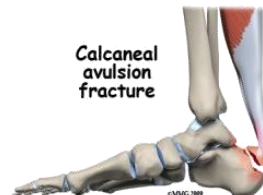


Fatigue and Fracture

- **Introduction to Fatigue**
- **Fatigue-Life Methods**
- **Fatigue Strength and Endurance Limits**
- **Endurance Limit Modifying Factors**
- **Stress Concentrations and Notch Sensitivity**
- **Fluctuating Stresses**
- **Combination of Loading Modes**
- **Varying, Fluctuating Stresses; Cumulative Fatigue Damage**

Examples of Fatigue



Patterns of Observed Fatigue Behavior

- **Fatigue Failure Results From REPEATED PLASTIC DEFORMATION**
 - Without repeated PLASTIC yielding, fatigue failures can not occur
 - Bending a paper clip by bending it back and forth (low cycle fatigue)
- **Fatigue Failures Typically occur after thousands or millions of cycles**
 - Minute yielding that exists on the MICROSCOPIC level
 - Fatigue failures can occur at stress levels far below conventional
 - Yield Point
 - Elastic Limit
- **Because of Highly Localized Yielding, Vulnerable Locations Include**
 - Holes, sharp corners, threads, keyways, surface scratches, material defects, and corrosion
 - *Strengthening vulnerable locations is often as effective as making the entire part from a stronger material*

Patterns of Observed Fatigue Behavior

- **Conditions that Accelerate Crack Initiation**
 - Residual Tensile Stress
 - Elevated Temperatures
 - Temperature Cycling
 - Corrosive Environments
 - High Frequency Cycling
- **Global Deformations Small**
 - Failure Surface Perpendicular to Load → Brittle Looking Failure Surface
 - Abrupt Final Propagation to Failure
- **Initial Crack Results in an increase in local stress concentration**
 - Crack root: Reverse Cycling → Crack Deepens → Cross Section Reduced → Increased Stress → Rate of Propagation Increases → Final Failure

Fatigue – Progressive Fracture

Poncelet (France) 1839

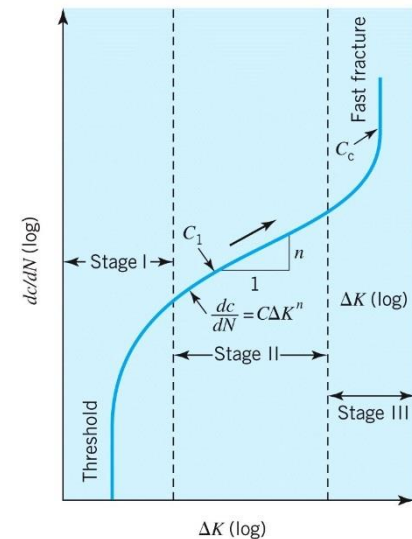
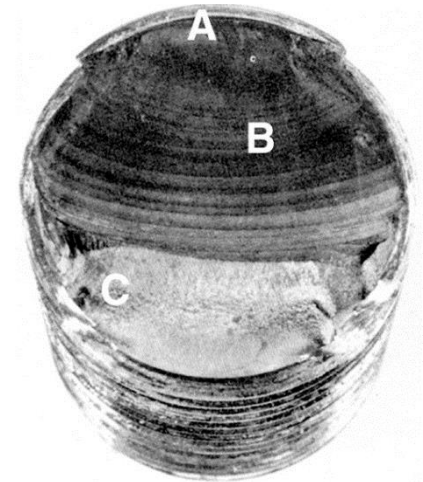
- **Stage I: Initiation (A)**

- Always at a Geometric Stress Riser
 - Minute material flaws
 - Preexisting Cracks
- Cracks form from Cyclic Plastic Deformation
 - Extremely Localized
- Typically Non-Visible Flaw

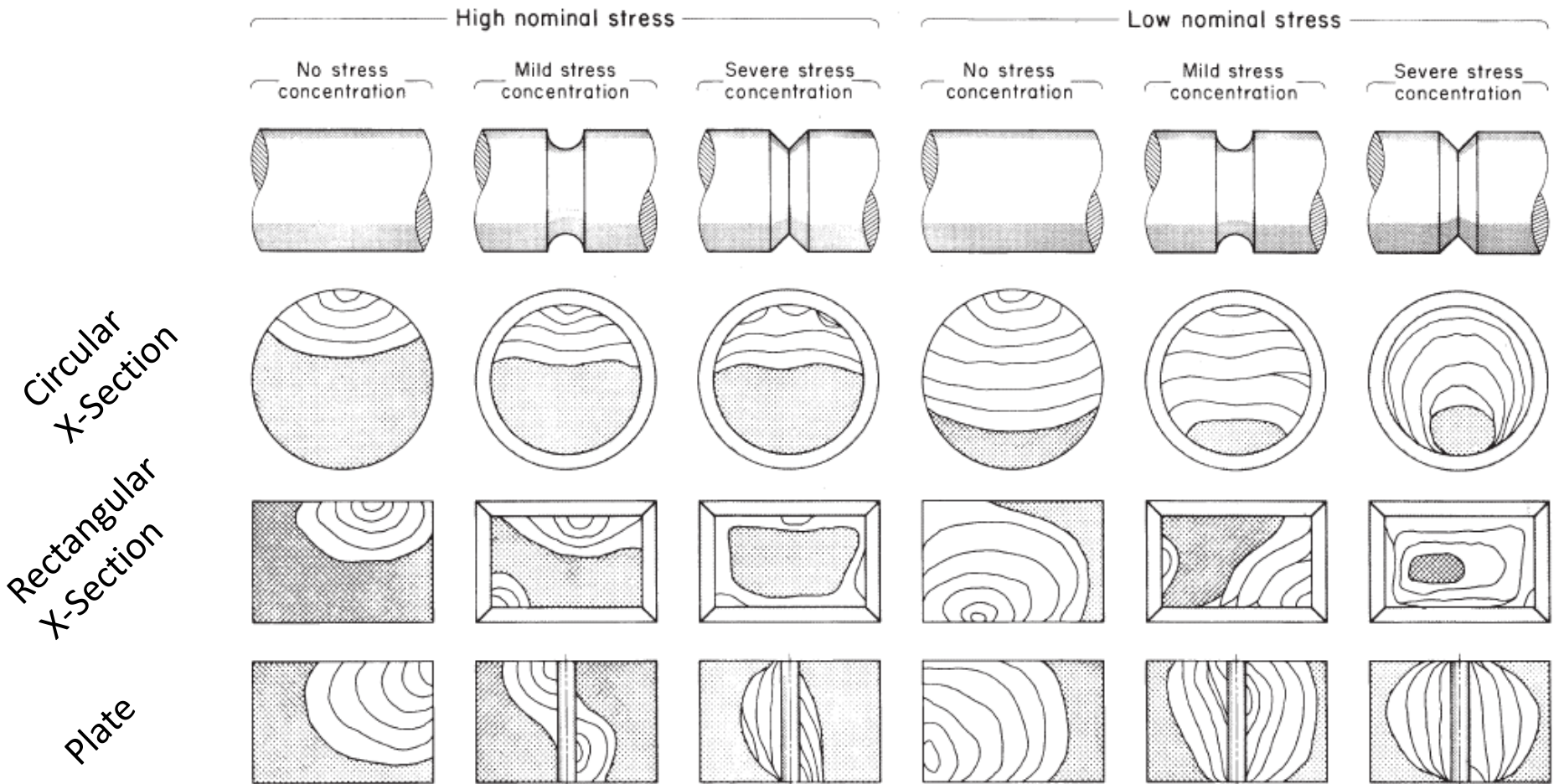
- **Stage II: Stable Propagation (B)**

- Micro- Cracks Coalesce into Macro-Cracks
- Macro-Cracks Open and Close with each Cycle
 - Cyclic plastic deformation at crack tip
 - Surface: Smooth Plateau & Longitudinal Ridges

