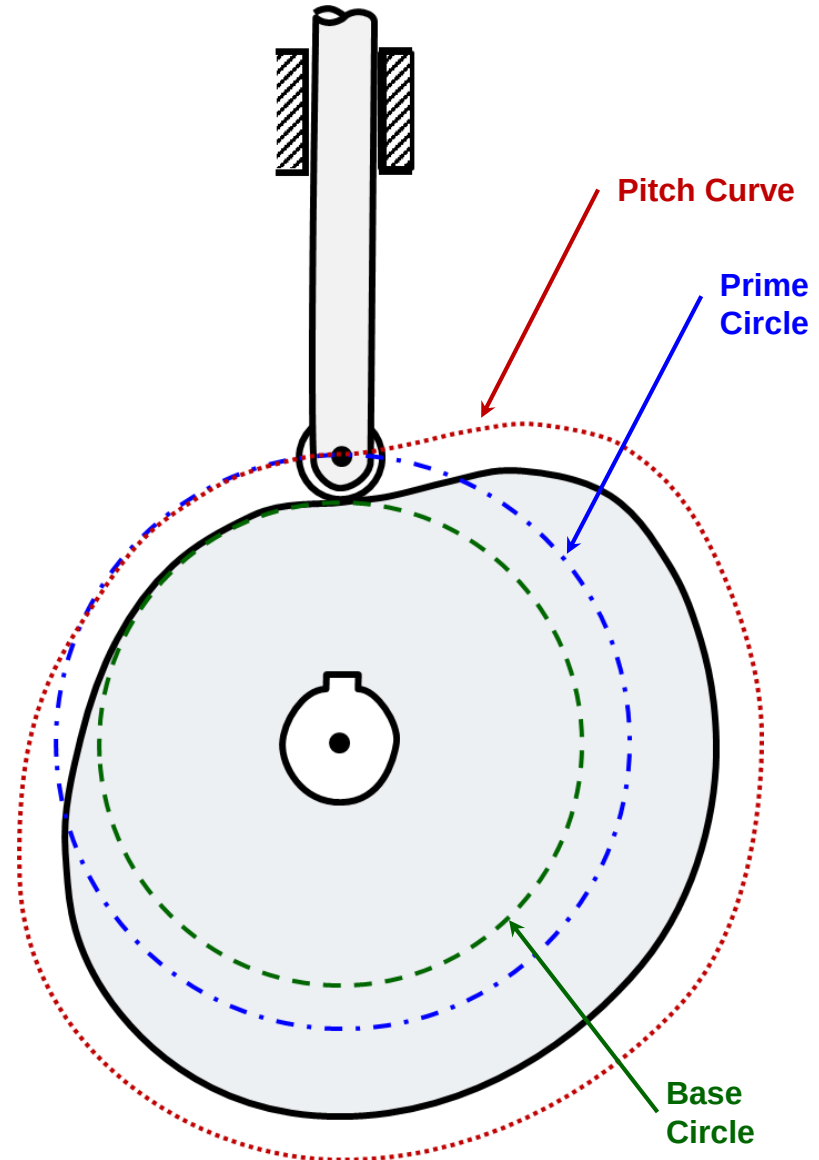
- Smallest circle which can be drawn tangent to the locus of the centerline of the follower
- Related to the **Pitch Curve**, locus of the centerline of the follower
- CAMs with **Roller** (or **Radius**) **Followers** are defined with respect to the **Pitch Curve**

Base Circle radius

- **FLAT FACED** Followers
- Smallest circle which can be drawn tangent to the physical CAM Surface

Major Factors affecting Cam Size

- Pressure Angle
- Radius of Curvature

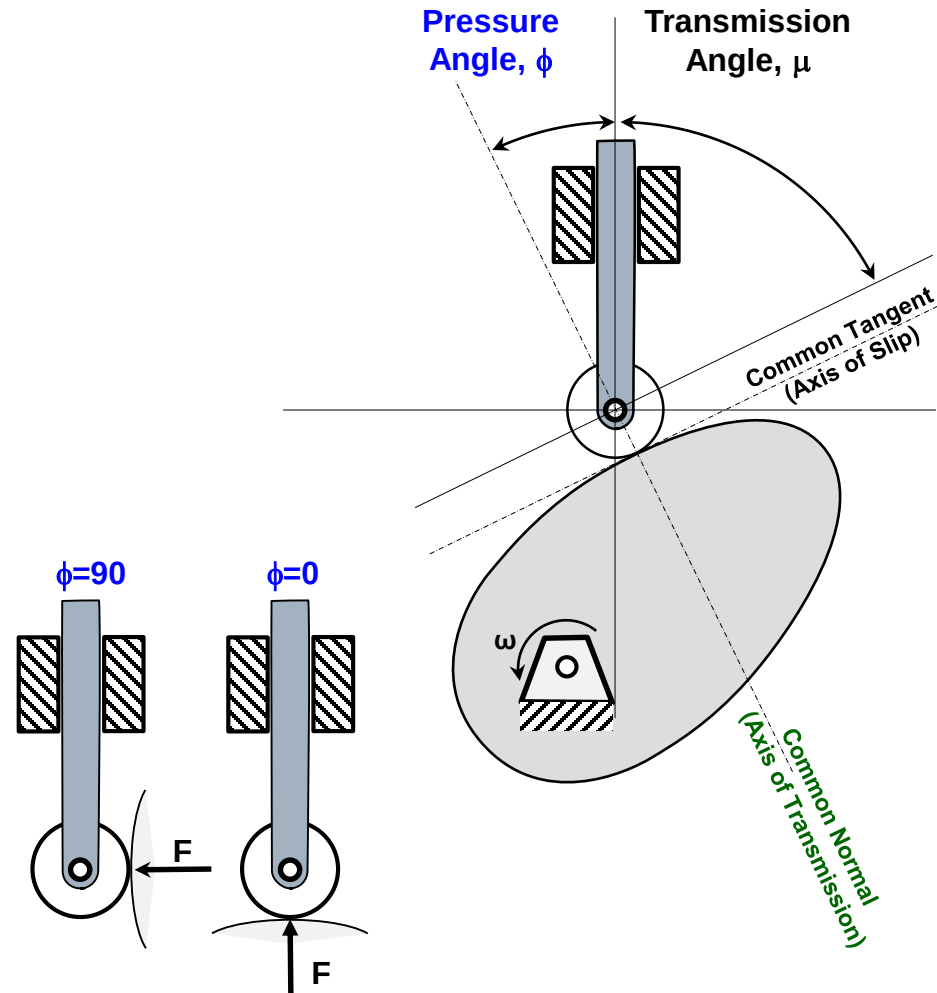


Roller (Radius) Follower:

PRESSURE ANGLE

PRESSURE ANGLE ϕ : Angle between the direction of motion (velocity) of the follower and the direction of the **Common Normal (Axis of Transmission)**

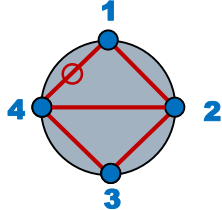
- Only valid for single DoF systems
- Extremes
 - $\phi=90^\circ$ no follower motion
 - $\phi=0^\circ$ all transmission force goes into motion of the follower
- Rule of Thumb
 - Translating Followers $0^\circ \leq \phi \leq 30^\circ$
 - Oscillating Followers $0^\circ \leq \phi \leq 35^\circ$



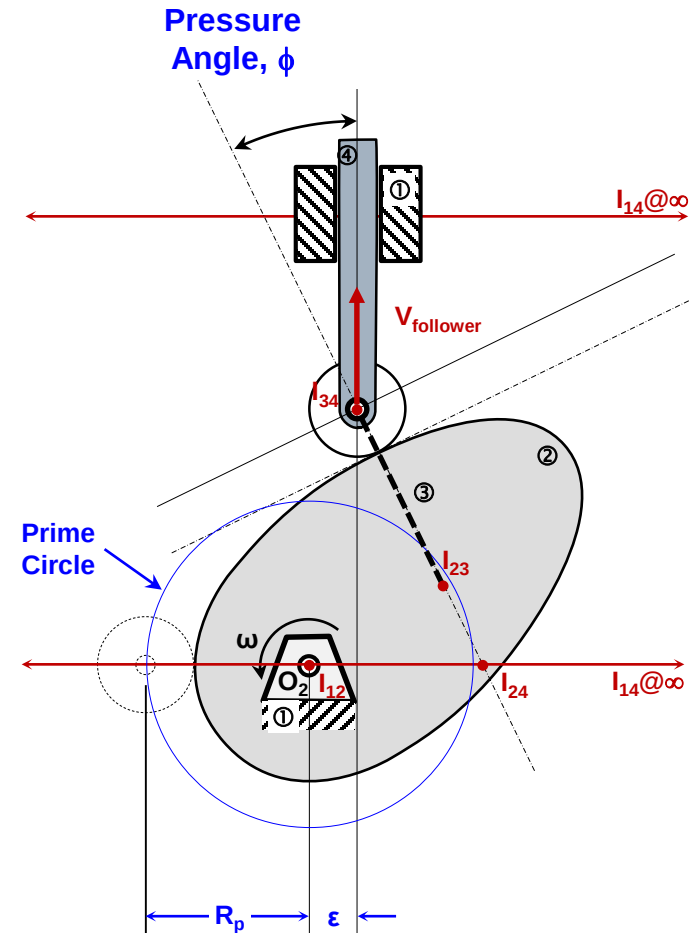
Calculating the Pressure Angle: Roller Follower Instant Center

