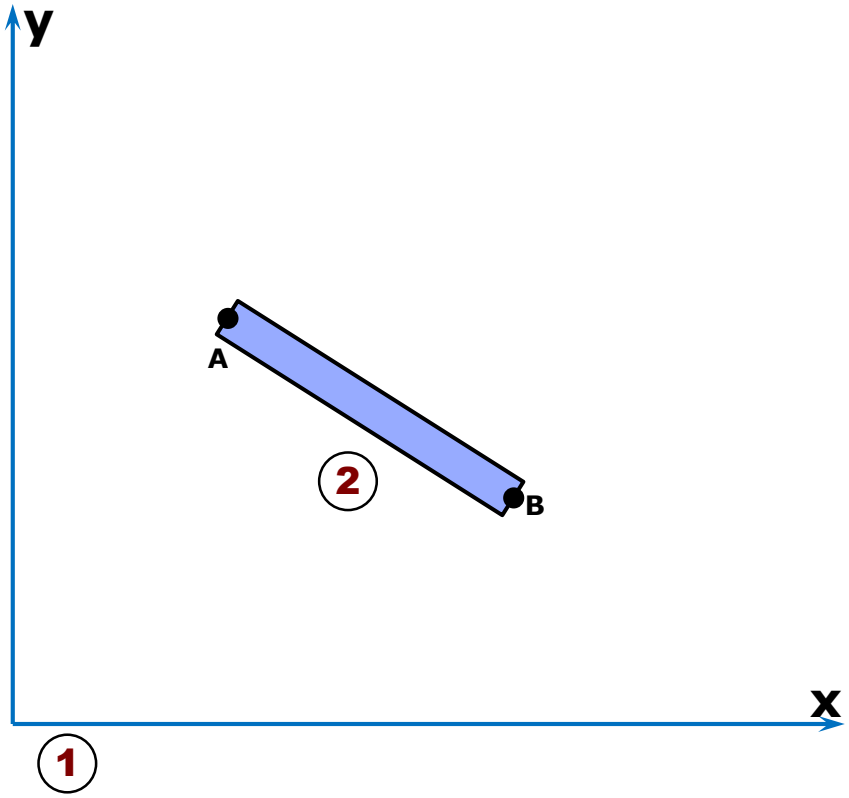
# Kinematics of Mechanisms

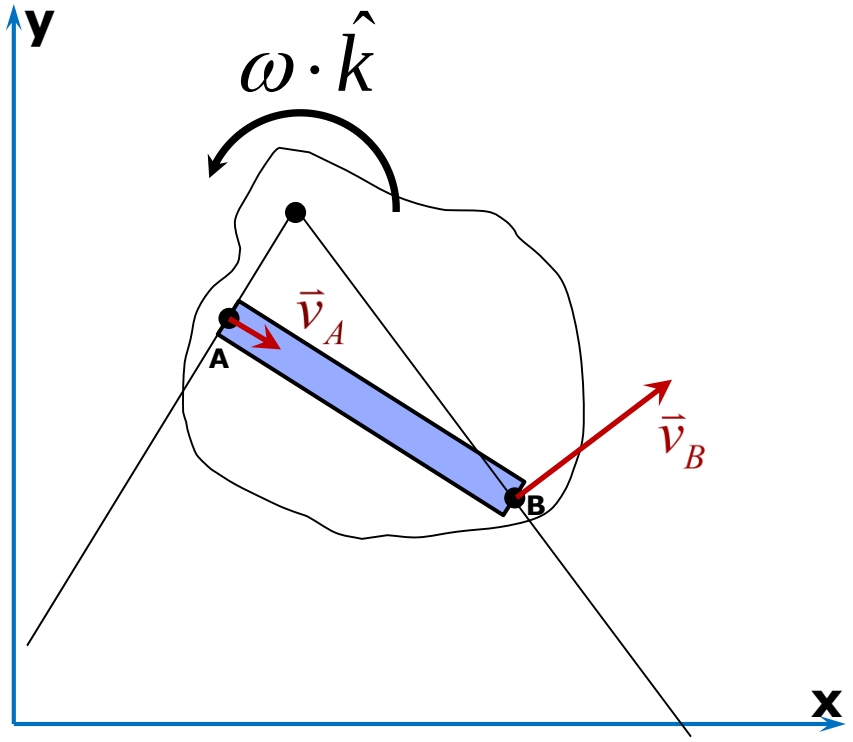
- Instant Centers
  - Primary
  - Secondary
- Aronhold-Kennedy Theorem

# Planar Body Motion Assumptions




- **Rigid Body Motion**
  - No axial deformation
  - No twisting
  - No bending
- **Planar Motion**
  - In a single plane
  - In parallel planes

# The Center of Rotation is Found from Known Velocities

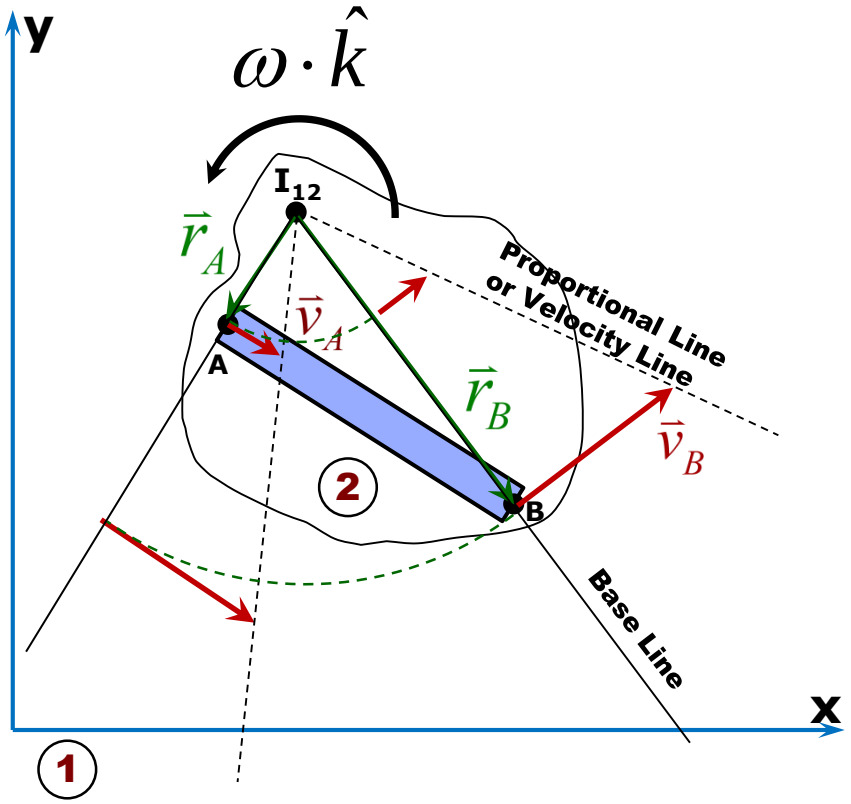



- **Rigid Body Motion**
  - No axial deformation
  - No twisting
  - No bending
- **Planar Motion**
  - In a single plane
  - In parallel planes
- **The Link appears to Rotate about a Point**

$$\vec{v} = \vec{\omega} \times \vec{r}$$

- **Link Expansion**
  - Any point in the plane

# Location of the Instant Centers of Velocity




- In General

$$\vec{v} = \vec{\omega} \times \vec{r}$$

- For Planar Problems

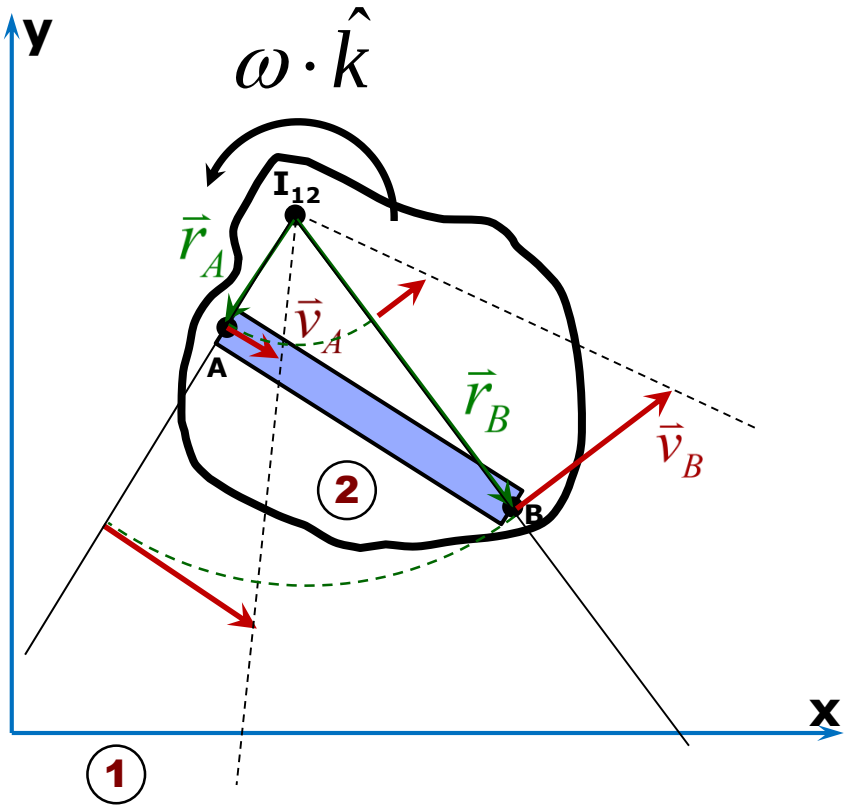
$$\vec{v} = \omega \cdot \hat{k} \times \vec{r}$$