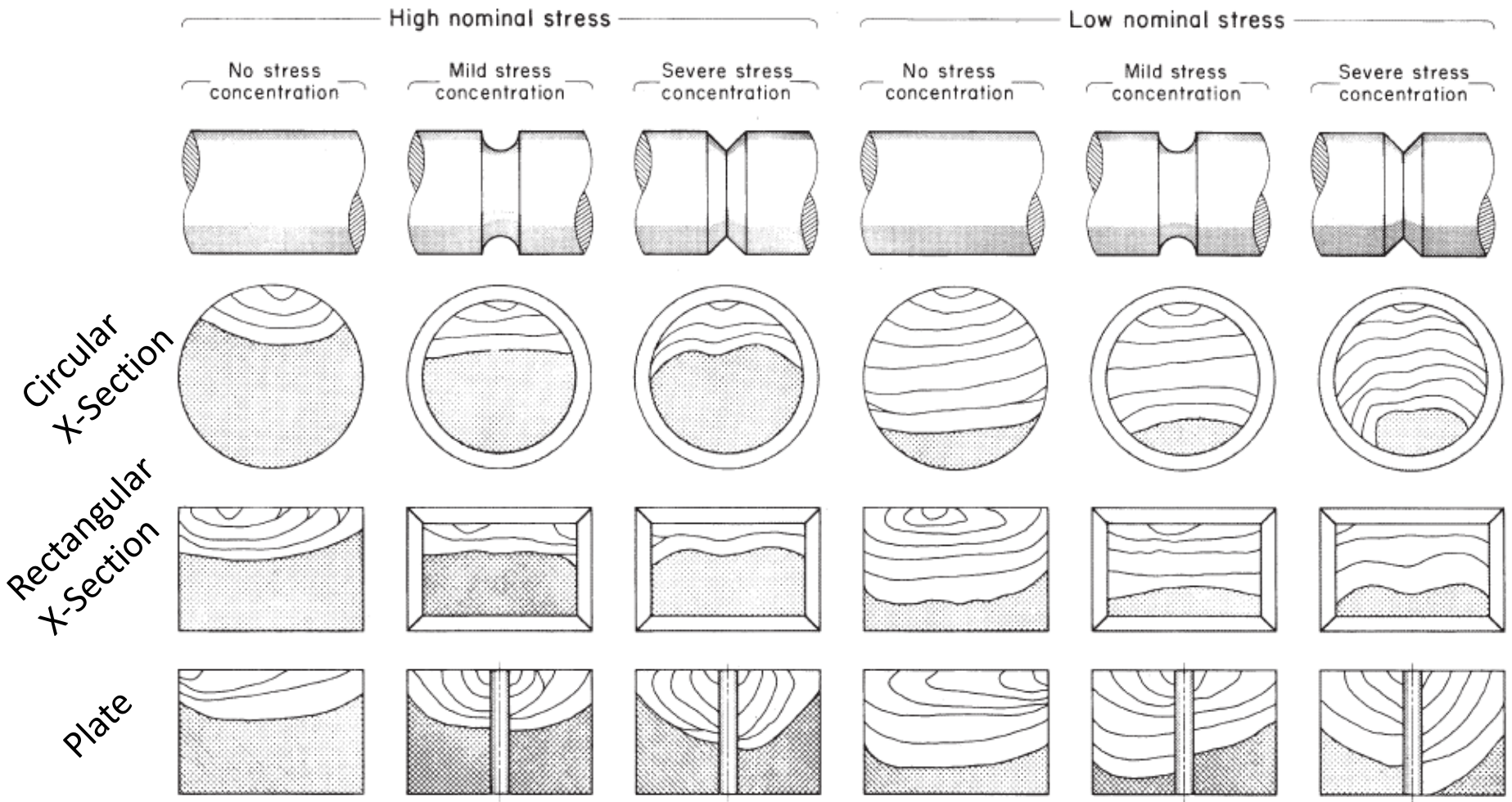
- **Stage III: Fast Fracture (C)**



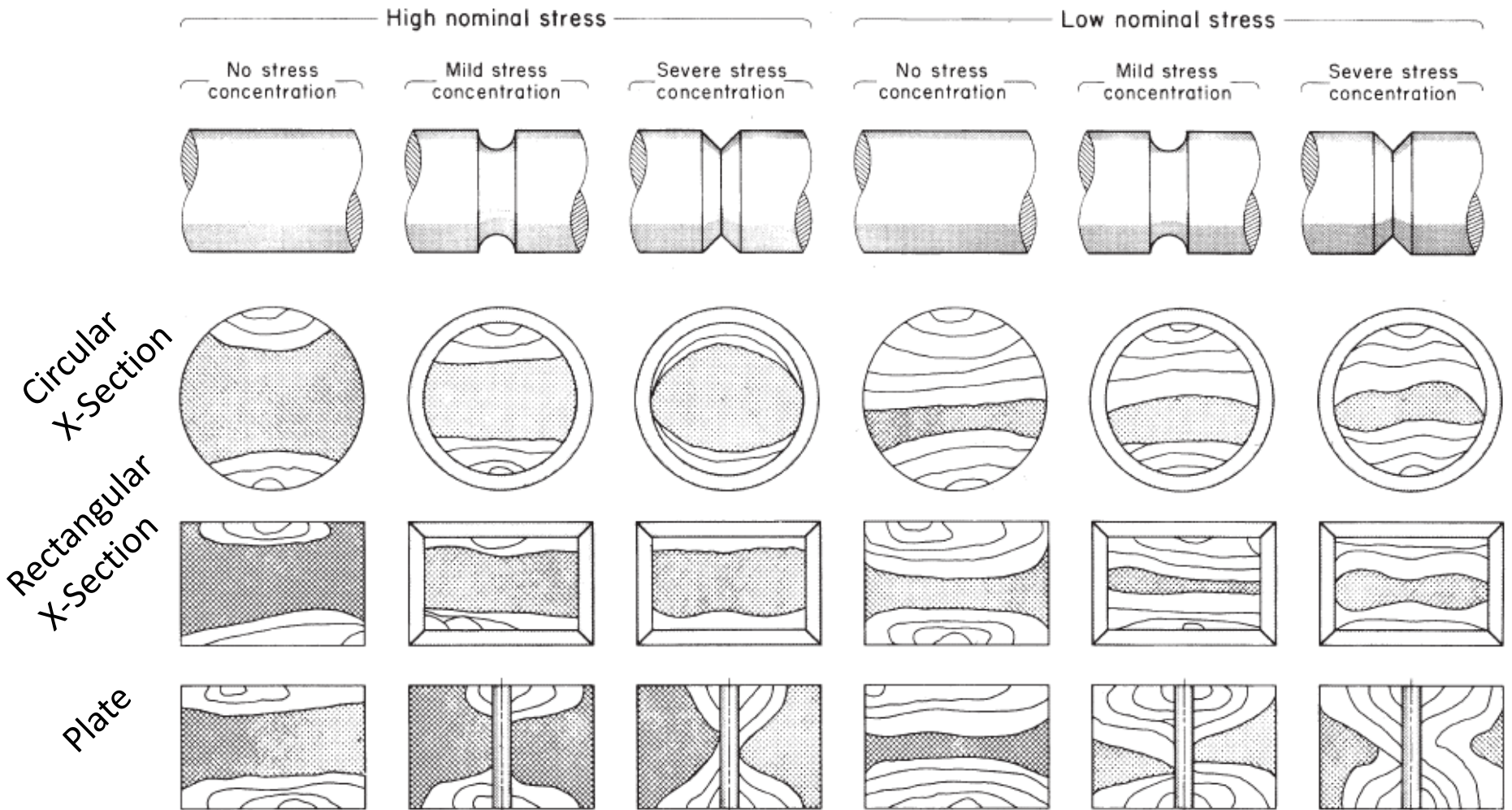
Fatigue: Tension-Tension or Tension-Compression