Consider the instant centers

- Roller is a Passive Degree of Freedom
- Contact between the Roller and CAM is a Half-Joint
- Follower offset ϵ from CAM center



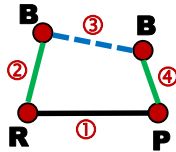
I_{24} is where the velocity of the CAM and follower have the same velocity (magnitude and direction)



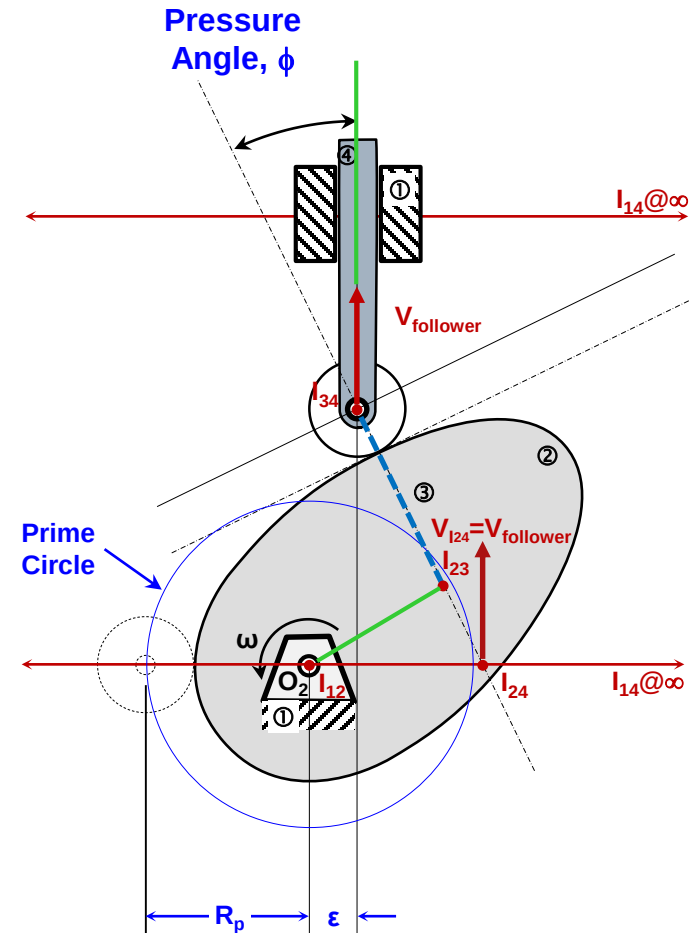
Calculating the Pressure Angle: Linkage Isomer

The Isomer for this mechanism can be drawn

- Four Bar (Catalog: $M=+1$, $B=4$, I)



- ① is the ground link
- ② is the CAM
- ③ is the imaginary link between the centers of curvature at the contact point
- ④ is the translating follower



Calculating the Pressure Angle: Finding Lowest Position of CAM $d(R_p, \varepsilon)$

Given the CAM angular velocity ω and eccentricity ε

- The velocity of I_{24} can be calculated

$$V_{I_{24}} = b \cdot \omega = \dot{s}$$

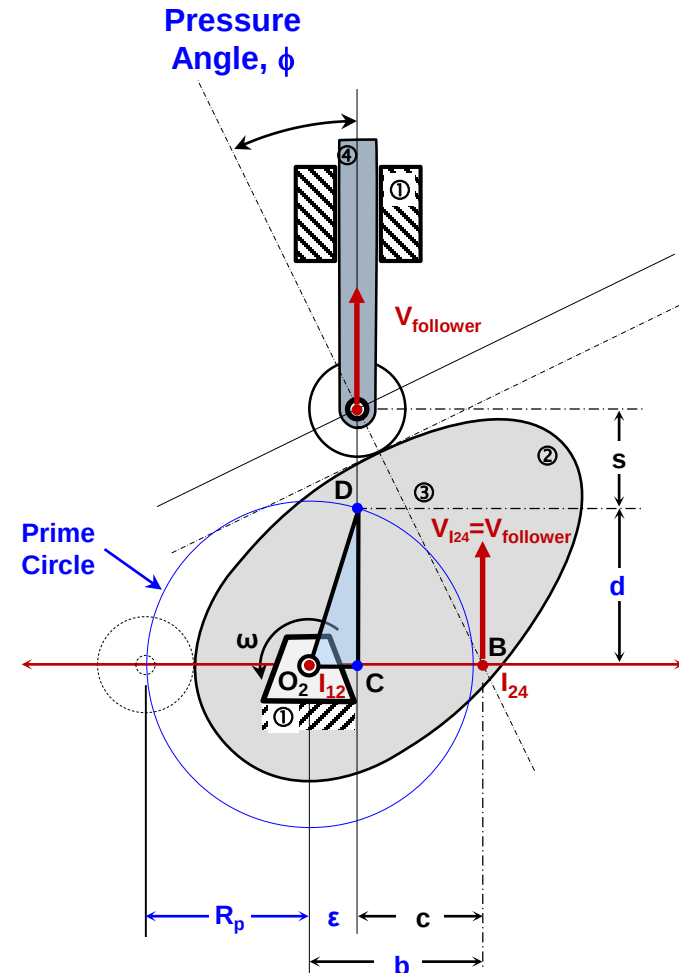
- b is determined knowing the relationship between s and its derivatives in the time and spatial domains.

$$b \cdot \omega = \dot{s} = s' \cdot \omega \Rightarrow b = s'$$

- From Geometry

$$d = \sqrt{R_p^2 - \varepsilon^2}$$

The lowest position allowed by the CAM,
This value does not change



Calculating the Pressure Angle ϕ

Once again using geometry

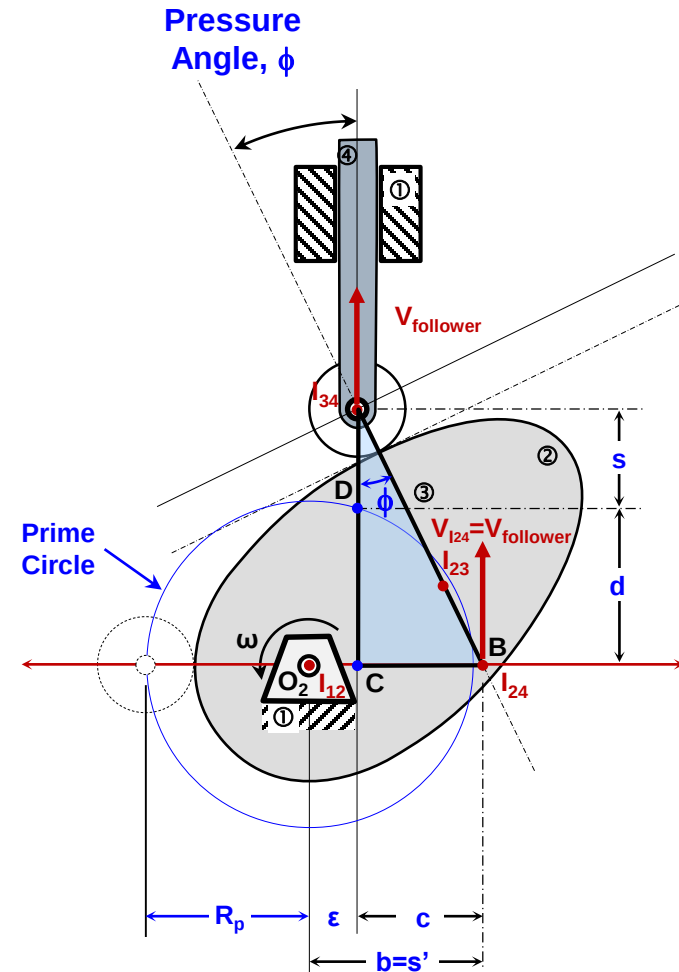
$$\phi = \tan^{-1}\left(\frac{c}{s+d}\right) = \tan^{-1}\left(\frac{s' - \varepsilon}{s + \sqrt{R_p^2 - \varepsilon^2}}\right)$$

s and s' are both functions of θ .