- If ω and r are perpendicular

$$v = \omega \cdot r$$

- $I_{12}$ is the instant center between the ground ① and body AB ②

# Instant Centers of Velocity Definition:

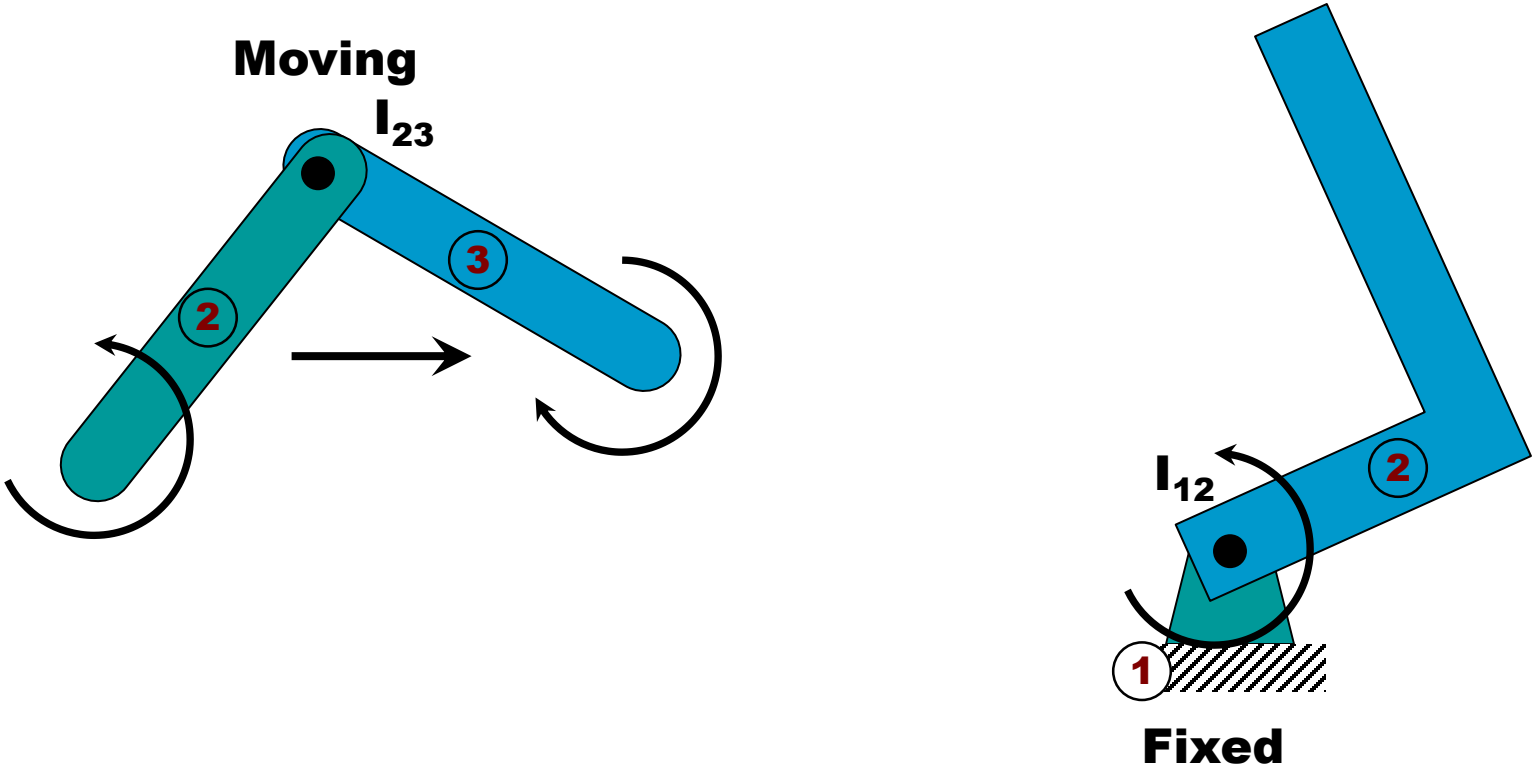



- Instant Centers often referred to as
  - Poles
  - Centros
  - Instantaneous Centers of Velocity
- Definition
  - A point on one body about which some other body is rotating either permanently or at an instant
  - A point common to two bodies having the same linear velocity in both magnitude and direction
- Functional Types
  - Primary
    - Found through Direct Observation
  - Secondary
    - Aronhold-Kennedy Theorem
- Kinematic Types
  - Fixed to the reference plane
  - Move with time
    - Instantaneous Location

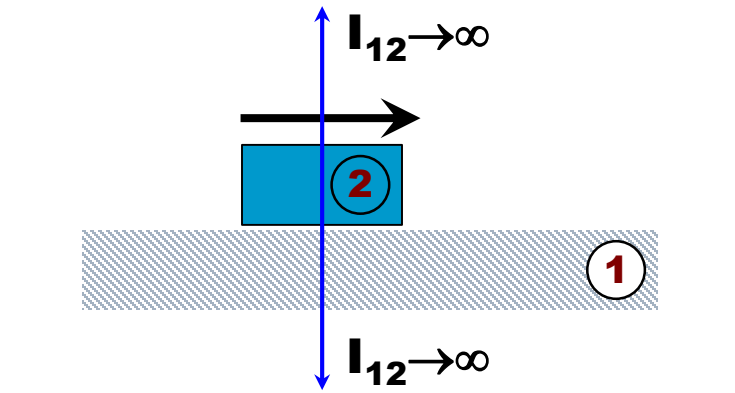
# Forms of Primary Instant Centers (Directly Observed)

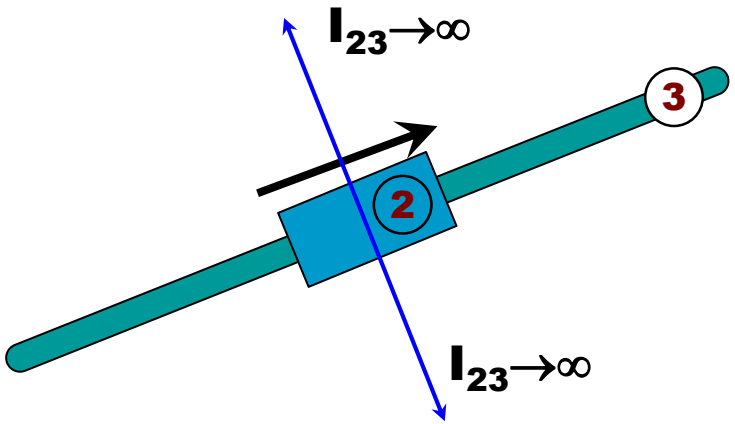
1. Instant Centers for Pin Connected Links
2. Instant Centers for Sliding Bodies
3. Instant Centers for Rolling Bodies
   - No-Slip

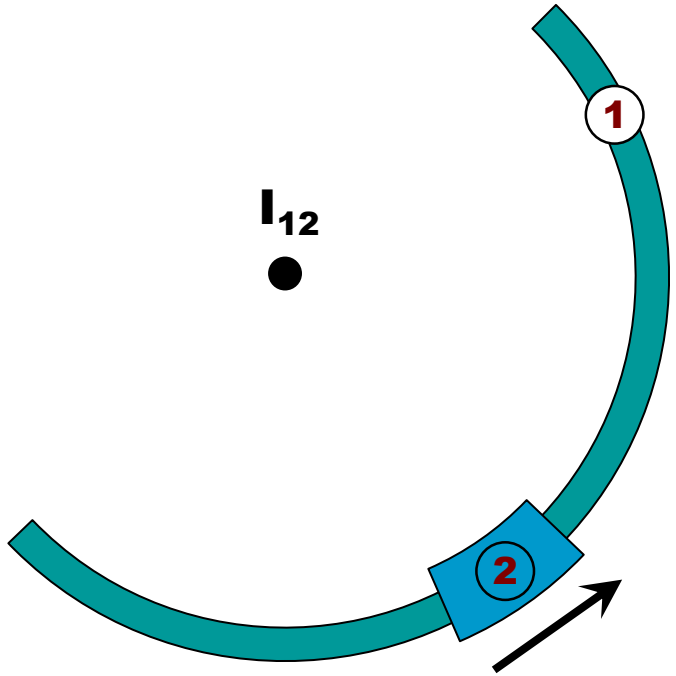
# Instant Center
# Revolute Joint/Pin Connections