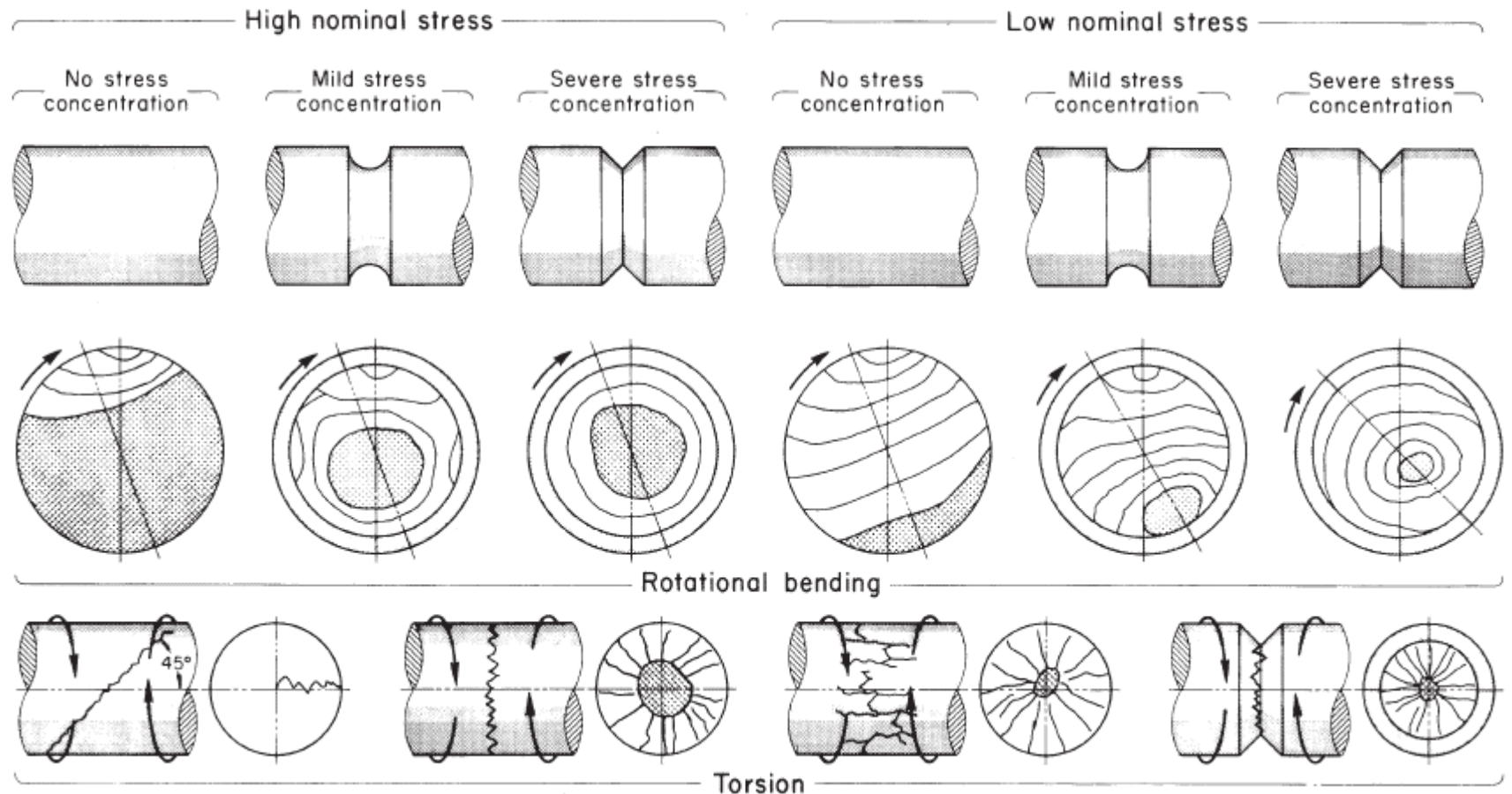
Fatigue: Unidirectional Bending



Fatigue: Reversed Bending



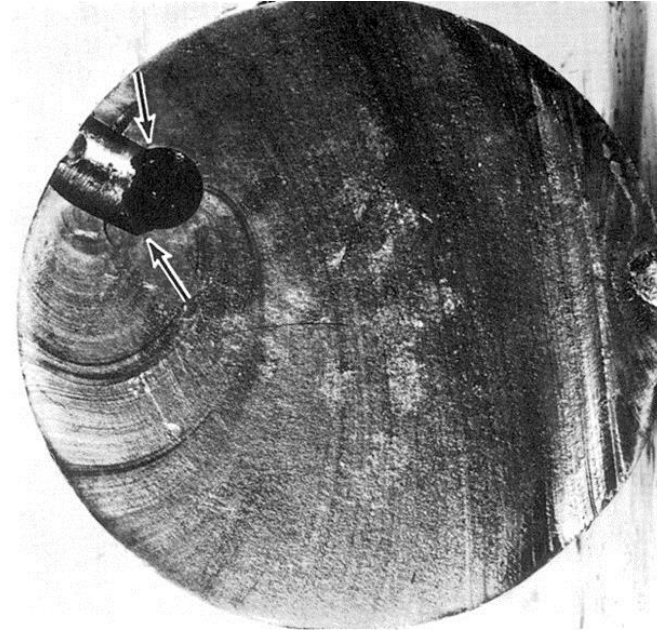
Fatigue: Rotational Bending and Torsion



Fatigue Fracture Examples



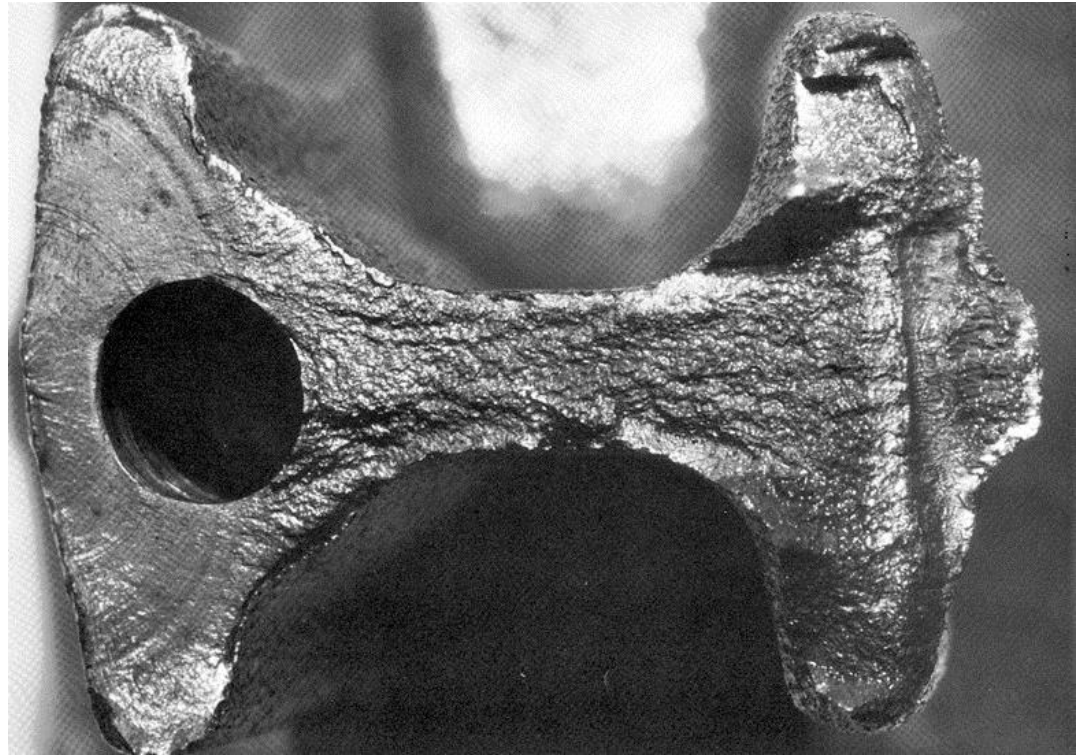
- AISI 4320 Drive Shaft
- B-Crack Initiation Stress Concentration in Keyway
- C-Final Brittle Failure



- Fatigue Failure Initiating at Mismatched Grease Holes
- Sharp Corners (at arrows) Provide Stress Concentration

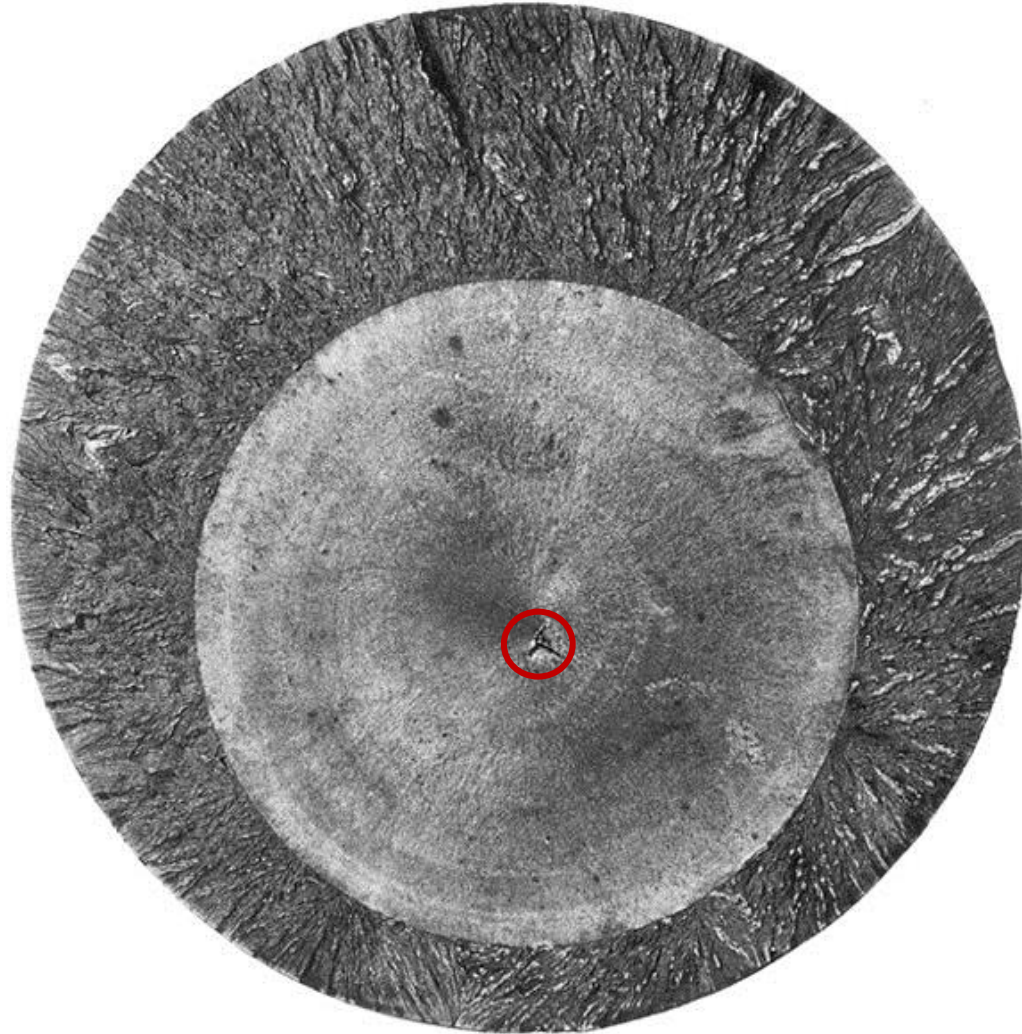
Fatigue Fracture Examples

- **Fatigue Failure of Forged Connecting Rod**
- **Crack Initiated at Flash Line of the Forging at the Left Edge of Picture**
- **Beach Marks Show Crack Propagation Halfway Around the Hole before Ultimate Fracture**



Fatigue Fracture Examples

- **Fatigue Failure of a 200 mm Diameter Piston Rod of an Alloy Steel Steam Hammer**
- **Loaded Axially**
- **Crack Initiated at a Forging Flake Internal to the Part**
- **Internal Crack Grew Outward Symmetrically**

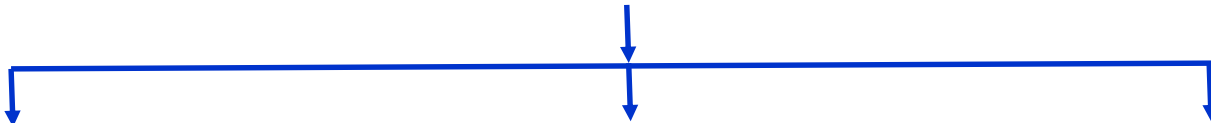


Fatigue and Fracture

- Introduction to Fatigue
- **Fatigue-Life Methods**
- Fatigue Strength and Endurance Limits
- Endurance Limit Modifying Factors
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- Fluctuating Stresses
- Combination of Loading Modes
- Varying, Fluctuating Stresses; Cumulative Fatigue Damage

Three Major Fatigue Life Models

Predict Life in Number of Cycles to Failure, N ,
for a Specified Level of Load



STRESS-LIFE METHOD

- Least Accurate (particularly for low cycle applications)
- Most Traditional
- Easiest to Implement

STRAIN-LIFE METHOD

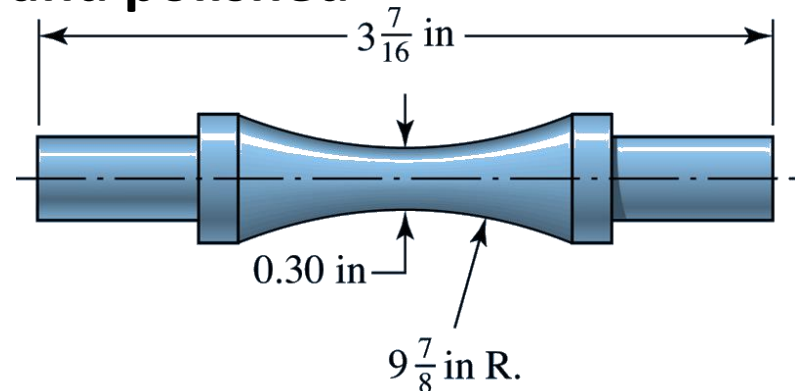
- Detailed Analysis of Plastic Deformation at Localized Region
- Several Idealizations Lead to Uncertainties in Results