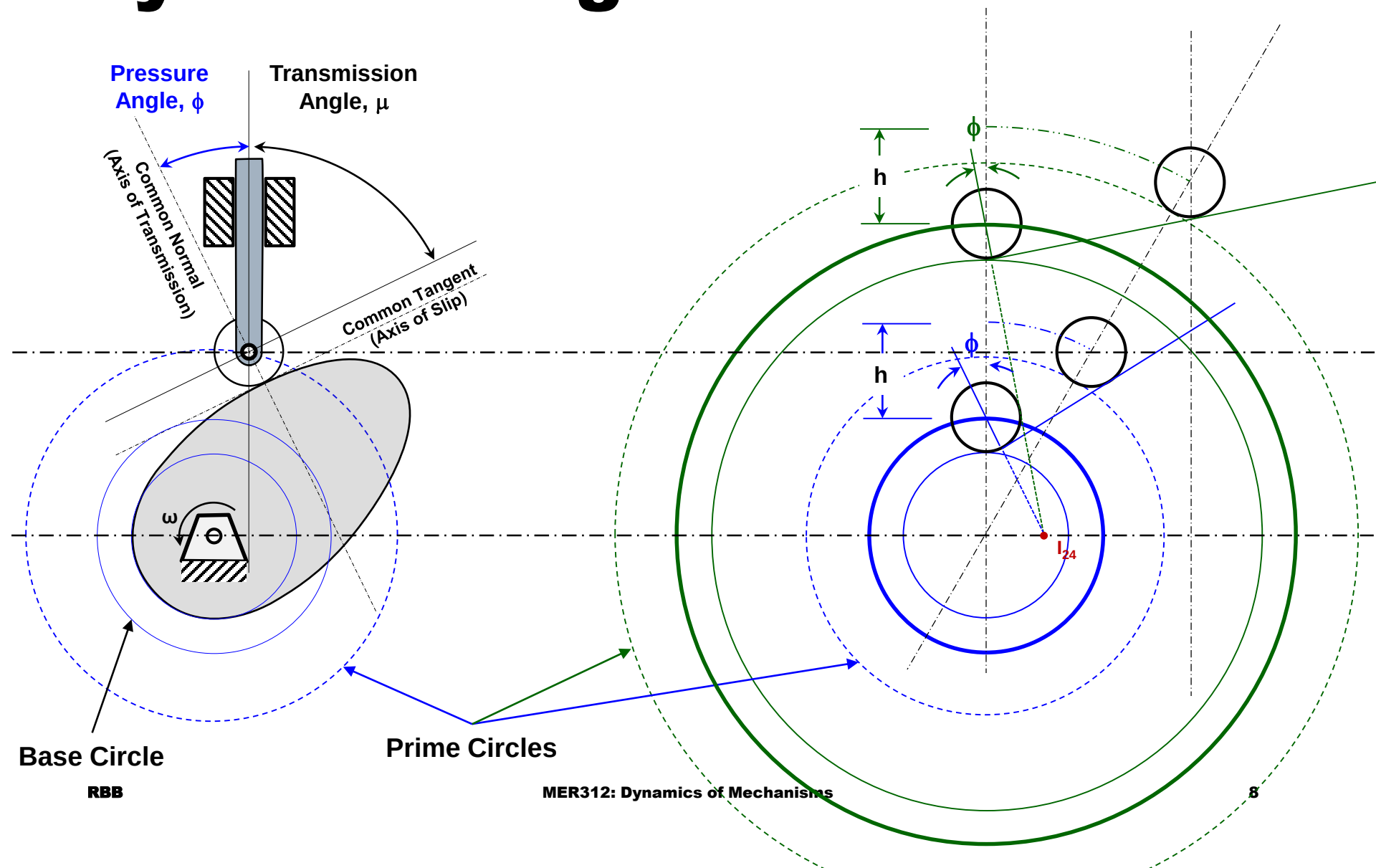
The x-y coordinate of the center of the follower is given by

$$\chi = \mathcal{E}$$

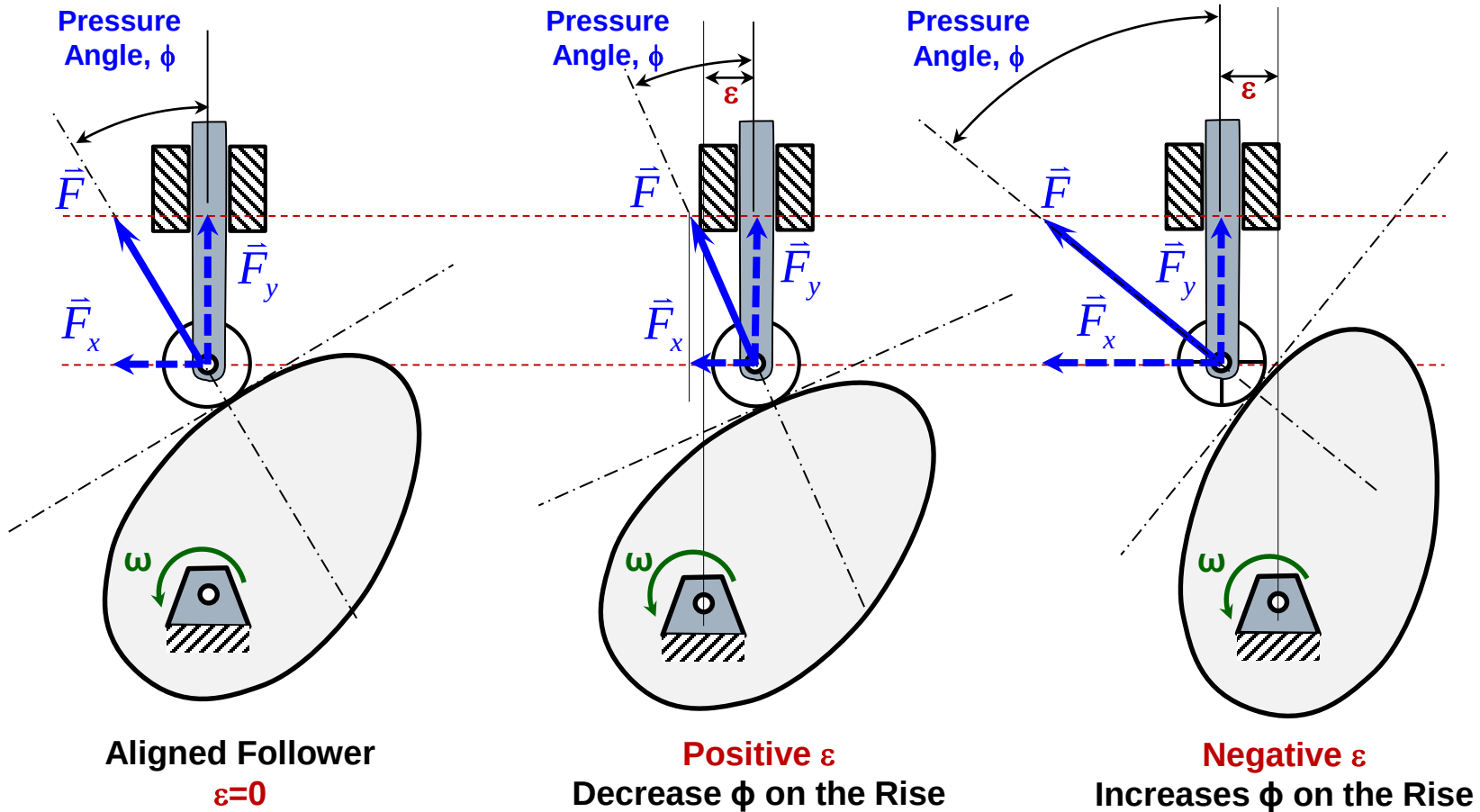
$$y = s + d = s + \sqrt{R_p^2 - \varepsilon^2}$$