# Instant Center
# Prismatic Joint/Sliding Body



# PRIMARY Instant Center

**Instant Centers that can be found through DIRECT Observation**

## Fixed Instant Centers

- Remains Fixed to the Frame
- Does Not Move

## Moving Instant Centers

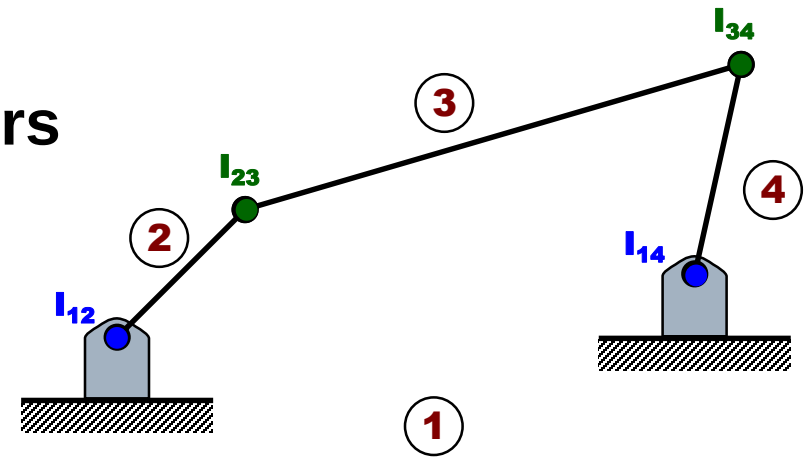
- Are not Fixed
- Can Move in Time

1 Ground Link

2 Drive/Crank Link

3 Coupler Link

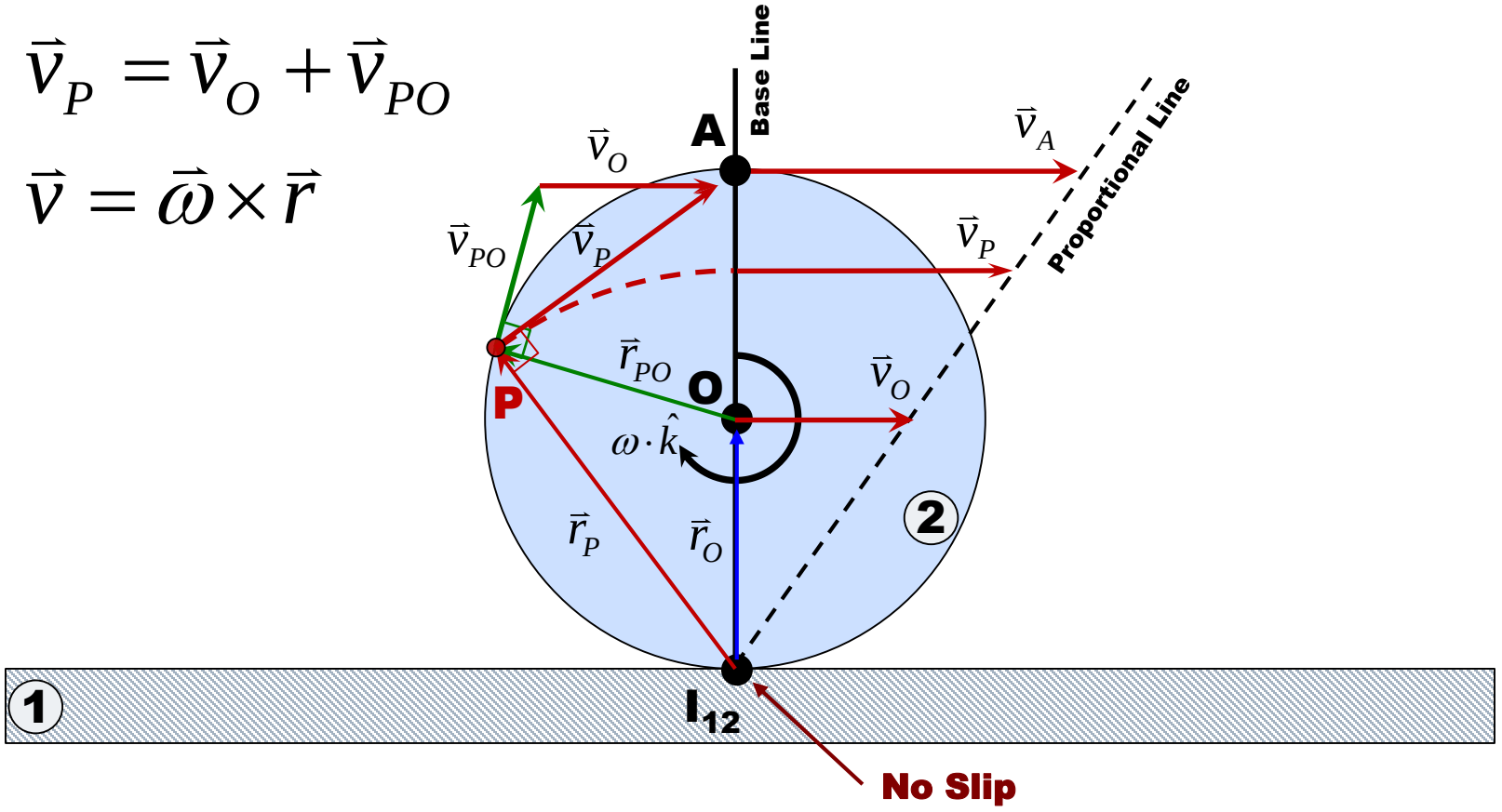
4 Follower Link



# Instant Center
# Rolling-NO Slip Contact

$$\vec{v}_P = \vec{v}_O + \vec{v}_{PO}$$

$$\vec{v} = \vec{\omega} \times \vec{r}$$



# The Number of Instant Centers Can Be Calculated

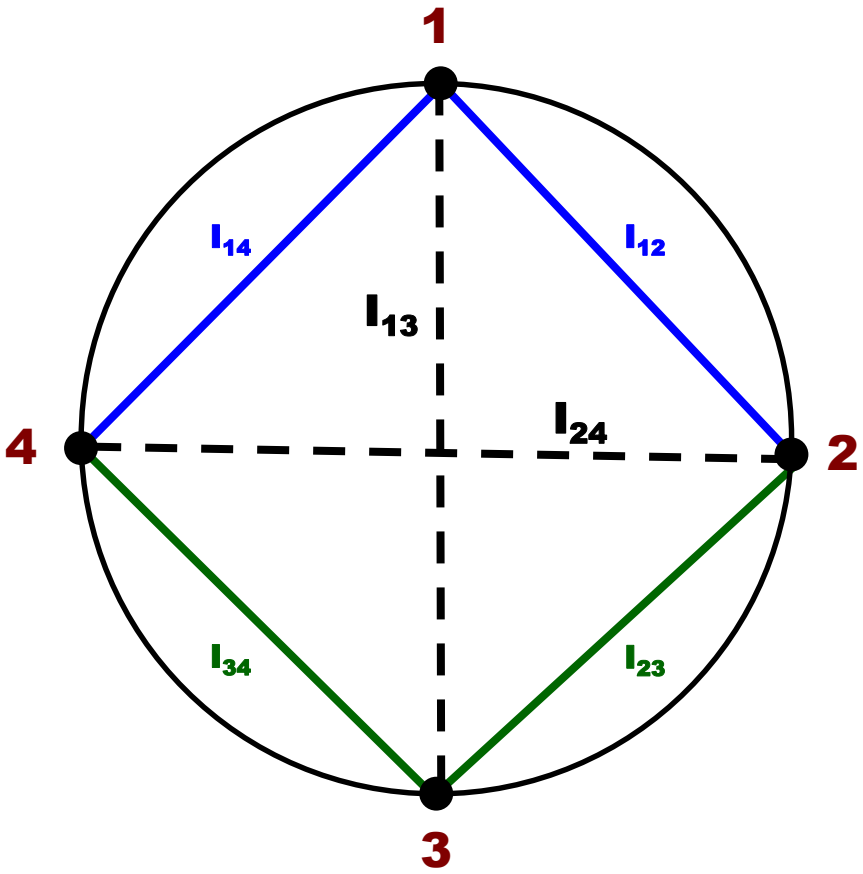
**Because any two links in a mechanism have motion relative to one another, they have a common Instant Center.**

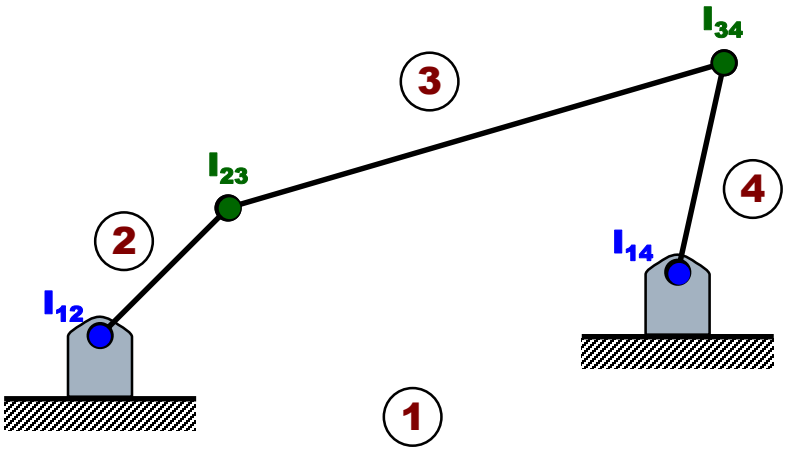
$$N = \binom{n}{2} = \frac{n!}{(n-2)!\cdot 2!} = \frac{n(n-1)}{2}$$