LINEAR-ELASTIC FRACTURE MECHANICS

- Assumes Cracks Exists
- Predicts crack Growth with Respect to Stress Intensity

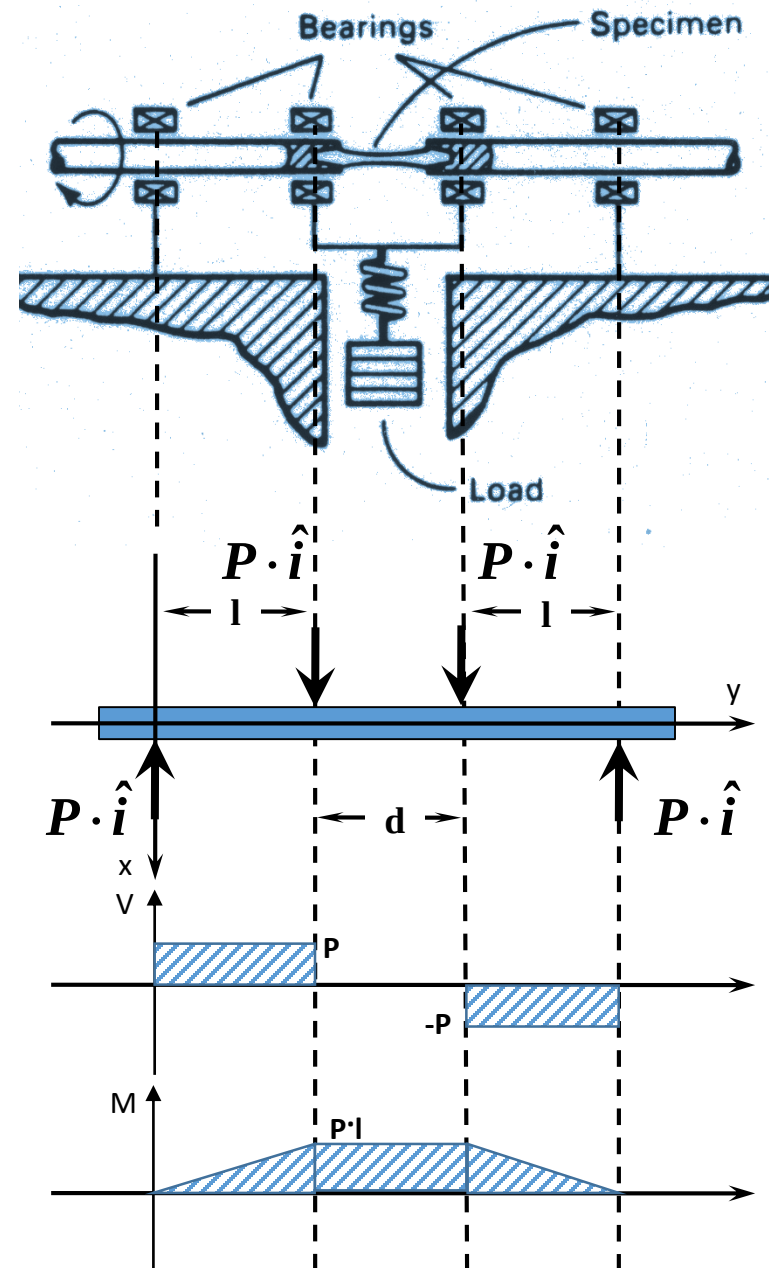
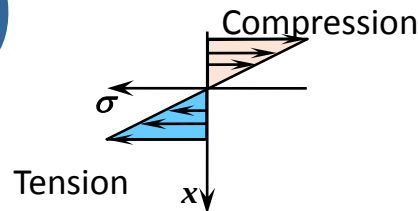
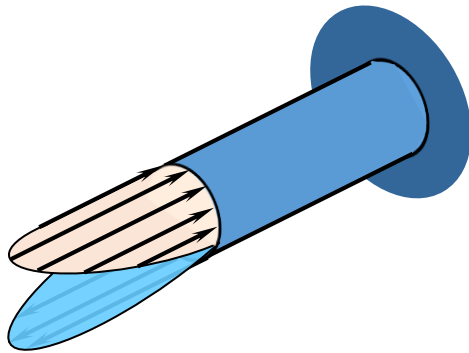
Stress-Life Method

- Test specimens are subjected to repeated stress while counting cycles to failure
- Most common test machine is R. R. Moore high-speed rotating-beam machine
- Subjects specimen to pure bending with no transverse shear
- As specimen rotates, stress fluctuates between equal magnitudes of tension and compression, known as *completely reversed* stress cycling
- Specimen is carefully machined and polished

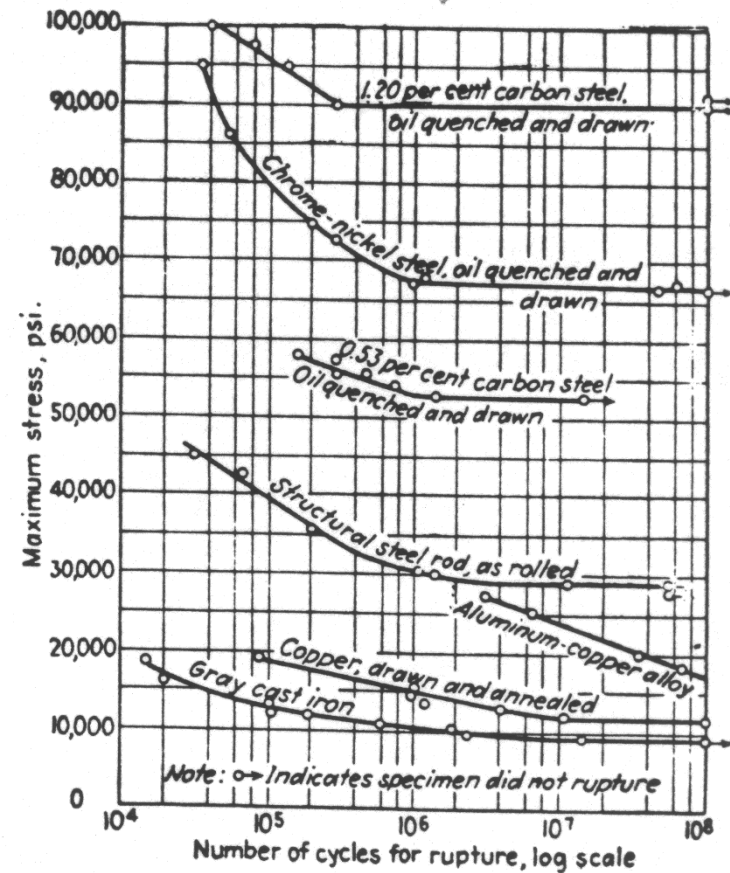
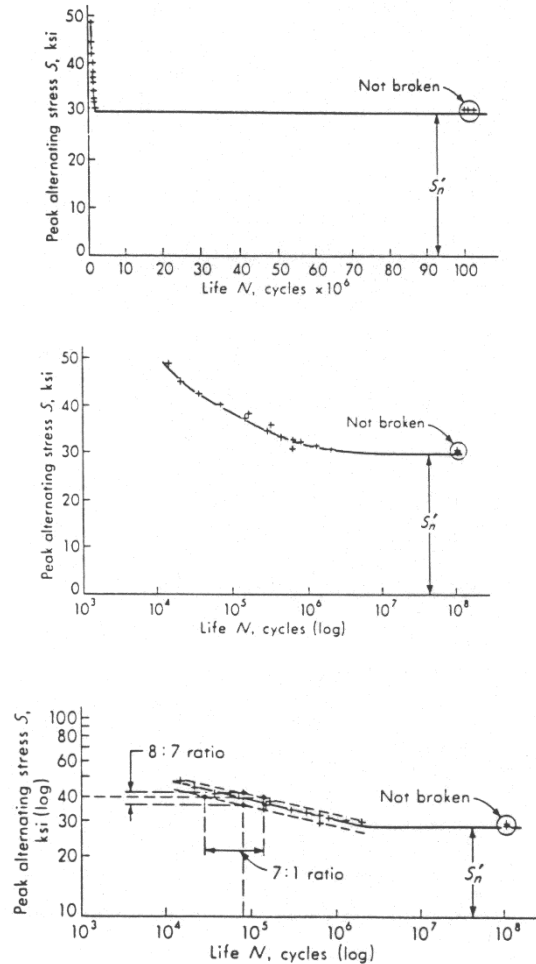


Stress-Life Method: R.R Moore Machine

- Repeated or varying loads
- Specified Magnitude



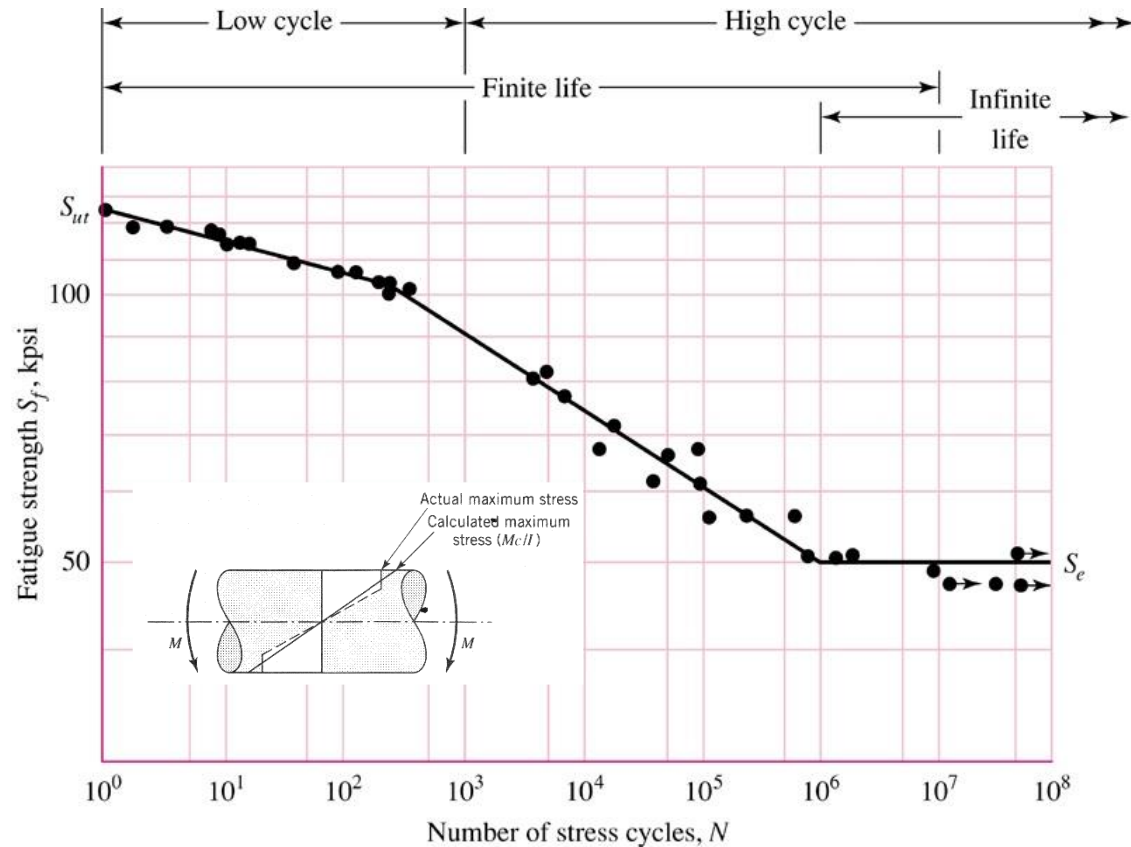
Stress-Life Method: Representing S-N Data