Decreasing **PRESSURE ANGLE** by Increasing the Prime Circle

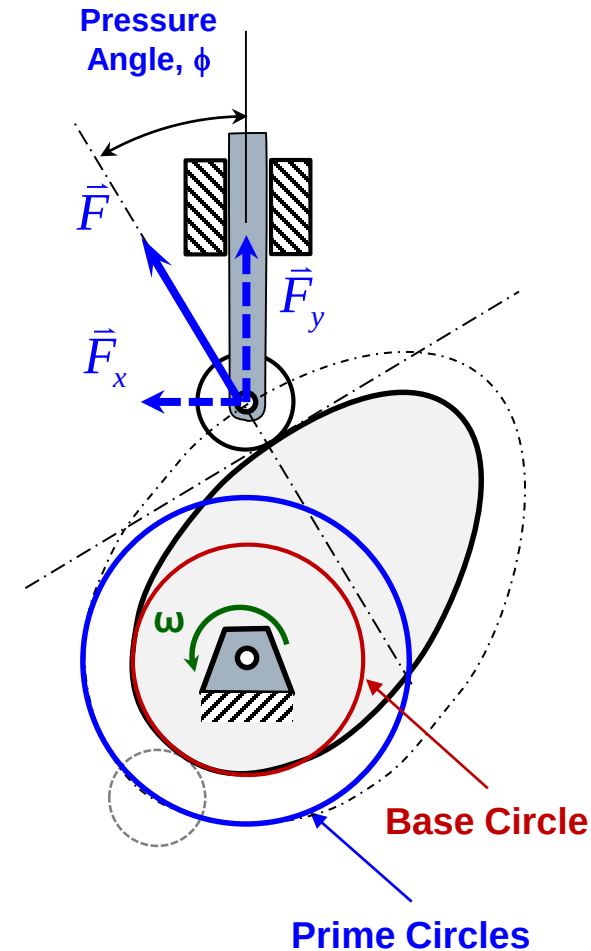


ECCENTRICITY ϵ Affect on Pressure Angle



Summary of Strategies to Reducing the Pressure Angle

1. Increase the Diameter of the Prime Circle
2. Decrease the total rise of the follower
3. Increase the amount of cam rotation for a given follower displacement
4. Change the type of follower motion
 - a. Harmonic
 - b. Polynomial
 - c. Other
5. Change the amount of Eccentricity, ε
 - a. Positive: **decrease** pressure angle on the rise, will **increase** it on the fall
 - b. Negative: **increase** pressure angle on the rise, will **decrease** it on the fall
 - c. Balance asymmetric pressure angle curves



Radius of Curvature Calculation Assists in Roller Sizing

$$r = r(\theta)$$

$$\rho = \frac{\left[r^2 + (r')^2 \right]^{3/2}}{\left| r^2 + 2 \cdot (r')^2 - r \cdot r'' \right|}$$

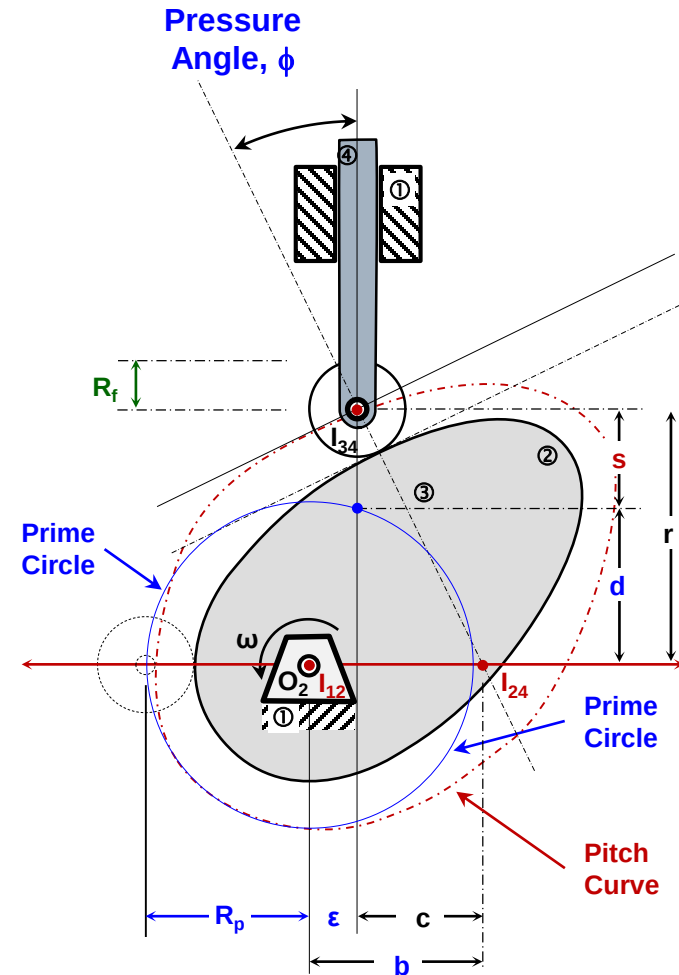
$$\rho_{pitch} = \frac{\left[\left(R_p + s \right)^2 + s'^2 \right]^{3/2}}{\left(R_p + s \right)^2 + 2 \cdot s'^2 - s'' \cdot \left(R_p + s \right)}$$

$\equiv$ Radius of Curvature of the Pitch Curve

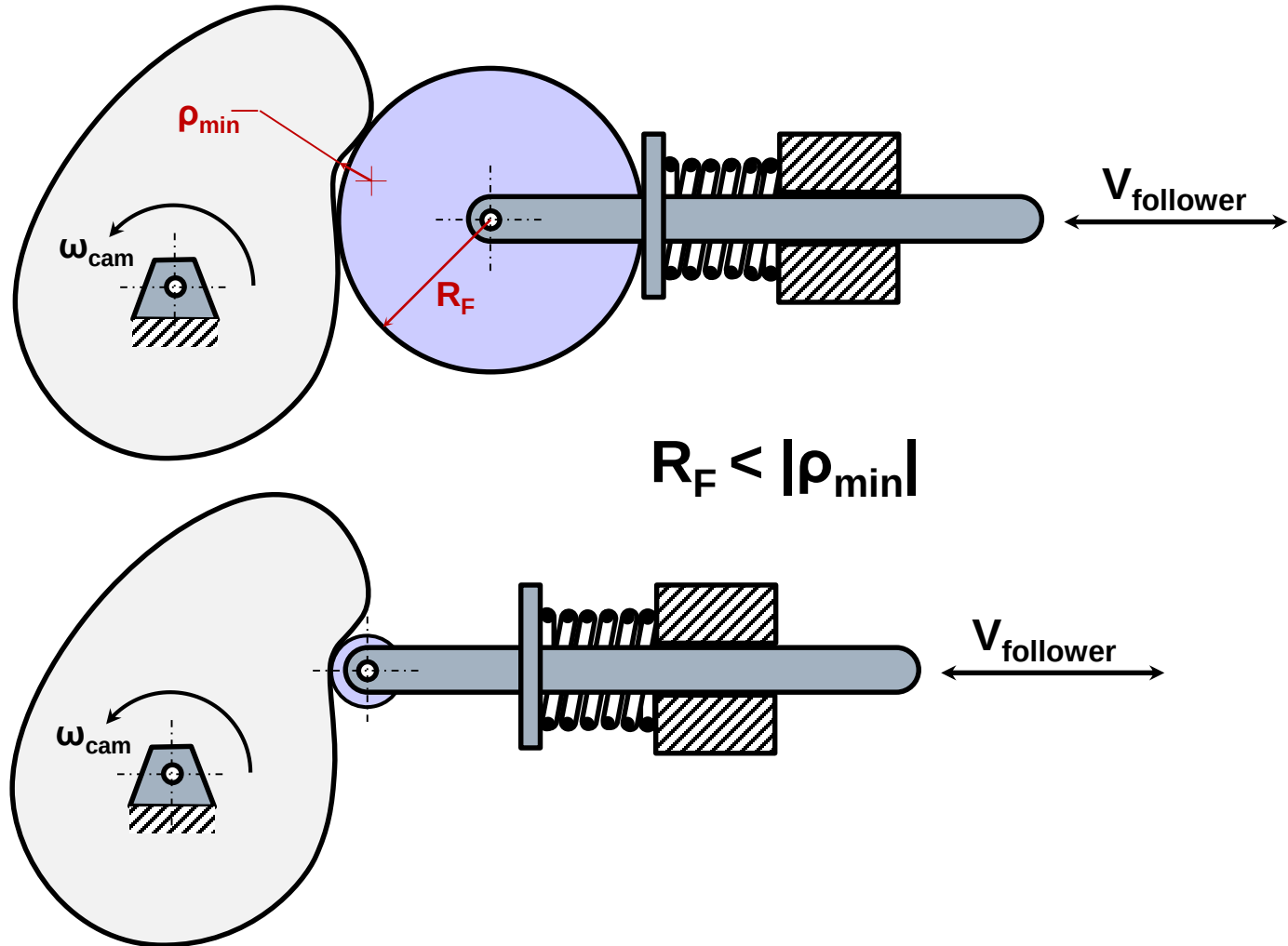
Rule of Thumb

$$|\rho_{\min}| \gg R_f$$

ρ At Least 2 to 3 Times R_f



Too Large a Follower for Concave Cam Surface

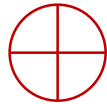


Follower Radius Affect on Convex Pitch Curve Surfaces

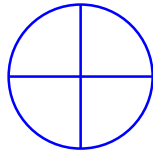
Consider the desired Pitch Curve shown.

Three radii for a translating roller follower are considered.