**The number of links in a mechanism taken two at a time. For a 4 Bar Mechanism:**

$$N = \frac{4(4-1)}{2} = 6$$

# Kennedy Circle

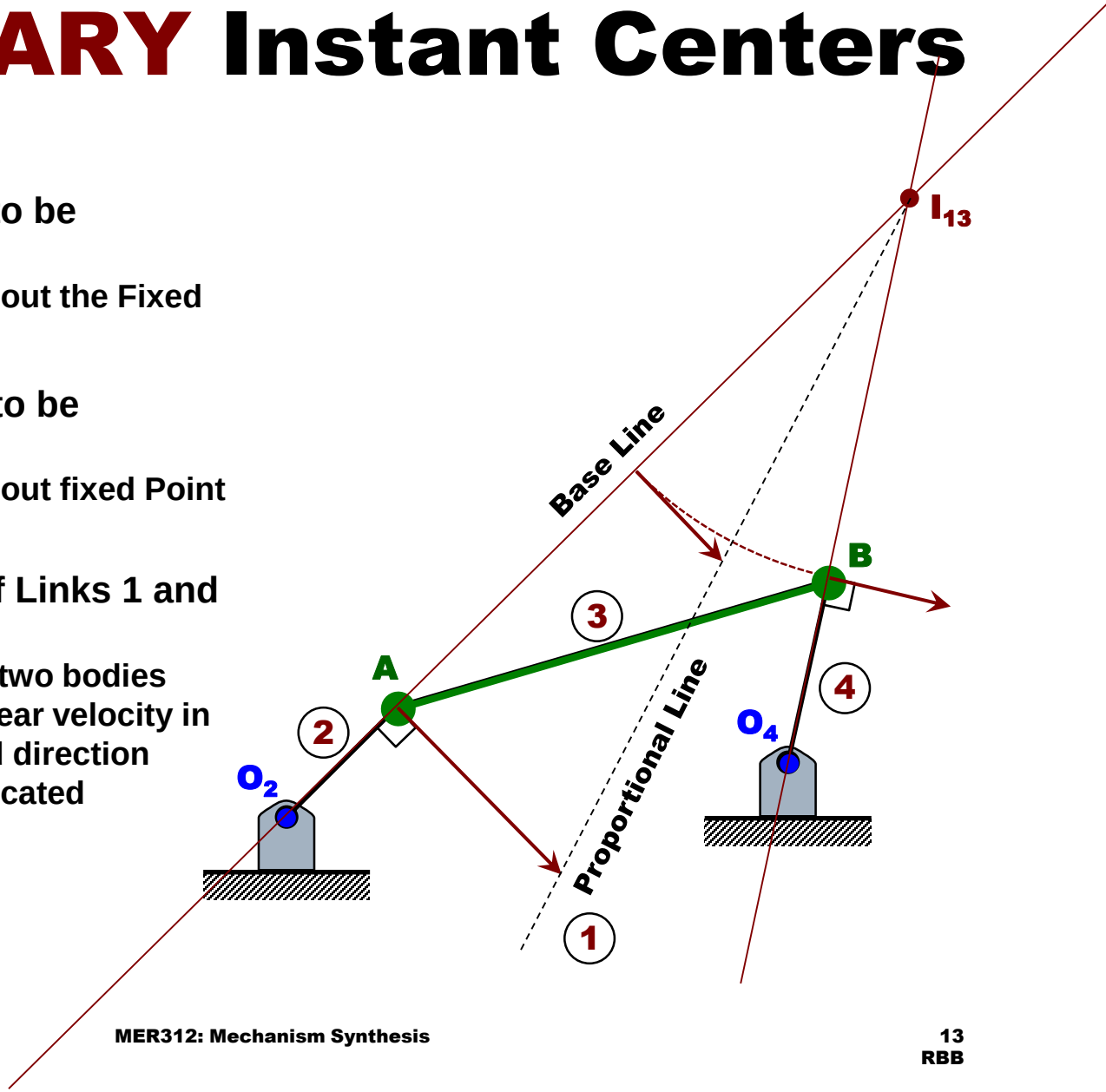



**Need to locate the SECONDARY Instant Centers $I_{13}$ and $I_{24}$**

# Where are the Other Two ICs SECONDARY Instant Centers

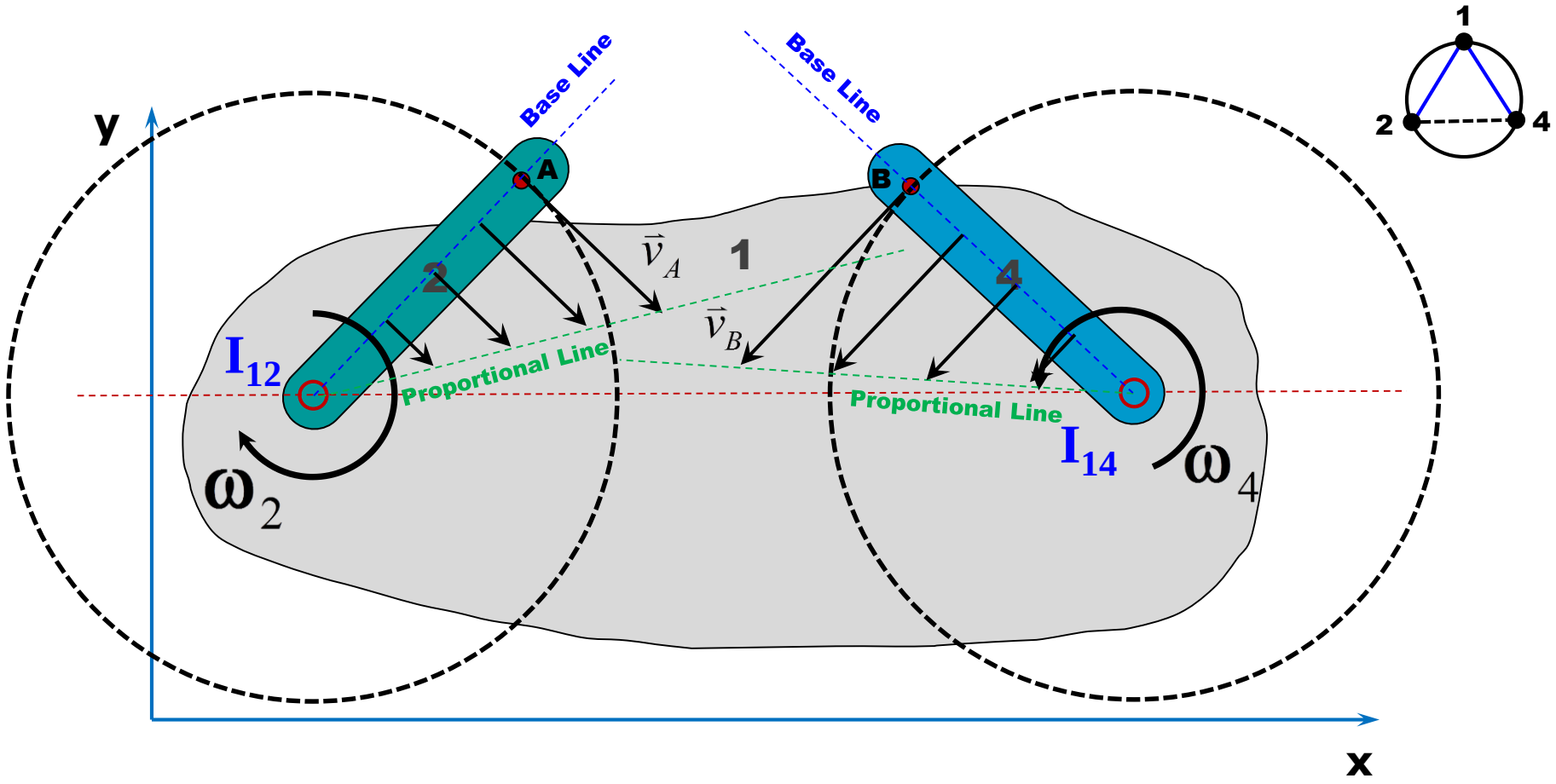
- The velocity at A has to be perpendicular to $O_2A$,
  - Link 2 is rotating about the Fixed Point $O_2$
- The velocity at B has to be perpendicular to $O_4B$,
  - Link 4 is rotating about fixed Point $O_4$
- Through Expansion of Links 1 and 3
  - A point common to two bodies having the same linear velocity in both magnitude and direction
  - Instant Center $I_{13}$ Located
  - $I_{13}$ is a MOVING IC
- Still need to find $I_{24}$



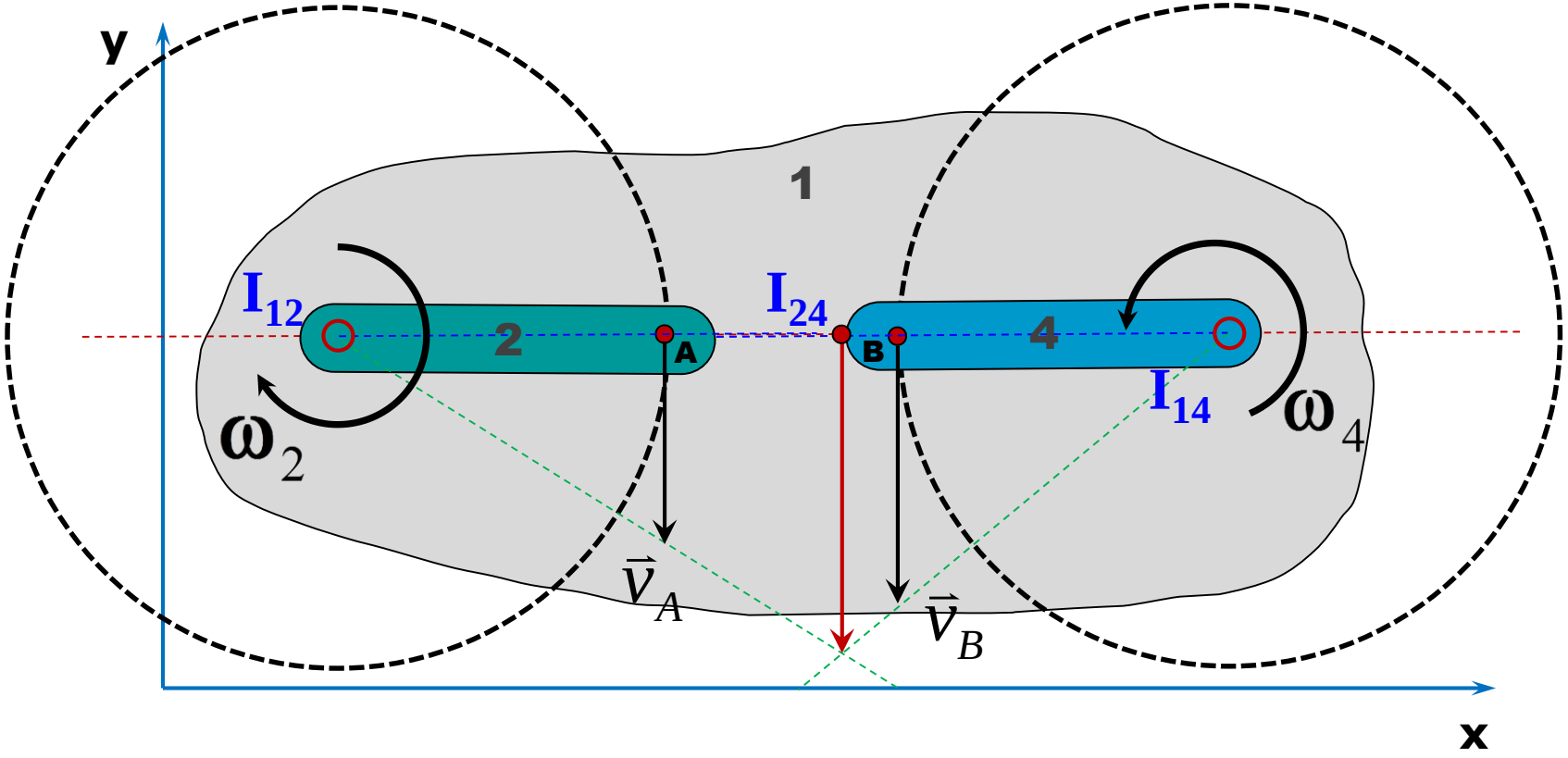
# Aronhold-Kennedy Theorem

- The three instant centers shared by three rigid bodies in relative motion to one another all lie on the same straight line
- The three bodies do not have to be connected
- Independently Discovered
  - S.H. Aronhold (1872) - German Speaking Countries
  - A.B.W. Kennedy (1886) - English Speaking Countries