Stress-Life Method:

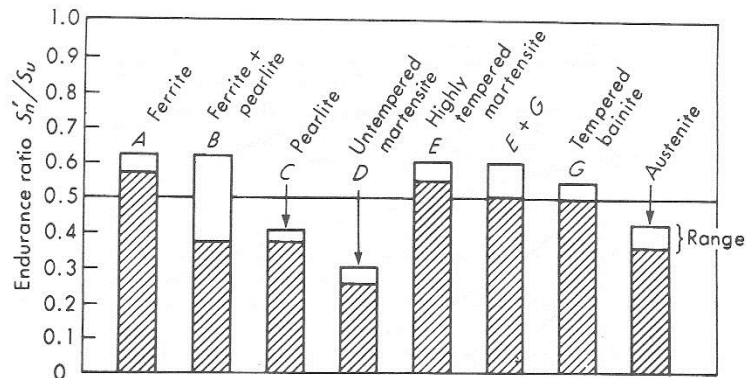
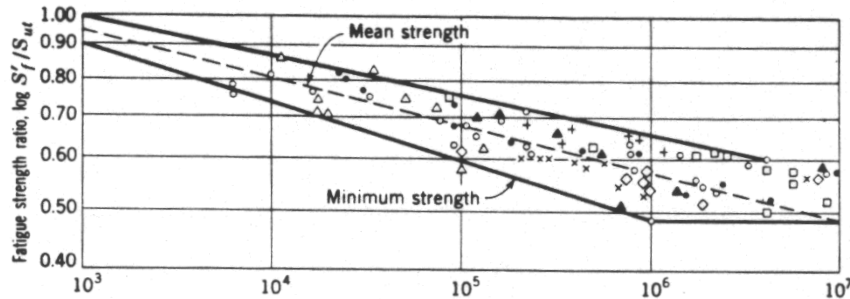
S-N Diagram – Log-Log Axes

- **Plotted on Log-Log Scale**
- **Three Regions Observed**
 - **Low Cycle**
 - Significant Plastic Deformation
 - Difficult to Predict
 - **Finite Life, S_f**
 - **Endurance Limit, S_e**
 - Mostly Ferrous Metals
 - Infinite Life

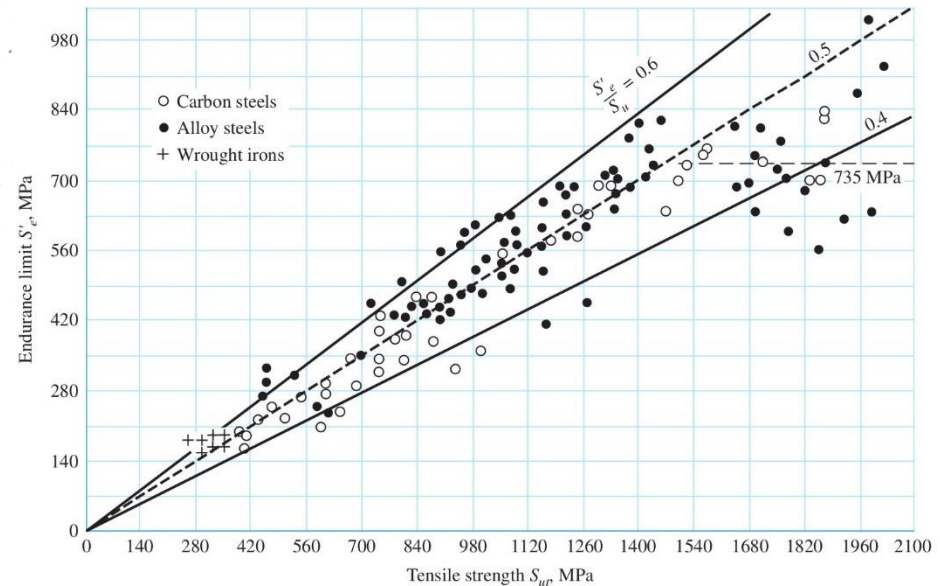


Stress-Life Method: S-N Curve for Steel

- The Endurance limit is approximately $0.5 S_{ut}$
- Cast Iron Behaves Similar to Steel
 - Except Endurance Limit is approximately $0.5 S_{ut}$



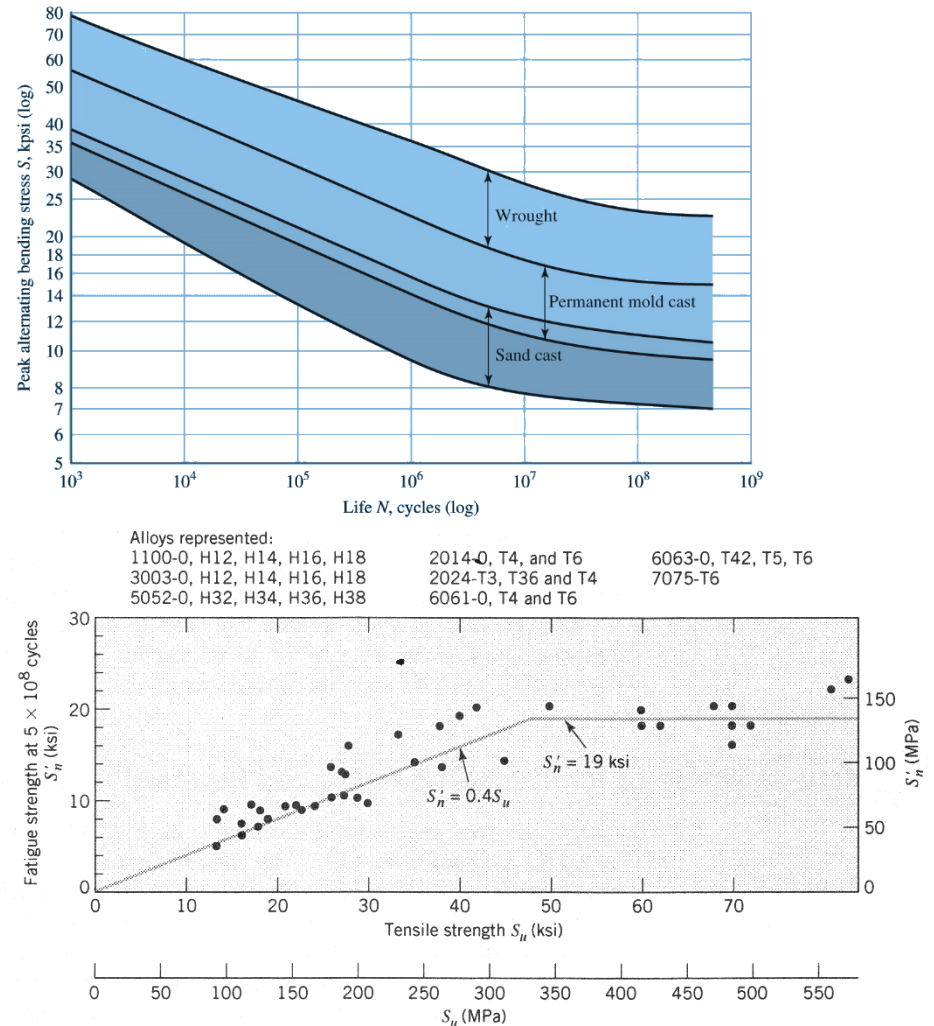
Effect of steel microstructure on endurance ratio.



Stress-Life Method:

S-N Curve for Aluminums

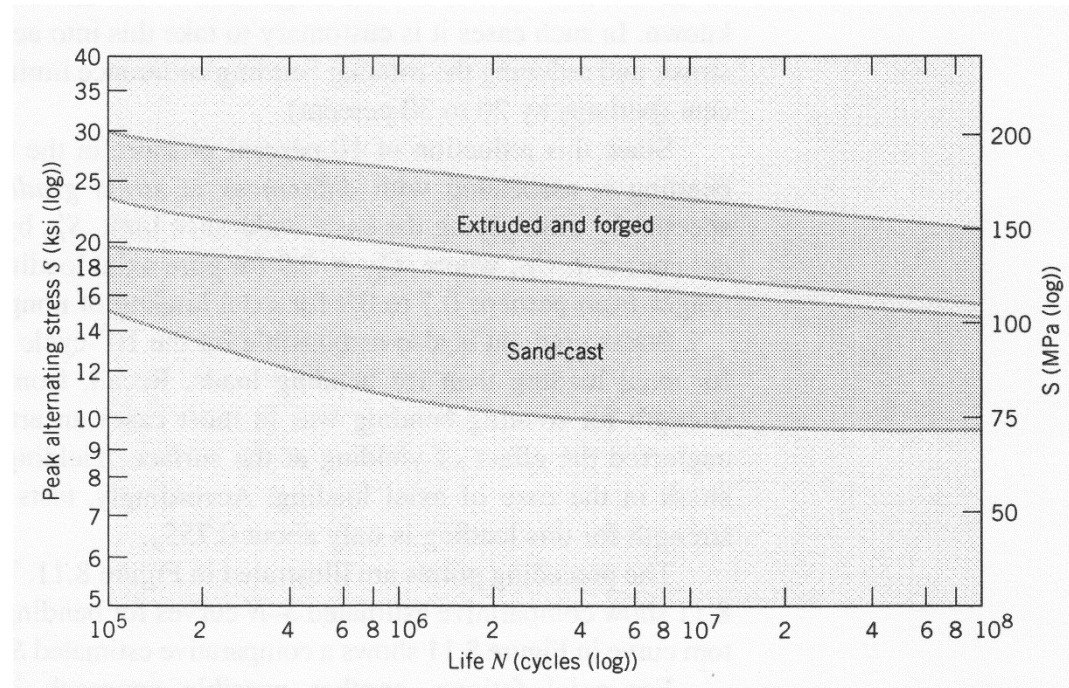
- Nonferrous Metals do not have an endurance limit
- Typically the Fatigue Strength at 5×10^8 cycles is reported at the Endurance Limit of $0.4 S_{ut}$