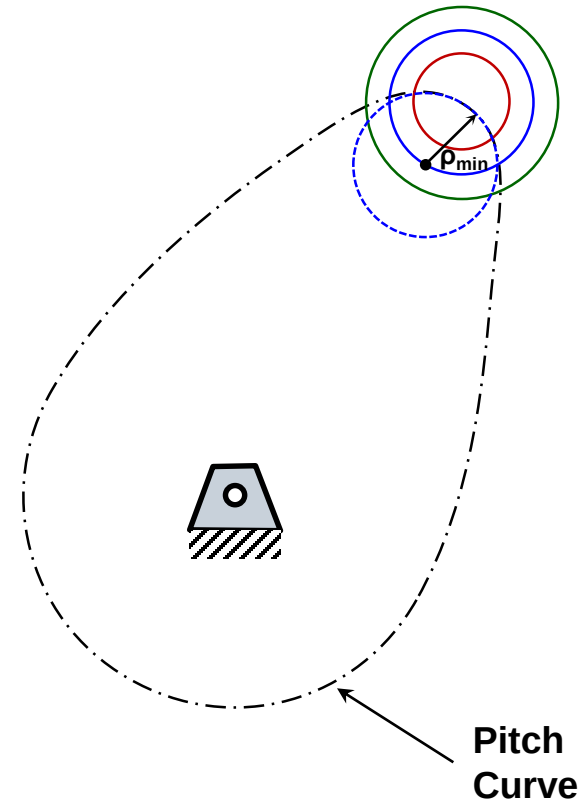
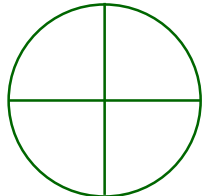
- $R_F = 0.5 \rho_{\min}$



- $R_F = \rho_{\min}$

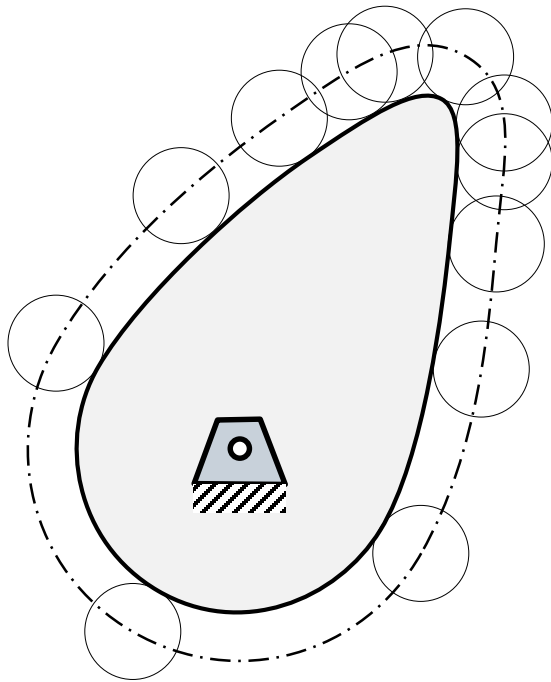


- $R_F = 1.5 \rho_{\min}$



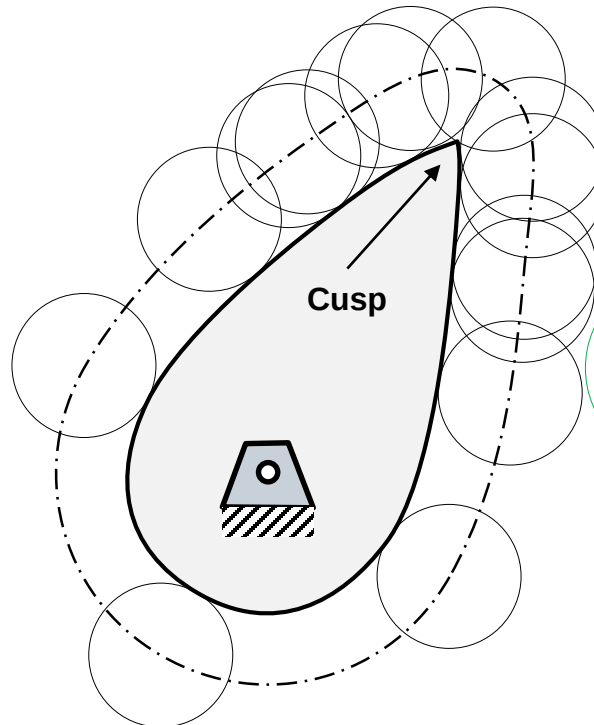
The Pitch Curve and Cam Surface Differ with Radius