# Finding $I_{24}$, Derivation of the Aronhold-Kennedy Theorem

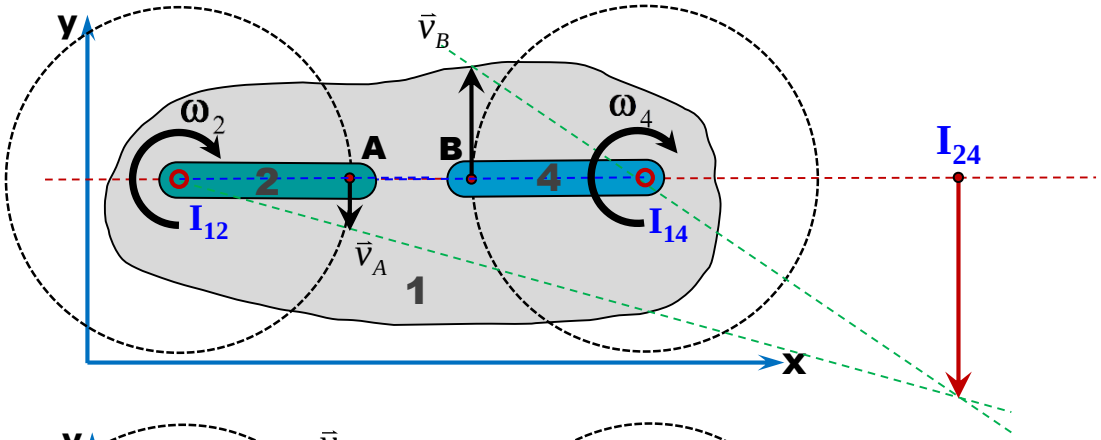


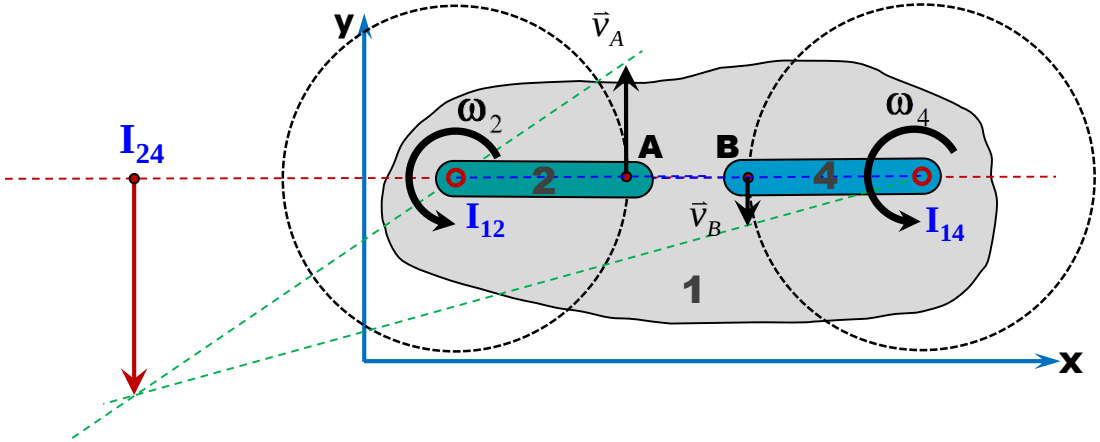
# Aronhold-Kennedy Theorem



# Aronhold-Kennedy Theorem Alternate Locations of $I_{24}$

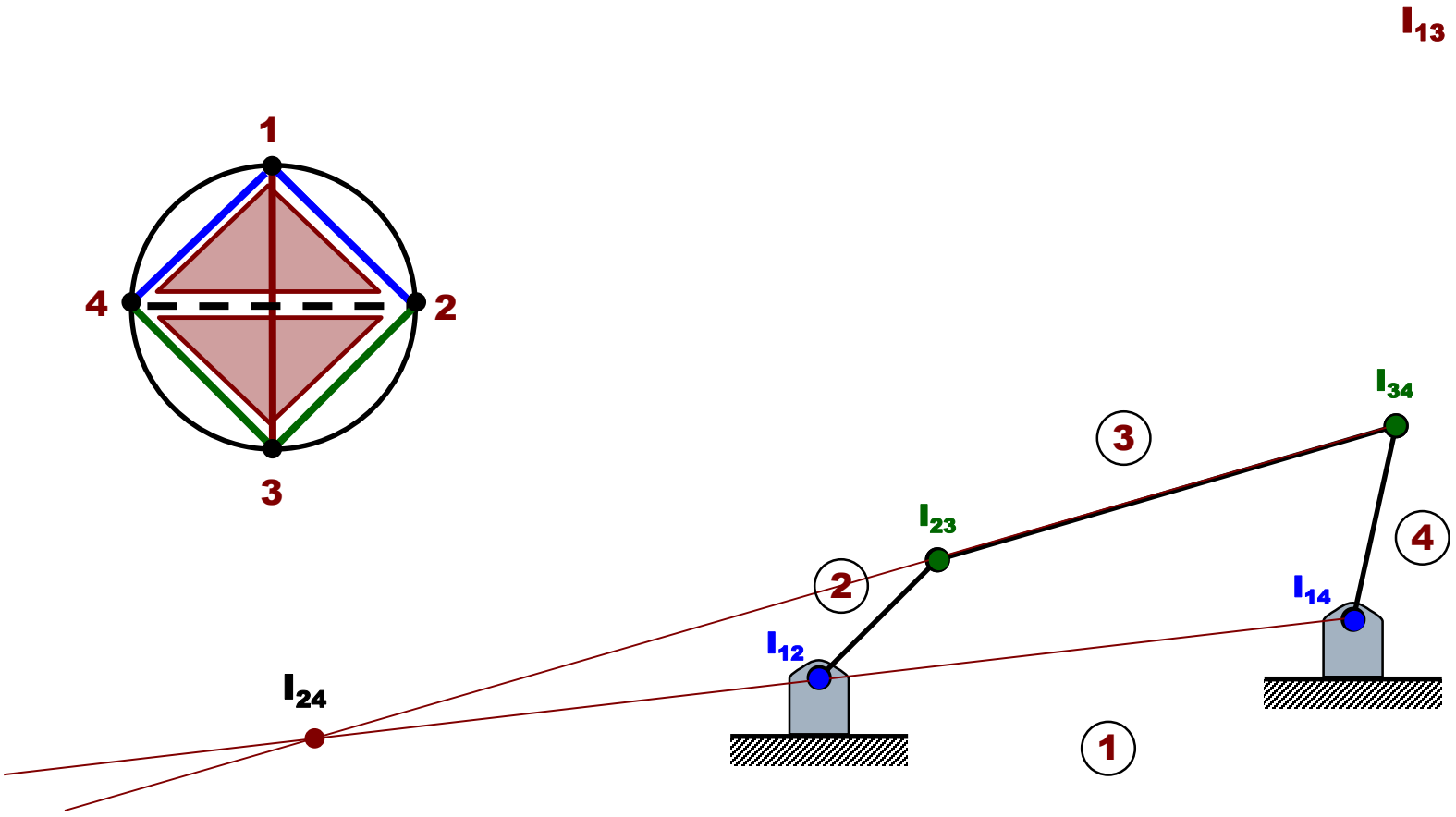
**Both Rotating Clockwise**



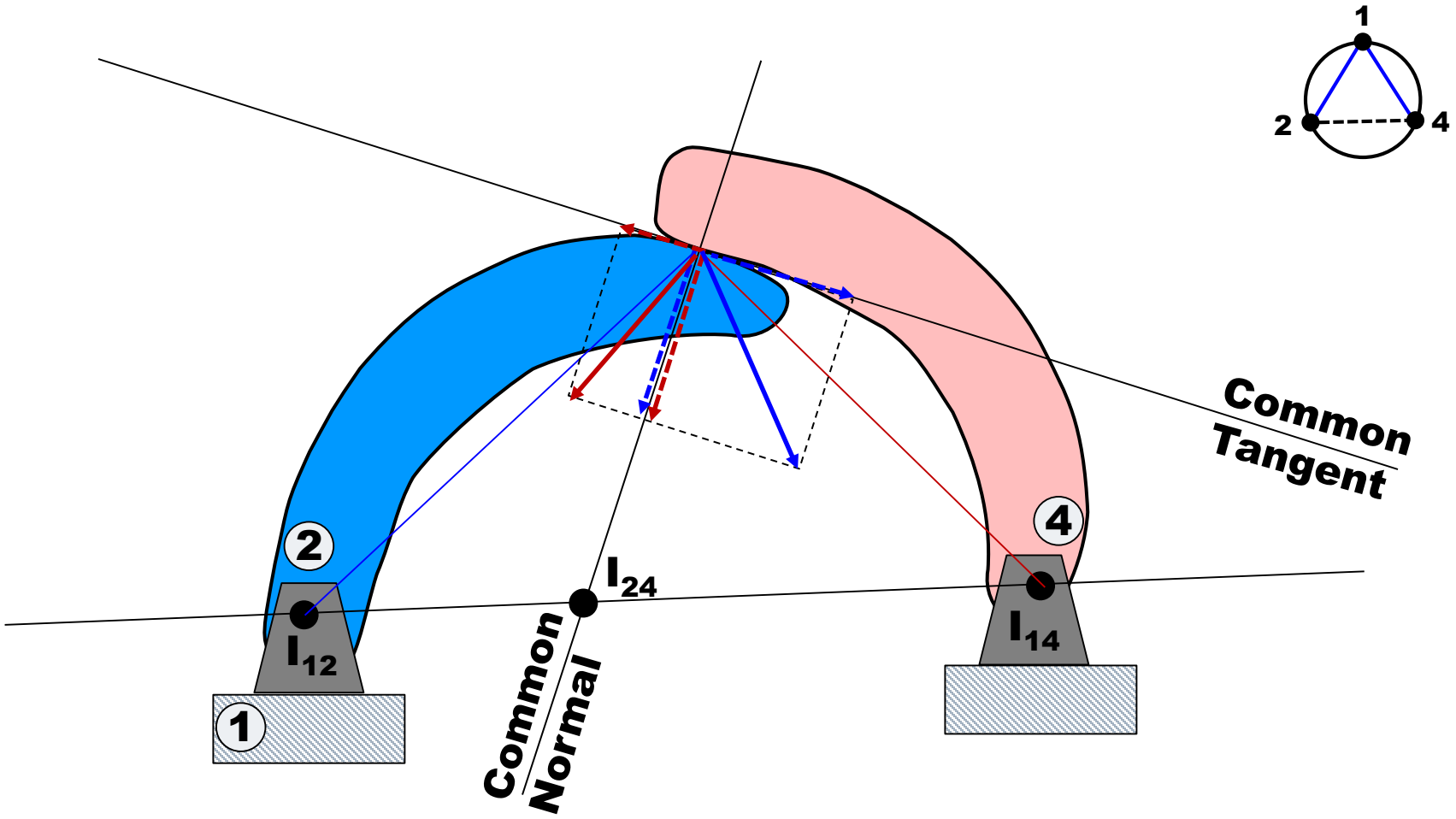




**Both Rotating Counter-Clockwise**

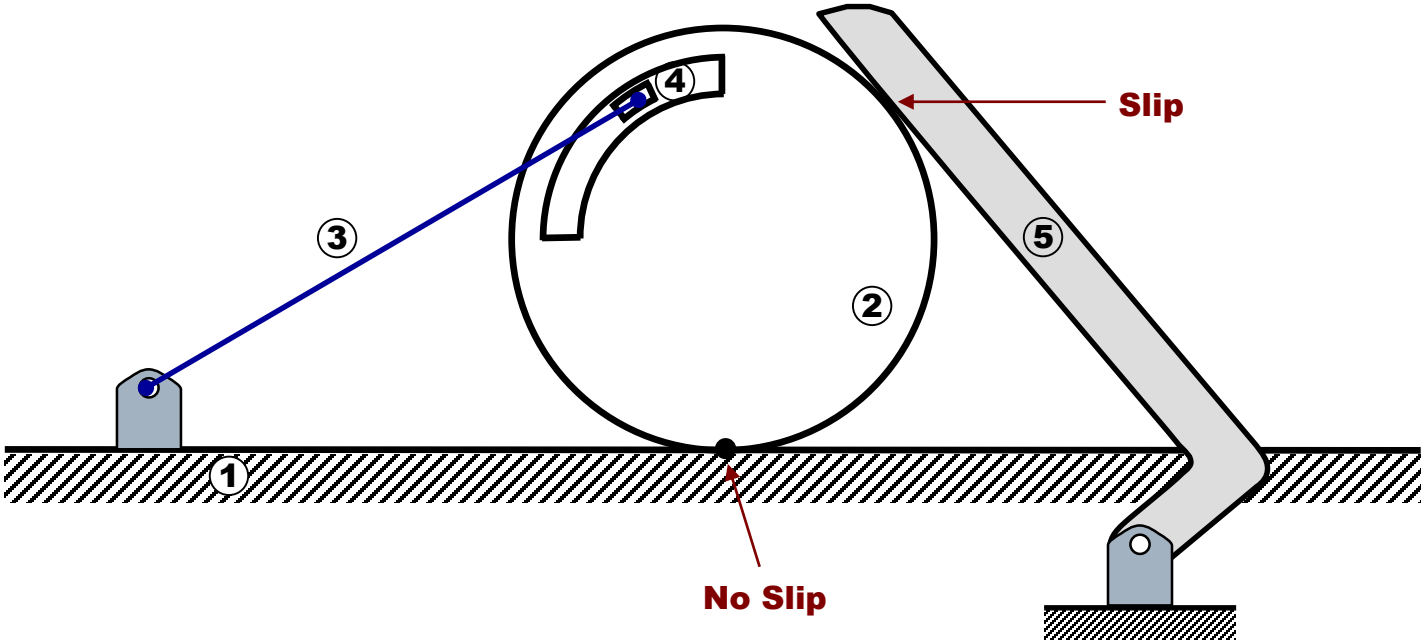
# SECONDARY Instant Centers Using Aronhold-Kennedy



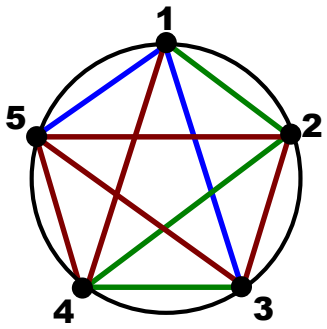
# Instant Center
# Rolling-Slip Contact



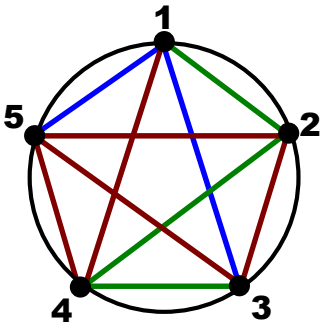
# Example: Locate the IC's for the Mechanism Below.