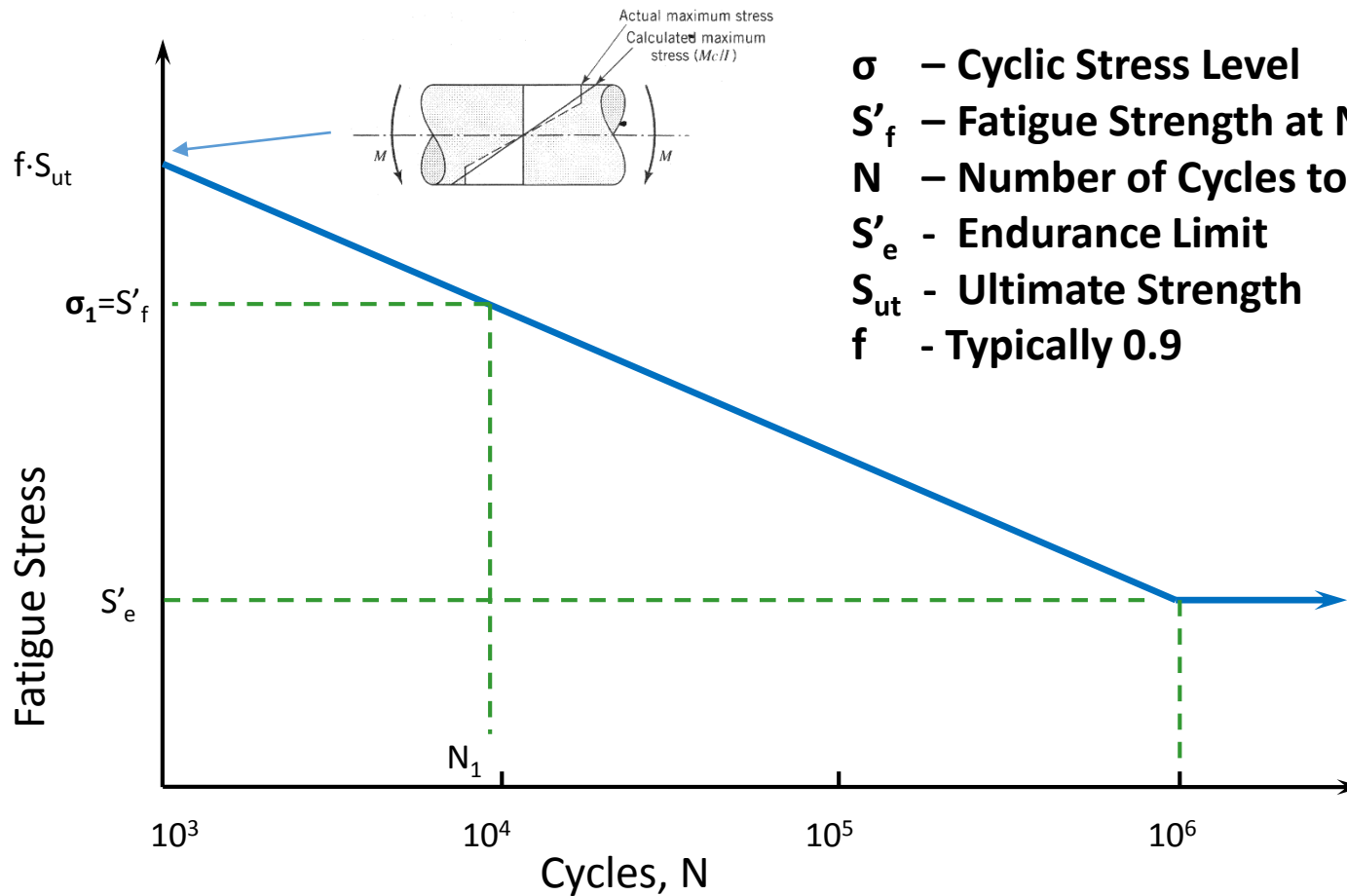
Stress-Life Method:

S-N Curve for Magnesium Alloys

- **Nonferrous Metals do not have an endurance limit**
- **Typically the Fatigue Strength at 10^8 cycles the reported Endurance Limit is approximately $0.35 S_{ut}$**



Stress-Life Method: Calculating Fatigue Life



σ – Cyclic Stress Level

S'_f – Fatigue Strength at N

N – Number of Cycles to Failure given S'_f

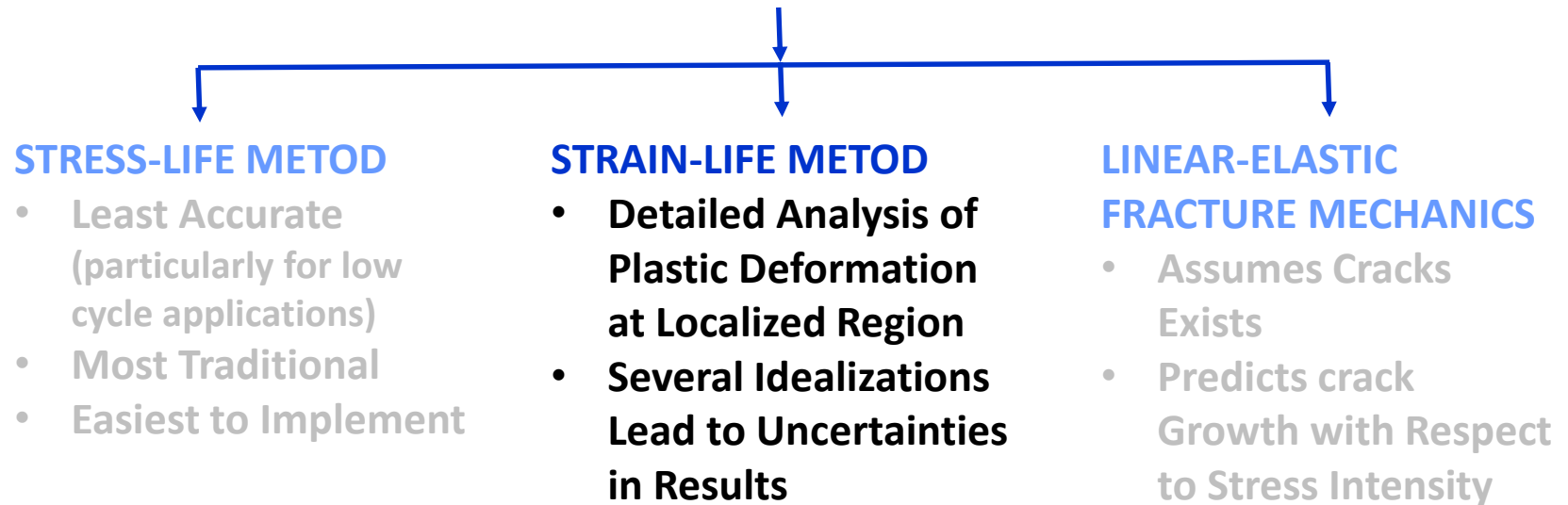
S'_e – Endurance Limit

S_{ut} – Ultimate Strength

f – Typically 0.9

Strain-Life Method:

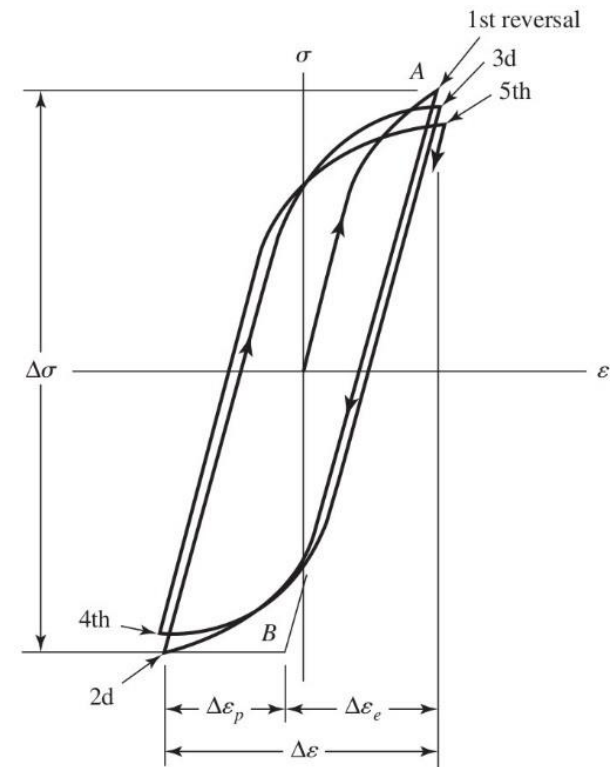
Predict Life in Number of Cycles to Failure, N ,
for a Specified Level of Load



Strain-Life Method:

Assists in understanding Fatigue

- Detailed analysis of Plastic Deformation at Localized Region
 - There must be CYCLIC PLASTIC STRAINS for Fatigue Fracture to Exist
- Cyclic Plastic Strains can change Elastic Limit, Leading to Fatigue
 - Elastic limits of *Annealed Steels* are likely to increase when subject to cycles of stress reversals
 - Elastic limits of *Cold-Drawn Steels* are likely to decrease when subject to cycles of stress reversals