$$R_F = 0.5 \rho_{\min}$$



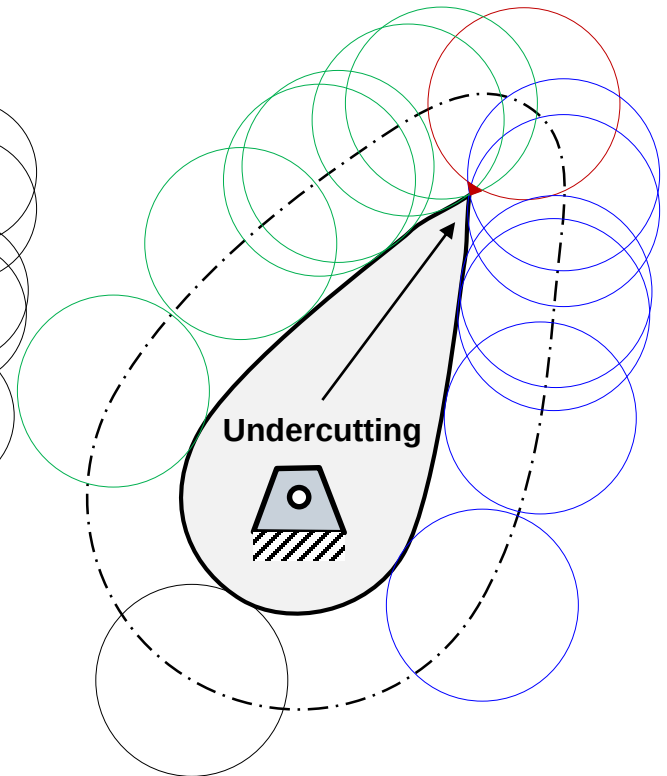
RBB

$$R_F = \rho_{\min}$$



MER312: Dynamics of Mechanisms

$$R_F = 1.5 \rho_{\min}$$

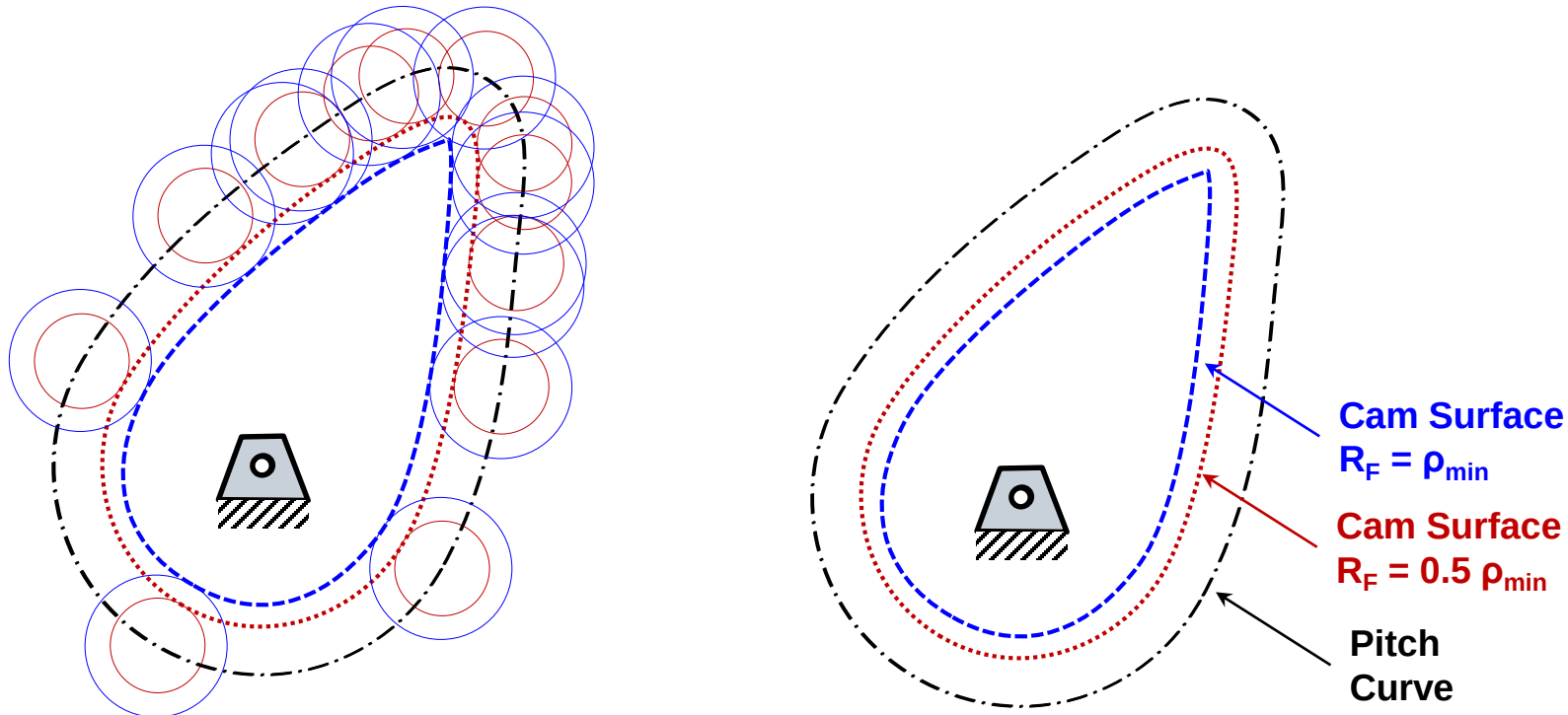


14

For $R_F \leq \rho_{\min}$ A CAM Surface is Produced that Traces Pitch Curve

Working Curve surfaces for $R_F = \rho_{\min}$ are not robust

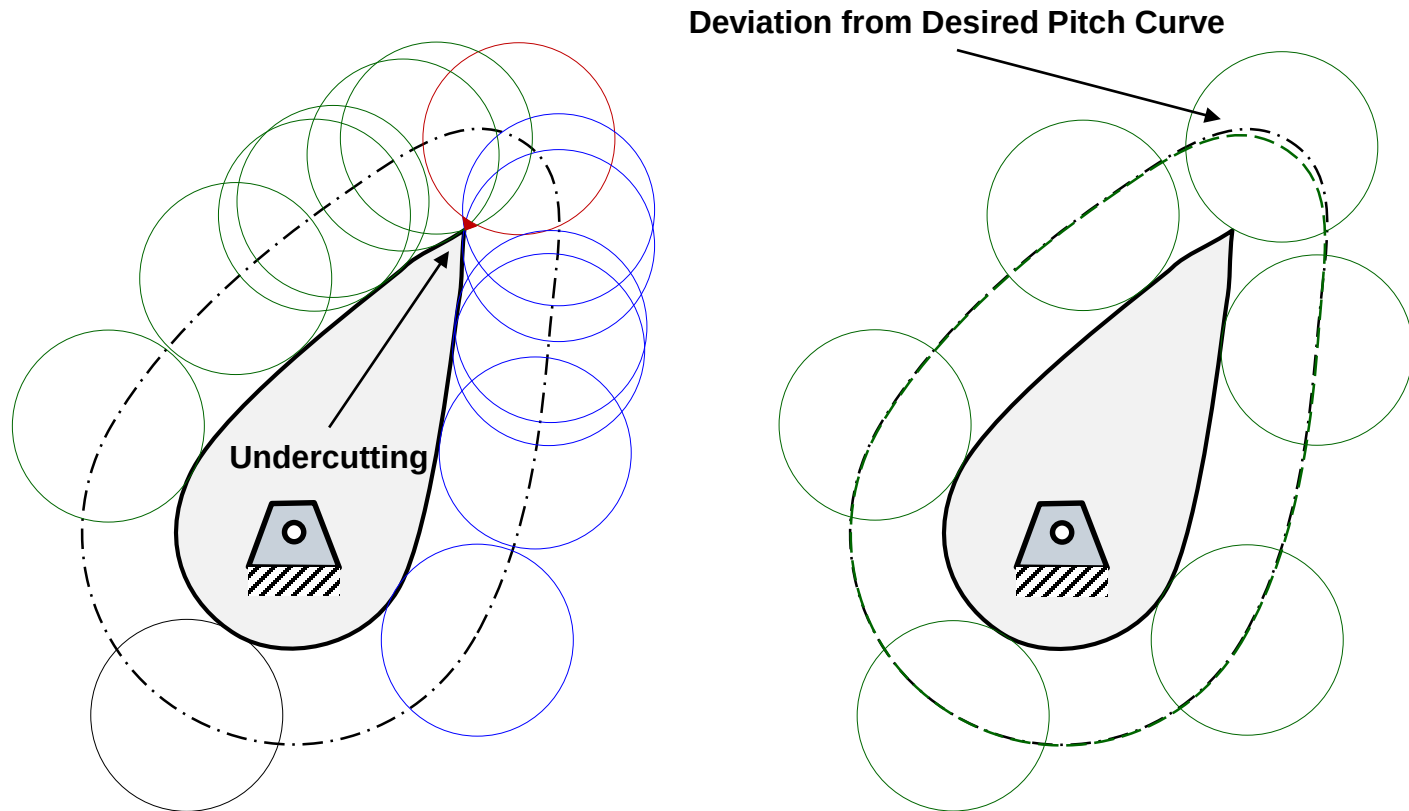
- High stresses at Cusp
- Cusp will wear rapidly



For $R_F > \rho_{\min}$ The CAM Surface will not Produced the Pitch Curve

Undercutting will occur on Working Curve when $R_F > \rho_{\min}$

- **Manufactured surface will not allow roller to trace Pitch Curve**



Pitch Curve Coordinates Used for Machining

Roller-Translating Follower Pitch Curve
Coordinates used for manufacturing CAM

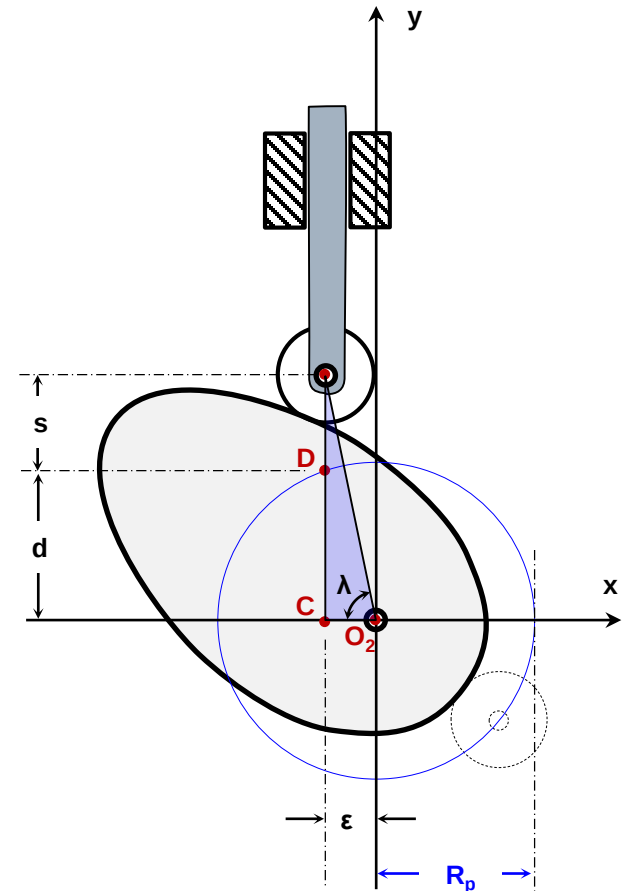
- $2\pi - \theta$ accounts for CAM being fixed and tool moving
 - followers path around stationary CAM

$$\lambda = (2\pi - \theta) - \tan^{-1} \left(\frac{\varepsilon}{d + s} \right)$$

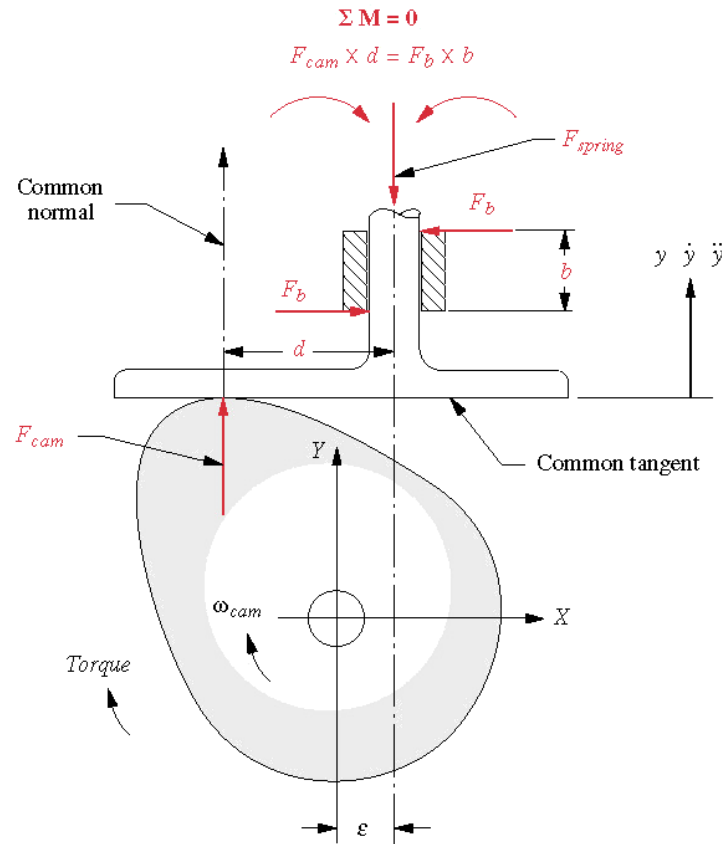
- Coordinates of the center of the tool radius