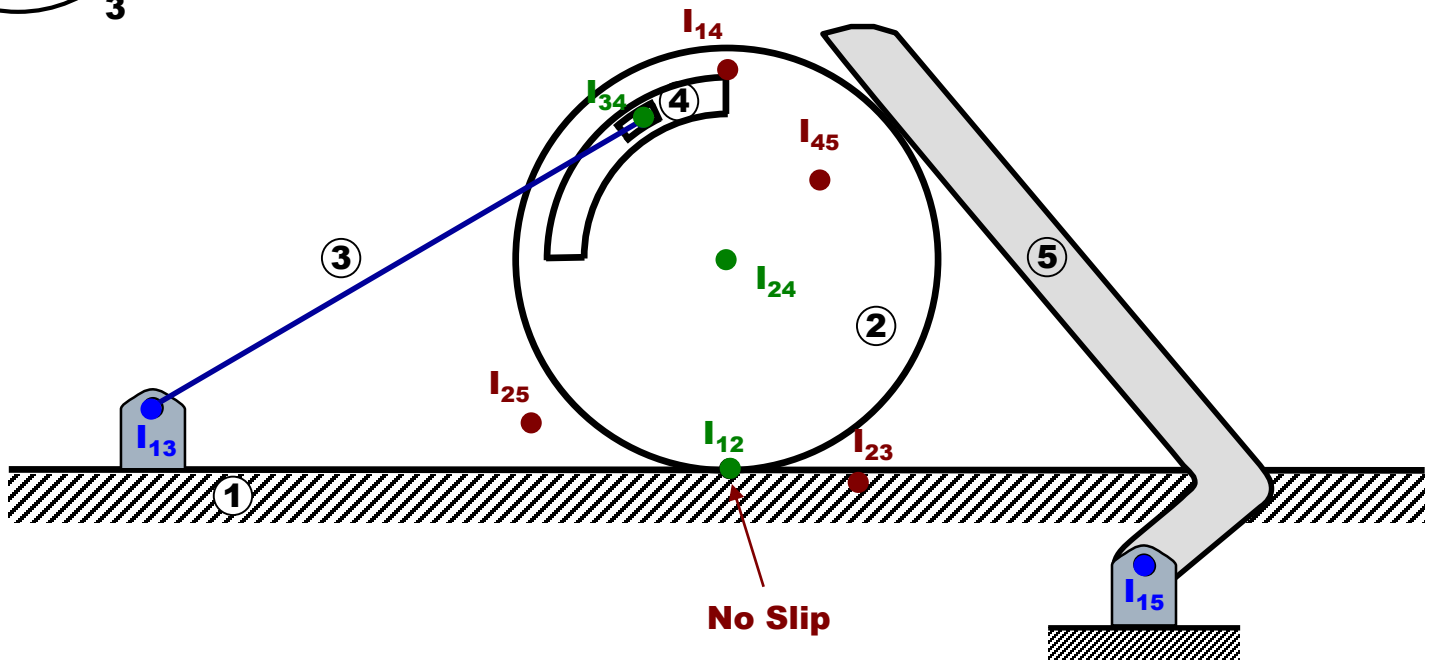


# Solution: Instant Centers Location Construction



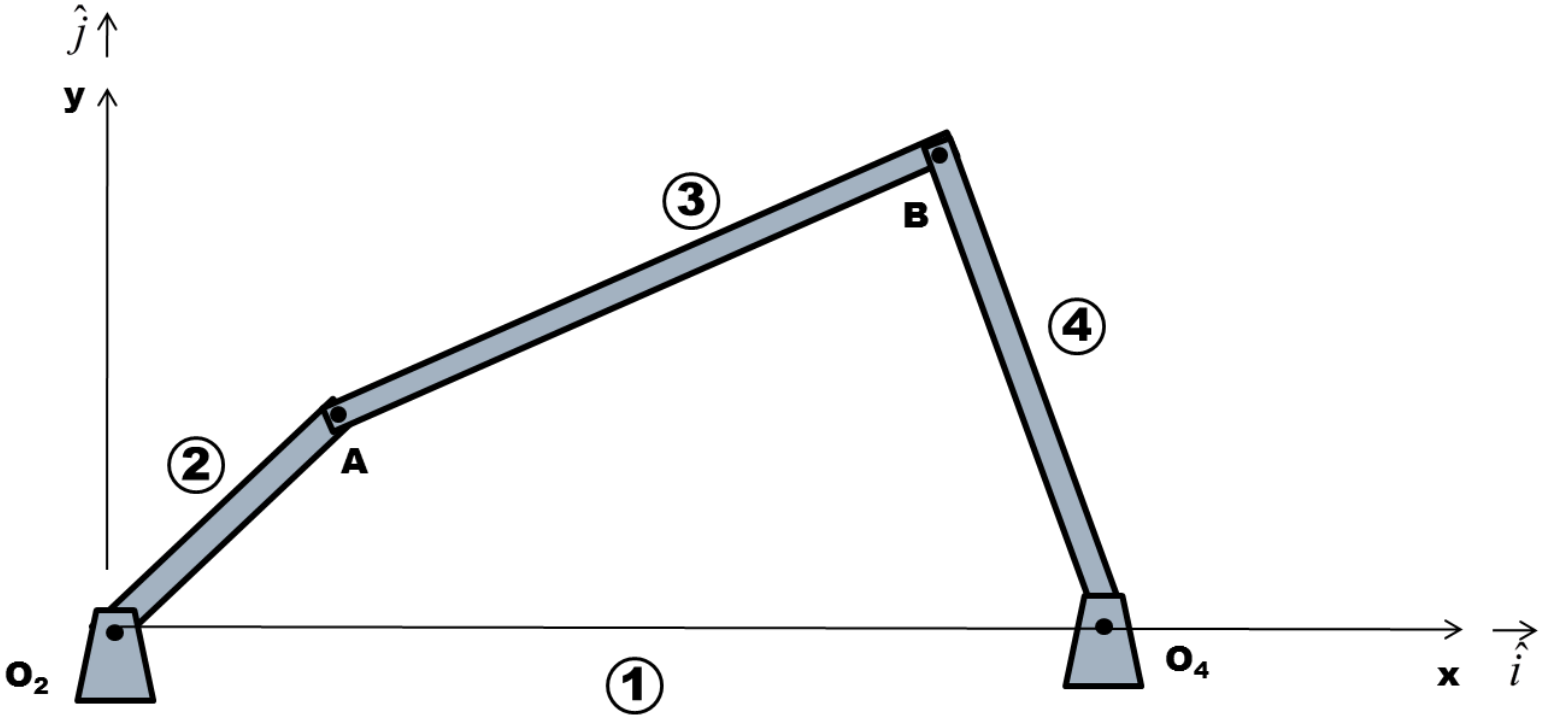
# Solution: Instant Centers



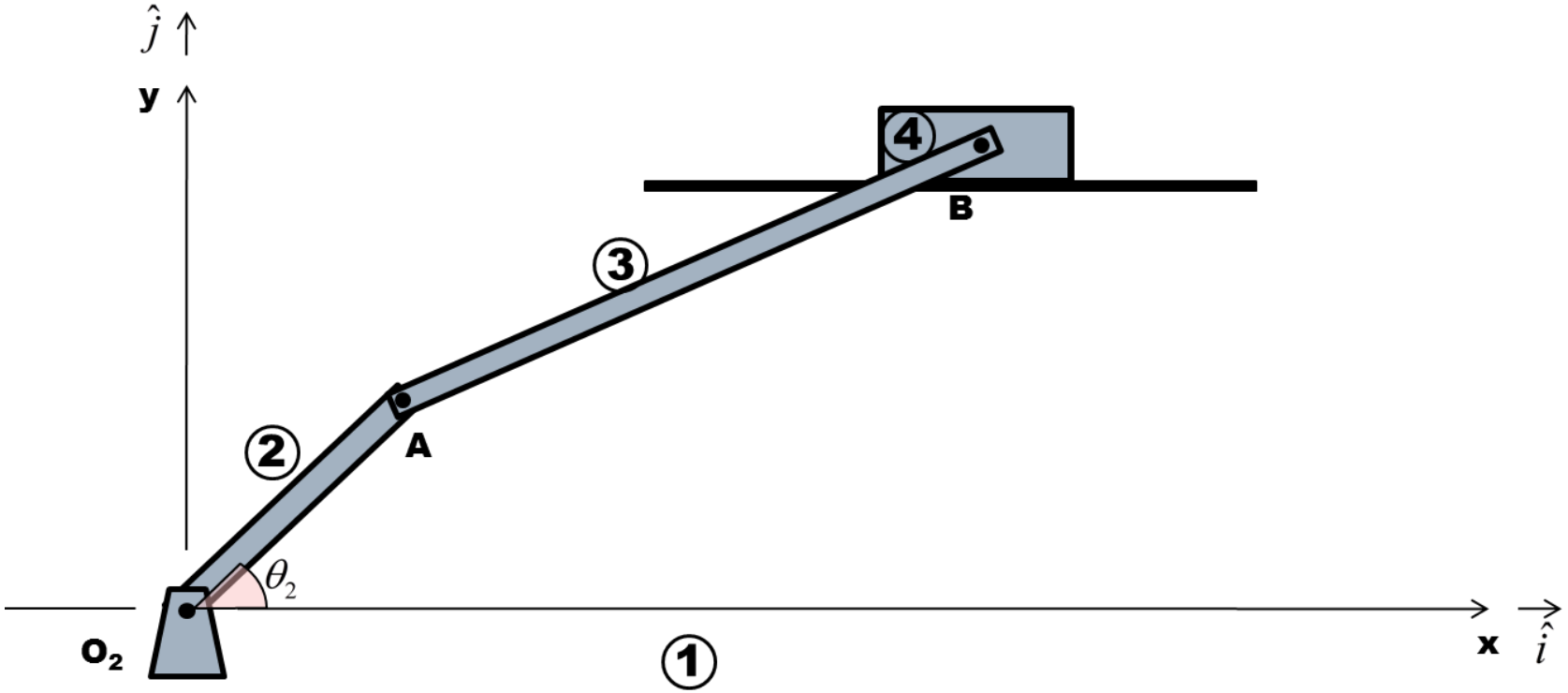




To I35→

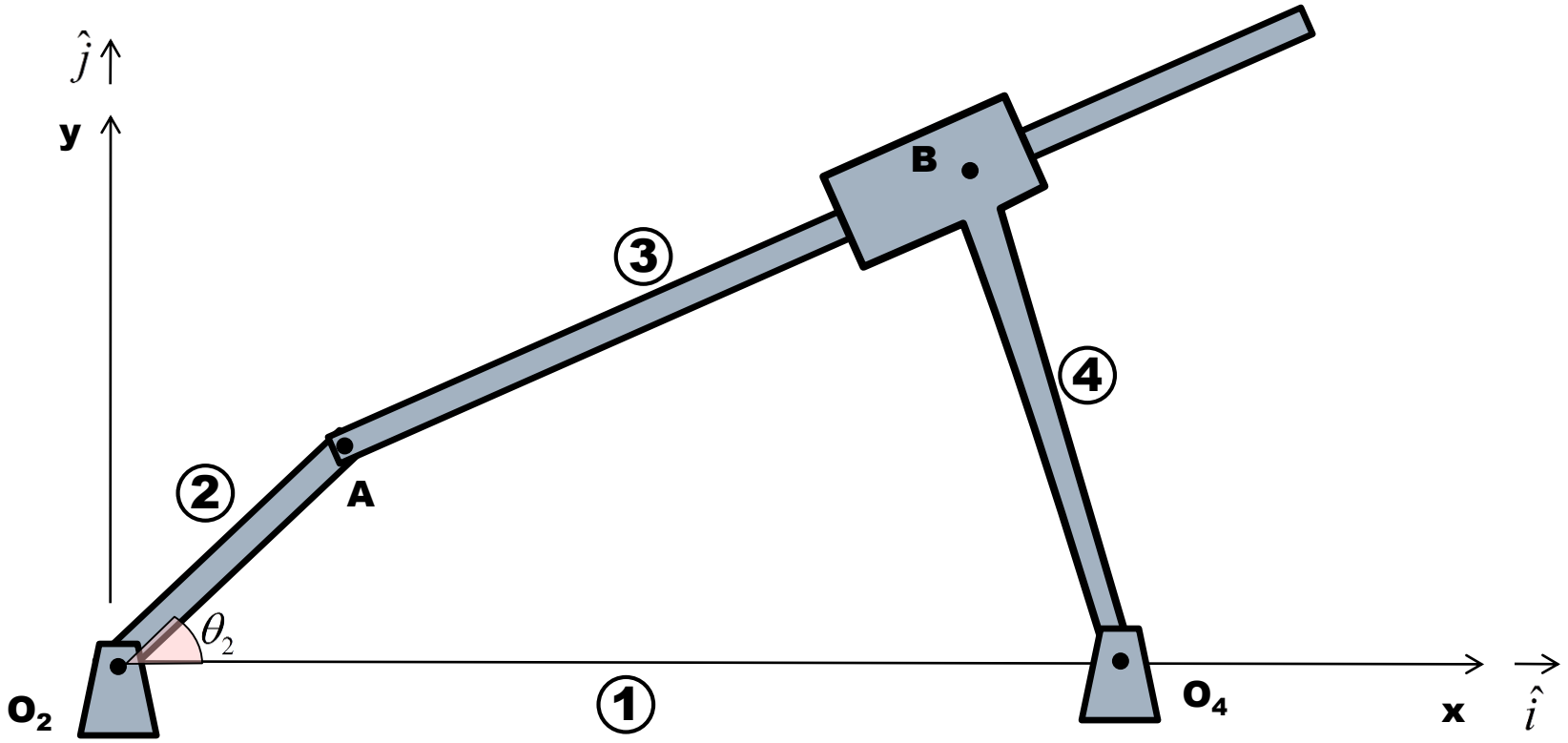