True Stress – True Strain Diagram for first few cycles of a High Strength Steel being loaded

Strain-Life Method:

Fatigue Life is Related to TRUE Strain Amplitude

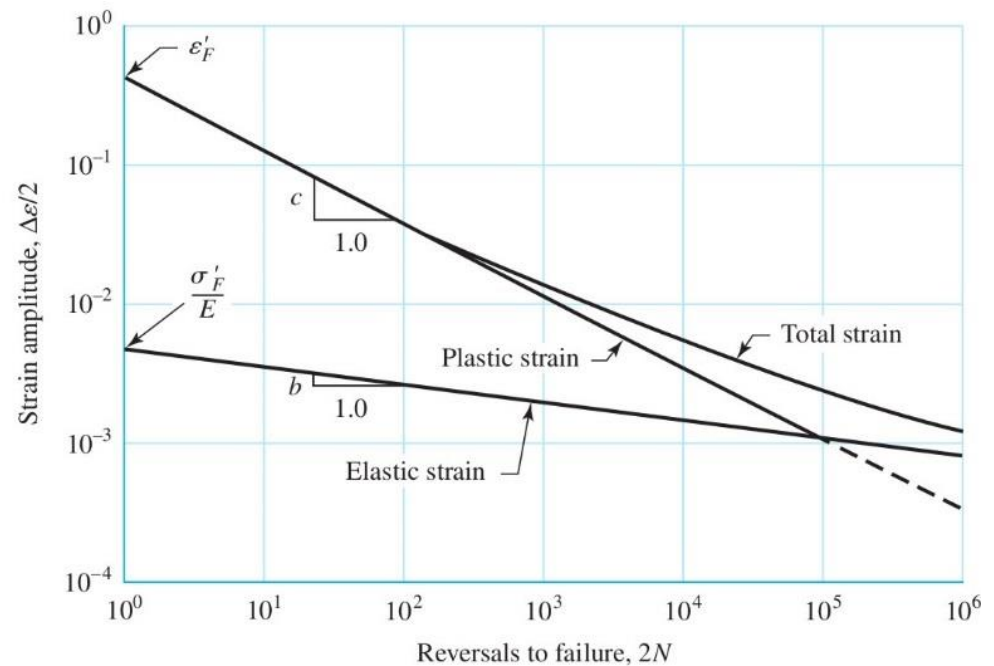
IMPORTANT TERMS

ε_f' $\equiv$ **Fatigue Ductility Coefficient**
True-Strain corresponding to fracture in one reversal

σ_f' $\equiv$ **Fatigue Strength Coefficient**
True-Stress Corresponding to fracture in one reversal

c $\equiv$ **Fatigue Ductility Exponent**
Slope of the plastic strain line

b $\equiv$ **Fatigue Strength Exponent**
Slope of the elastic strain line



Strain-Life Method:

Total Strain is Sum of Elastic and Plastic Strain

Strain Amplitude is Half the Total
Strain Range

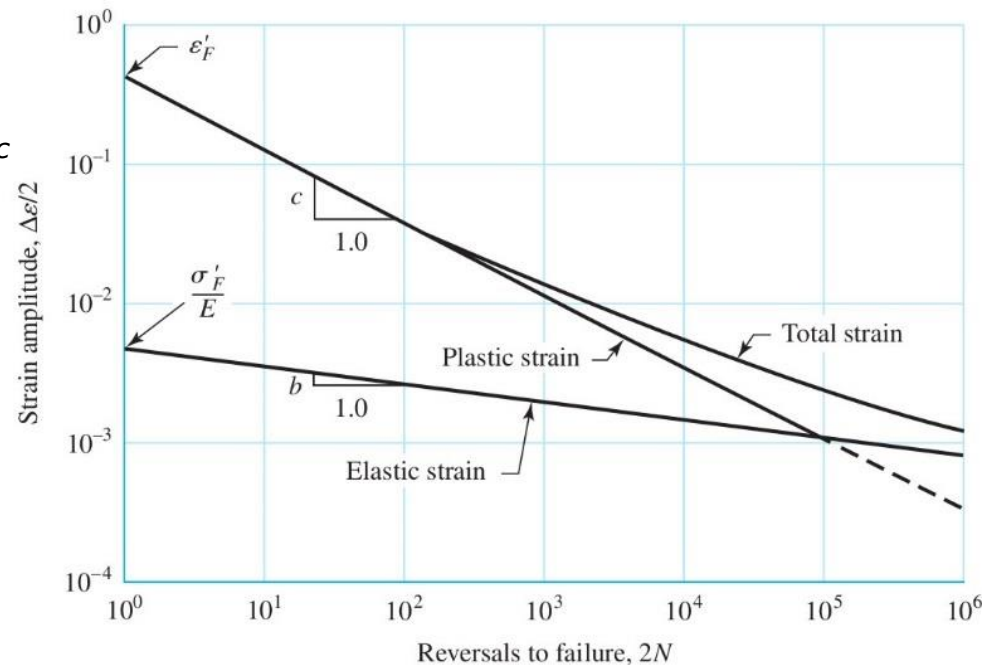
$$\begin{aligned}\frac{\Delta \varepsilon}{2} &= \frac{\Delta \varepsilon_e}{2} + \frac{\Delta \varepsilon_p}{2} \\ &= \frac{\sigma'_f}{E} \cdot (2 \cdot N)^b + \varepsilon'_f \cdot (2 \cdot N)^c\end{aligned}$$

Plastic Strain Line Equation

$$\frac{\Delta \varepsilon_p}{2} = \varepsilon'_f \cdot (2 \cdot N)^c$$

Elastic Strain Line Equation

$$\frac{\Delta \varepsilon_e}{2} = \frac{\sigma'_f}{E} \cdot (2 \cdot N)^b$$



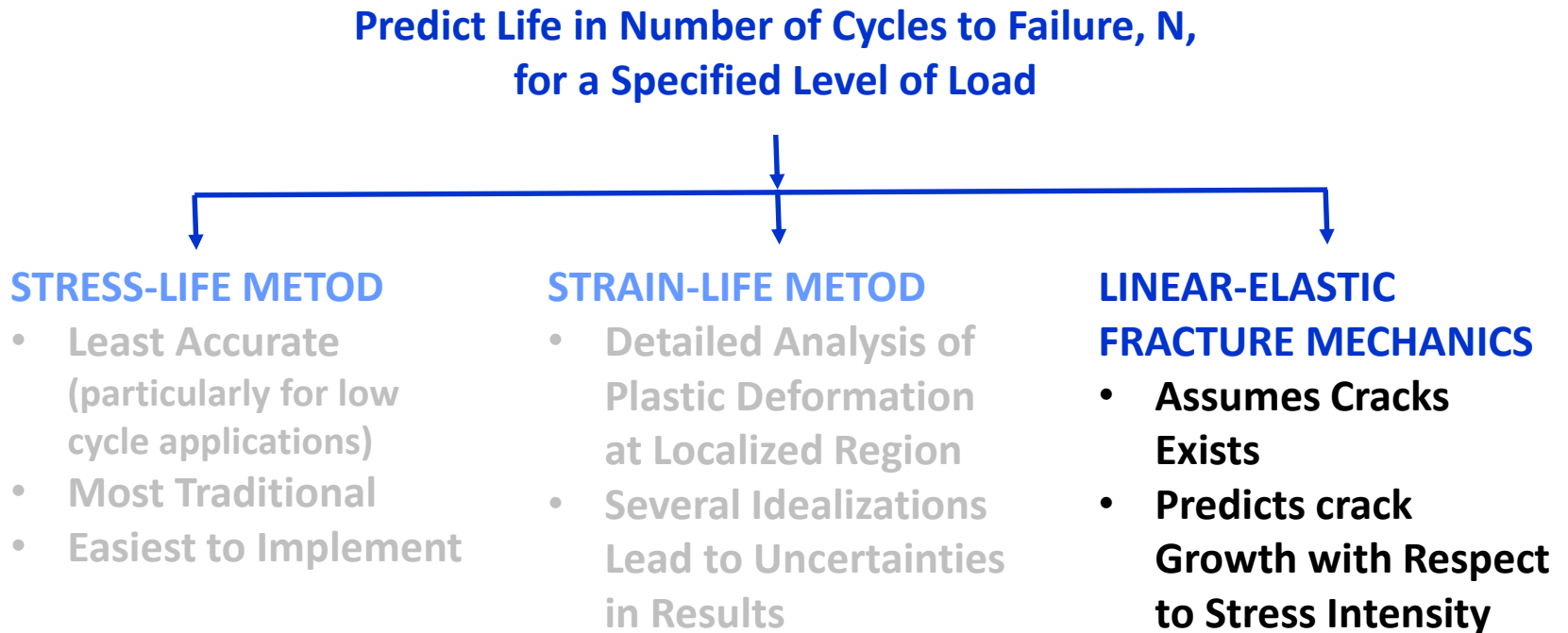
Strain-Life Method: $\frac{\Delta \varepsilon}{2} = \frac{\sigma'_f}{E} \cdot (2 \cdot N)^b + \varepsilon'_f \cdot (2 \cdot N)^c$