$$x = \cos \lambda \cdot \sqrt{(d + s)^2 + \varepsilon^2}$$

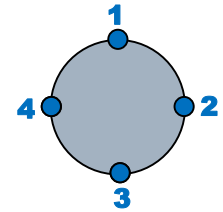
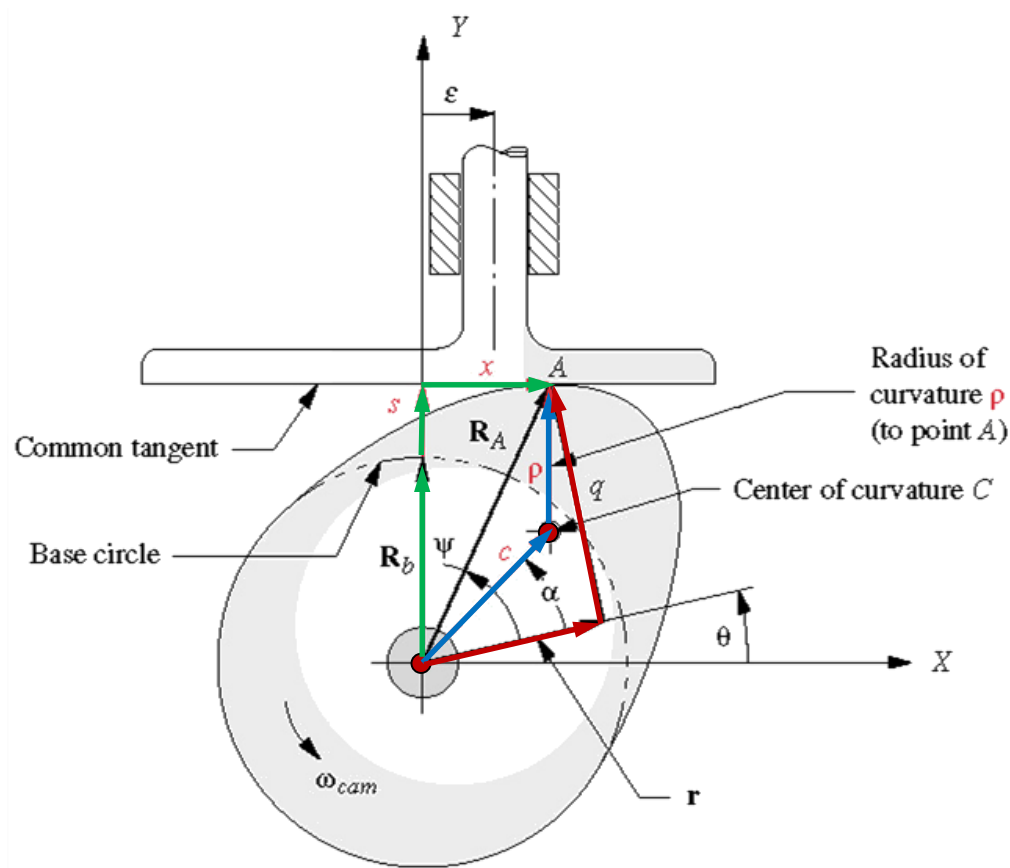
$$y = \sin \lambda \cdot \sqrt{(d + s)^2 + \varepsilon^2}$$



Flat Faced Follower



Determining the Shape of the Cam



$$\frac{ds/dt}{r_c \cdot \omega} = \frac{q}{r_c}$$

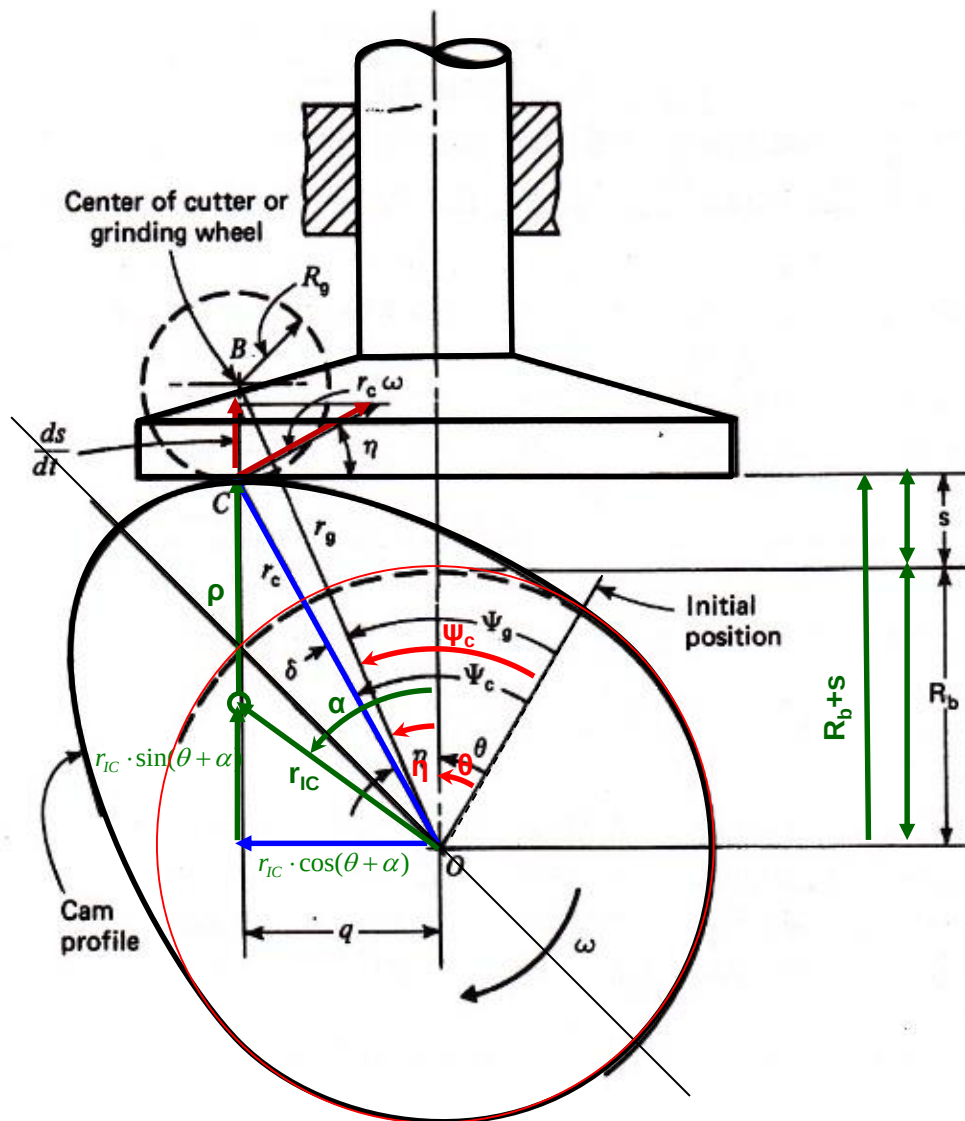
$$q = \frac{ds/dt}{\omega} = \frac{\dot{s}}{\omega} = s' = \frac{ds}{d\theta}$$

$$r_c = \left[(R_b + s)^2 + q^2 \right]^{1/2}$$

$$\tan \eta = \frac{q}{R_b + s}$$

$$\text{Rise: } \psi_c = \theta + \eta$$

$$\text{Fall: } \psi_c = \theta - \eta$$



Determining Face Width

Using Similar Triangles

$$\frac{ds/dt}{r_c \cdot \omega} = \frac{x}{r_c}$$

Therefore, x can be related to s'

$$x = \frac{ds/dt}{\omega} = \frac{\dot{s}}{\omega} = s' = \frac{\partial s}{\partial \theta}$$