# TYPE I (RRRR): 4 - BAR



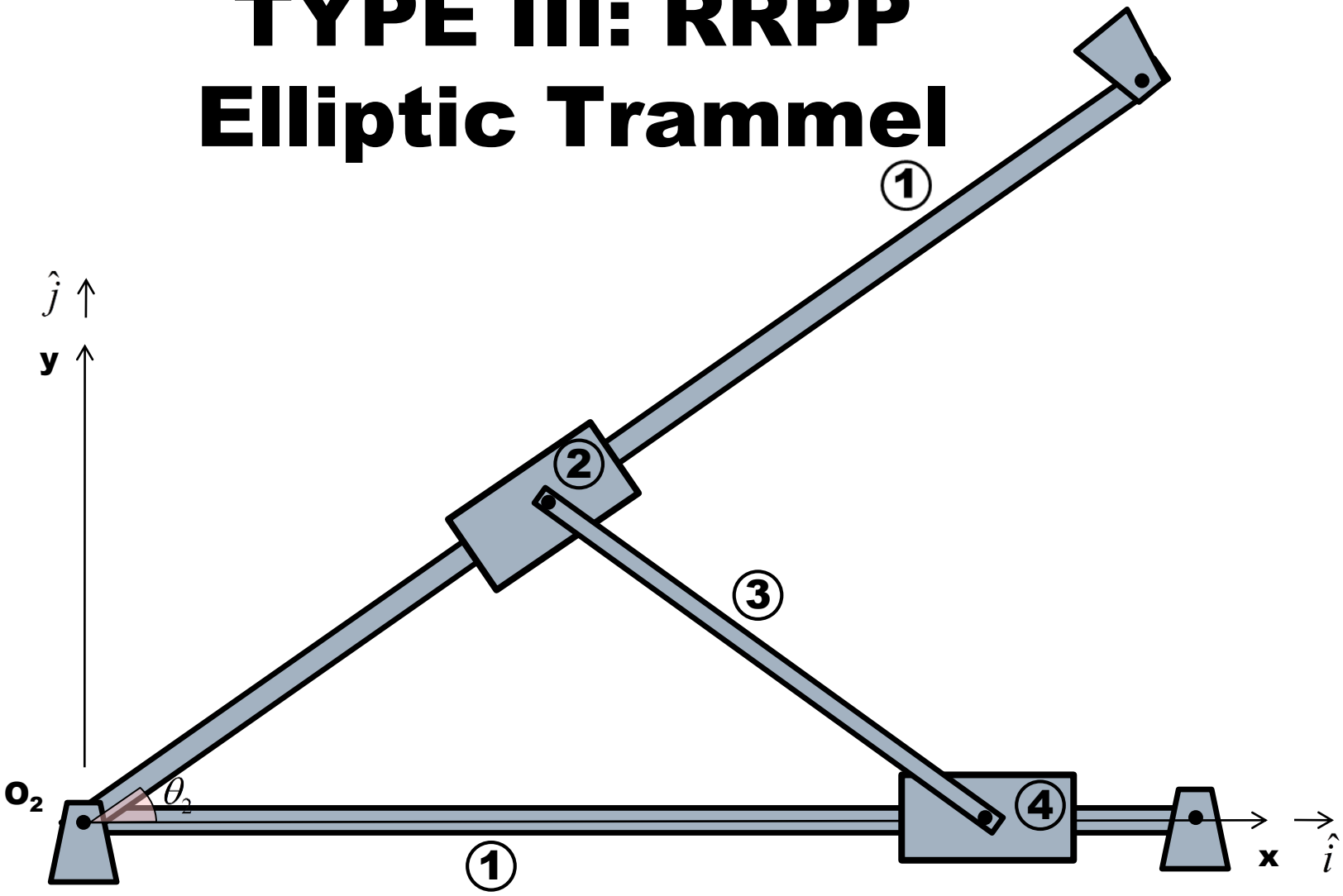
# TYPE II: (RRRP): SLIDER CRANK



# TYPE II: (RRRP)



# TYPE III: RRPP
# Elliptic Trammel



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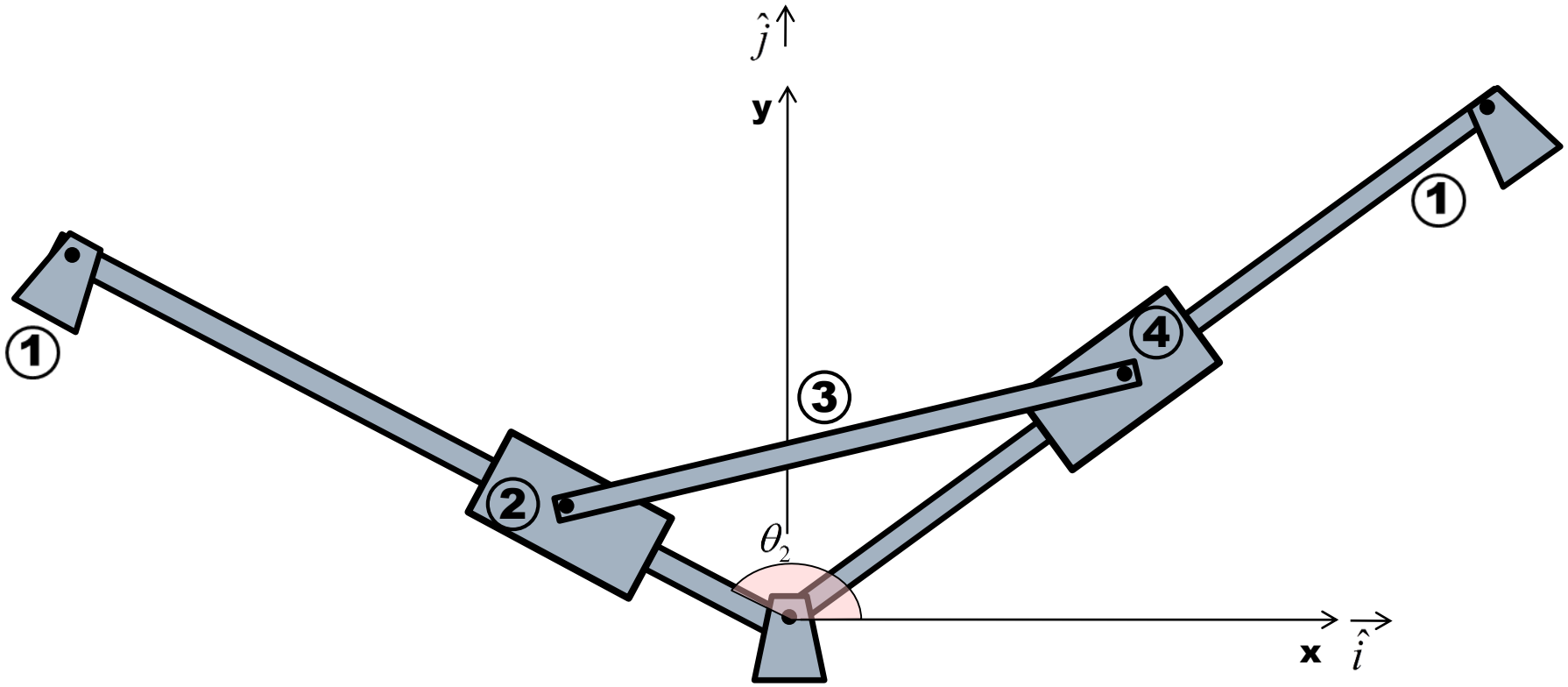