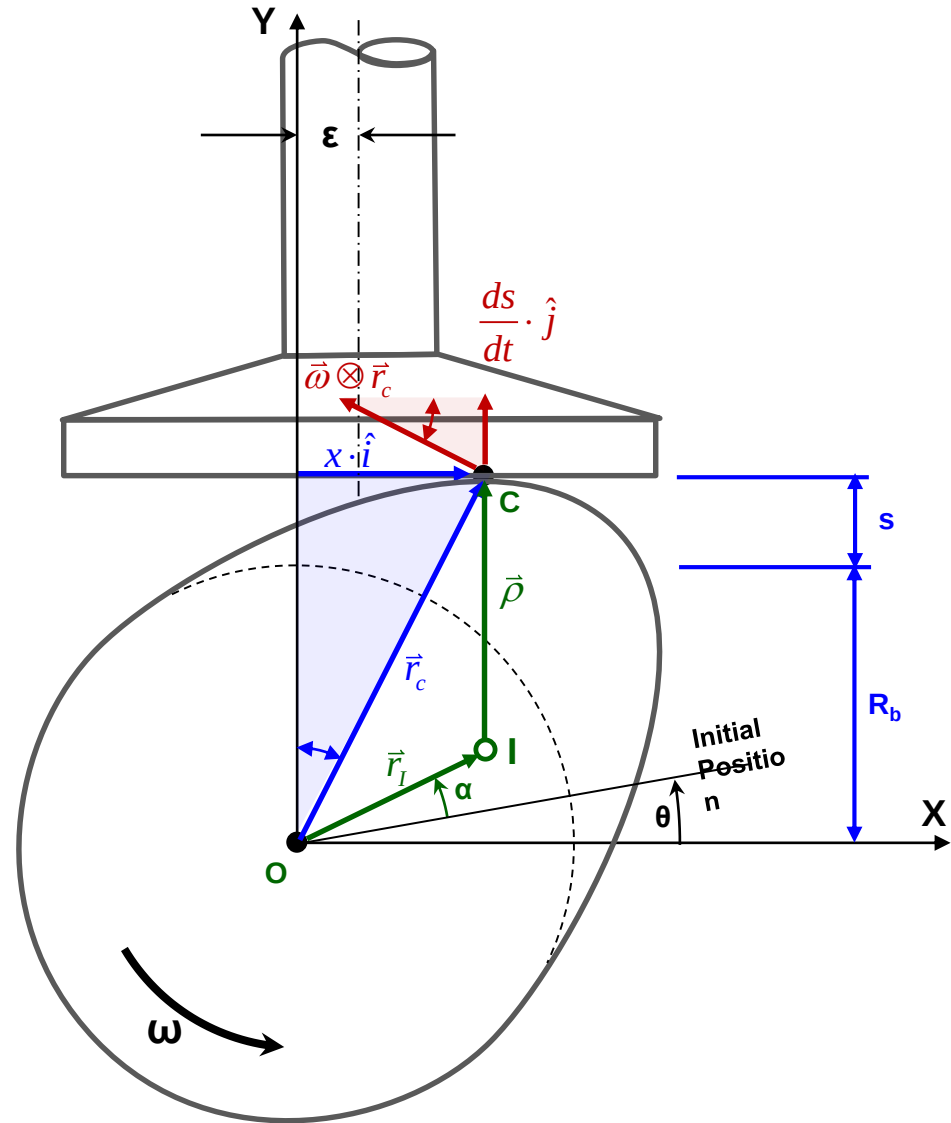
$$x = s' \quad \textcircled{1}$$

From the s' diagram the width of the CAM can be computed

$$\begin{aligned} \text{facewidth} &> x_{\max} - x_{\min} \\ &> s'_{\max} - s'_{\min} \end{aligned}$$

Taking the derivative of $\textcircled{1}$

$$\frac{\partial x}{\partial \theta} = \frac{\partial s'}{\partial \theta} \Rightarrow x' = s'' \quad \textcircled{2}$$



Determining Radius of Curvature

Vector Loop for OC

$$x \cdot \hat{i} + (R_b + s) \cdot \hat{j} = r_I \cdot \cos(\theta + \alpha) \cdot \hat{i} + [r_I \cdot \sin(\theta + \alpha) + \rho] \cdot \hat{j}$$

Resolving the components (C's coordinates)

$$x = r_I \cdot \cos(\theta + \alpha) \quad (3)$$

$$y = R_b + s = r_I \cdot \sin(\theta + \alpha) + \rho \quad (4)$$

I is stationary, for small θ ,
 r_I , ρ , and α are constant.
 Taking the derivative of (3) with respect to θ .

$$\frac{\partial x}{\partial \theta} = x' = -r_I \cdot \sin(\theta + \alpha)$$

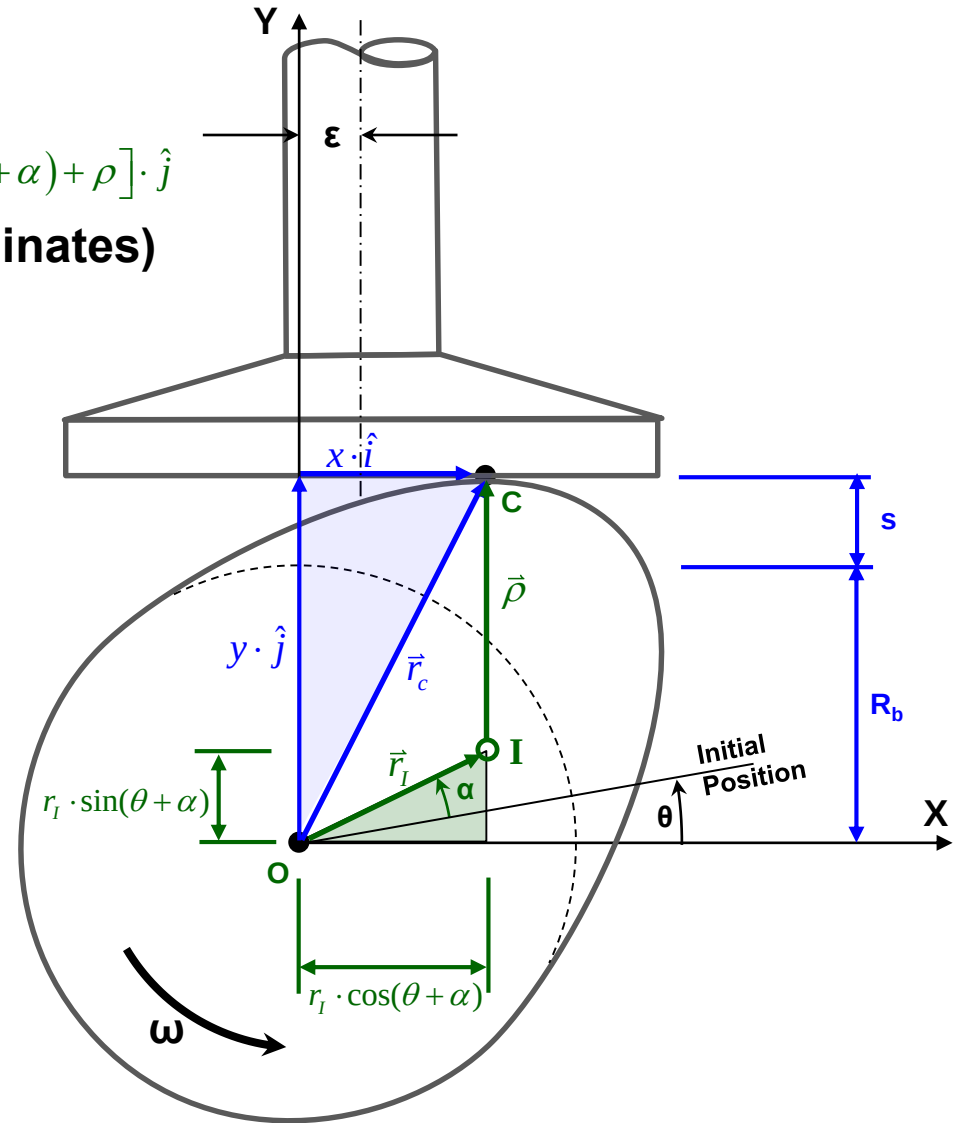
Substituting in (2)

$$x' = s'' = -r_I \cdot \sin(\theta + \alpha) \quad (5)$$

Substituting in (5) into (4)

$$R_b + s = -s'' + \rho$$

$$\rho = R_b + s + s''$$



Determining Tool Positioning

Vector Loop for OC

$$x \cdot \hat{i} + (R_b + s) \cdot \hat{j} = r_l \cdot \cos(\theta + \alpha) \cdot \hat{i} + [r_l \cdot \sin(\theta + \alpha) + \rho] \cdot \hat{j}$$

Resolving the components

$$x = r_l \cdot \cos(\theta + \alpha) \quad \textcircled{3}$$

$$R_b + s = r_l \cdot \sin(\theta + \alpha) + \rho \quad \textcircled{4}$$

I is stationary, for small θ ,
 r_l , ρ , and α are constant.
 Taking the derivative of $\textcircled{3}$ with
 respect to θ .

$$\frac{\partial x}{\partial \theta} = x' = -r_l \cdot \sin(\theta + \alpha)$$

Substituting in $\textcircled{2}$

$$x' = s'' = -r_l \cdot \sin(\theta + \alpha) \quad \textcircled{5}$$

Substituting in $\textcircled{5}$ into $\textcircled{4}$

$$R_b + s = -s'' + \rho$$

$$\rho = R_b + s + s''$$