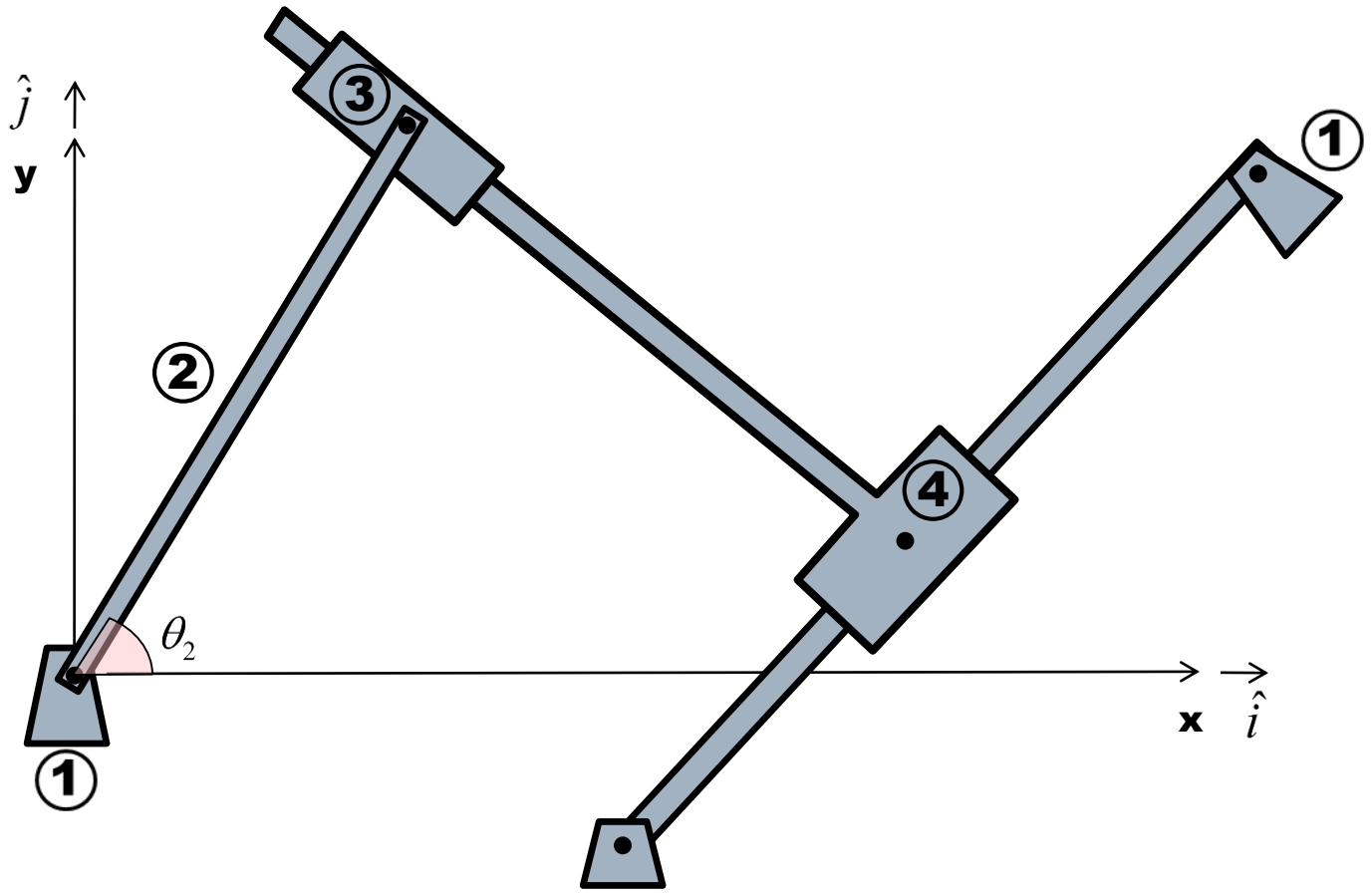
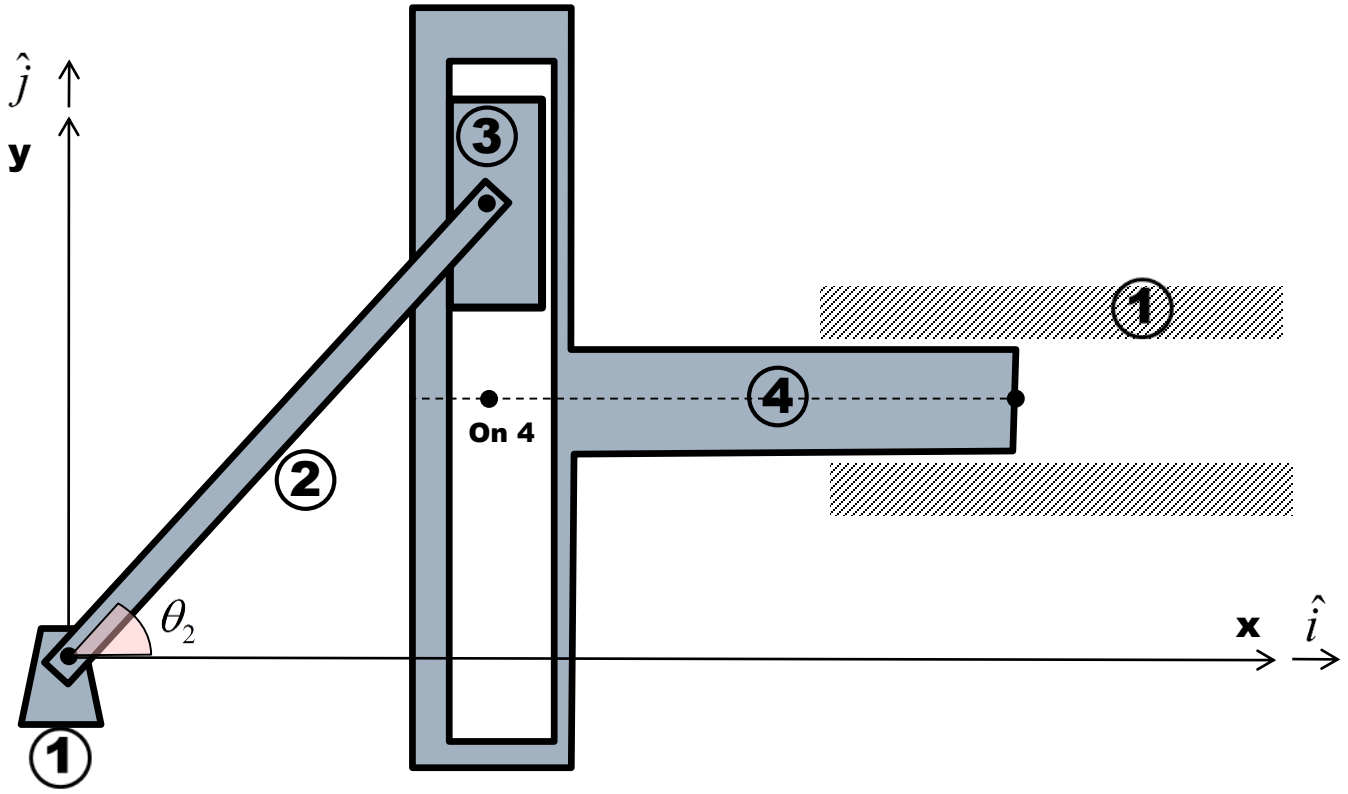


# Type III: RRPP
# Elliptic Trammel



# Type III: RRPP
# Elliptic Trammel: Scotch Yoke



# Type IV: RPRP
# Rapson Slide Linkage