Manson-Coffin Relationship For Steel

Grade (a)	Orienta- tion (e)	Description (f)	Hard- ness HB	Tensile Strength S_{ut}		Reduction in Area %	True Strain at Fracture ε_f	Modulus of Elasticity E		Fatigue Strength Coefficient σ'_f		Fatigue Strength Exponent b	Fatigue Ductility Coefficient ε'_f	Fatigue Ductility Exponent c
				MPa	ksi			GPa	10 ⁶ psi	MPa	ksi			
A538A (b)	L	STA	405	1515	220	67	1.10	185	27	1655	240	-0.065	0.30	-0.62
A538B (b)	L	STA	460	1860	270	56	0.82	185	27	2135	310	-0.071	0.80	-0.71
A538C (b)	L	STA	480	2000	290	55	0.81	180	26	2240	325	-0.07	0.60	-0.75
AM-350 (c)	L	HR, A		1315	191	52	0.74	195	28	2800	406	-0.14	0.33	-0.84
AM-350 (c)	L	CD	496	1905	276	20	0.23	180	26	2690	390	-0.102	0.10	-0.42
Gainex (c)	LT	HR sheet		530	77	58	0.86	200	29.2	805	117	-0.07	0.86	-0.65
Gainex (c)	L	HR sheet		510	74	64	1.02	200	29.2	805	117	-0.071	0.86	-0.68
H-11	L	Ausformed	660	2585	375	33	0.40	205	30	3170	460	-0.077	0.08	-0.74
RQC-100 (c)	LT	HR plate	290	940	136	43	0.56	205	30	1240	180	-0.07	0.66	-0.69
RQC-100 (c)	L	HR plate	290	930	135	67	1.02	205	30	1240	180	-0.07	0.66	-0.69
10B62	L	Q&T	430	1640	238	38	0.89	195	28	1780	258	-0.067	0.32	-0.56
1005-1009	LT	HR sheet	90	360	52	73	1.3	205	30	580	84	-0.09	0.15	-0.43
1005-1009	LT	CD sheet	125	470	68	66	1.09	205	30	515	75	-0.059	0.30	-0.51
1005-1009	L	CD sheet	125	415	60	64	1.02	200	29	540	78	-0.073	0.11	-0.41
1005-1009	L	HR sheet	90	345	50	80	1.6	200	29	640	93	-0.109	0.10	-0.39
1015	L	Normalized	80	415	60	68	1.14	205	30	825	120	-0.11	0.95	-0.64
1020	L	HR plate	108	440	64	62	0.96	205	29.5	895	130	-0.12	0.41	-0.51
1040	L	As forged	225	620	90	60	0.93	200	29	1540	223	-0.14	0.61	-0.57
1045	L	Q&T	225	725	105	65	1.04	200	29	1225	178	-0.095	1.00	-0.66
1045	L	Q&T	410	1450	210	51	0.72	200	29	1860	270	-0.073	0.60	-0.70
1045	L	Q&T	390	1345	195	59	0.89	205	30	1585	230	-0.074	0.45	-0.68
1045	L	Q&T	450	1585	230	55	0.81	205	30	1795	260	-0.07	0.35	-0.69
1045	L	Q&T	500	1825	265	51	0.71	205	30	2275	330	-0.08	0.25	-0.68
1045	L	Q&T	595	2240	325	41	0.52	205	30	2725	395	-0.081	0.07	-0.60
1144	L	CDSR	265	930	135	33	0.51	195	28.5	1000	145	-0.08	0.32	-0.58

Linear-Elastic Fracture Mechanics:

Predict Life in Number of Cycles to Failure, N ,
for a Specified Level of Load



STRESS-LIFE METHOD

- Least Accurate (particularly for low cycle applications)
- Most Traditional
- Easiest to Implement

STRAIN-LIFE METHOD

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- Several Idealizations Lead to Uncertainties in Results

LINEAR-ELASTIC FRACTURE MECHANICS

- Assumes Cracks Exists
- Predicts crack Growth with Respect to Stress Intensity

Linear-Elastic Fracture Mechanics: Crack Growth

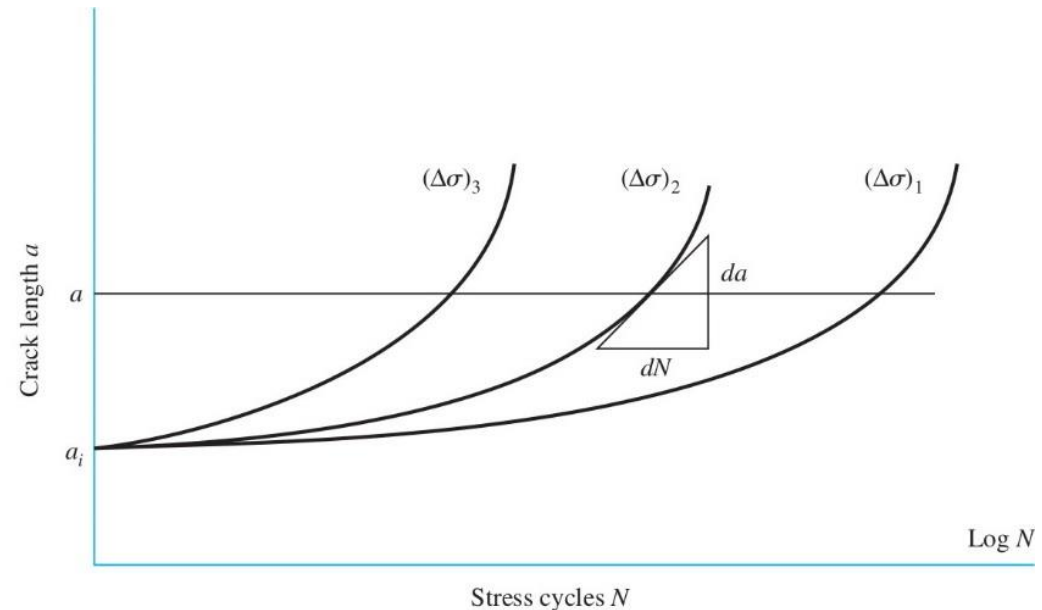
Stress Intensity Factor

$$K_I = \beta \cdot \sigma \cdot \sqrt{\pi \cdot a}$$

For Stress Range $\Delta\sigma$

$$\begin{aligned}\Delta K_I &= \beta \cdot (\sigma_{\max} - \sigma_{\min}) \cdot \sqrt{\pi \cdot a} \\ &= \beta \cdot \Delta\sigma \cdot \sqrt{\pi \cdot a}\end{aligned}$$

Figure Based on

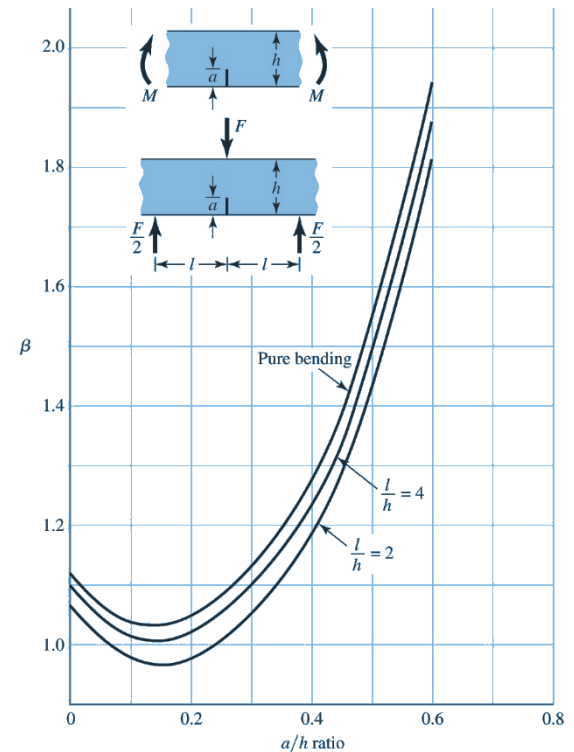
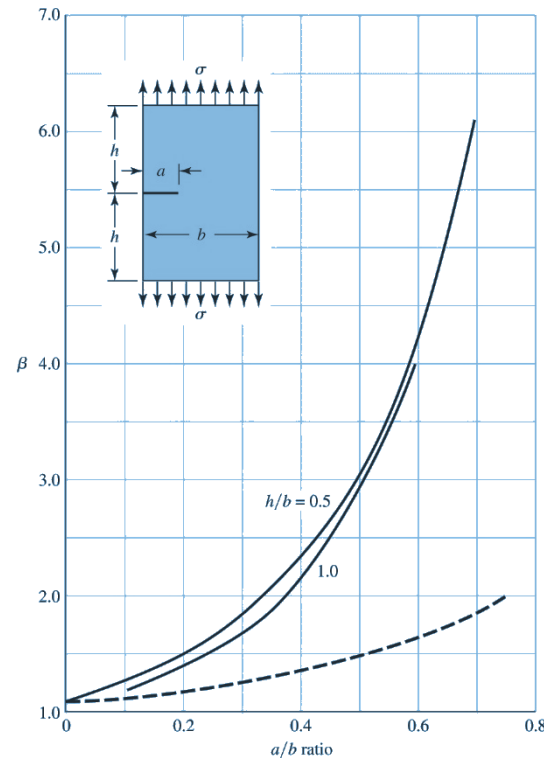
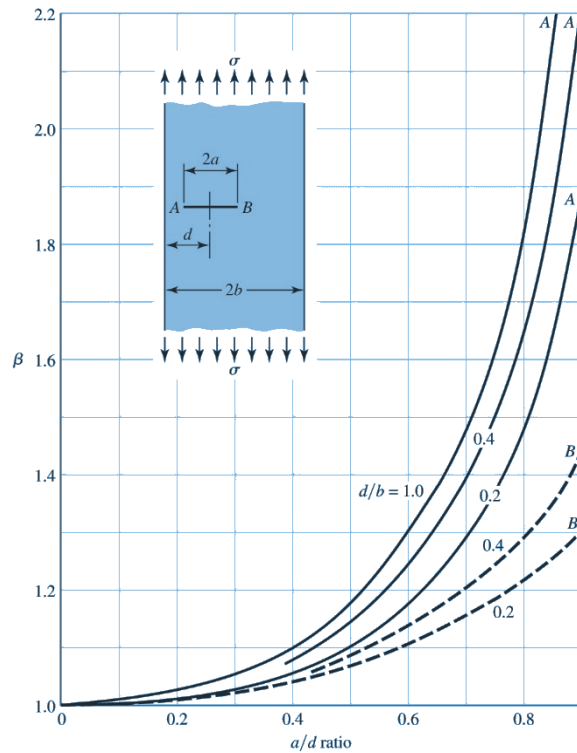


$$(\Delta K_I)_c > (\Delta K_I)_3 > (\Delta K_I)_2 > (\Delta K_I)_1 > (\Delta K_I)_{th} \Rightarrow (\Delta\sigma)_3 > (\Delta\sigma)_2 > (\Delta\sigma)_1$$

Linear-Elastic Fracture Mechanics:

β Dependent on Geometry

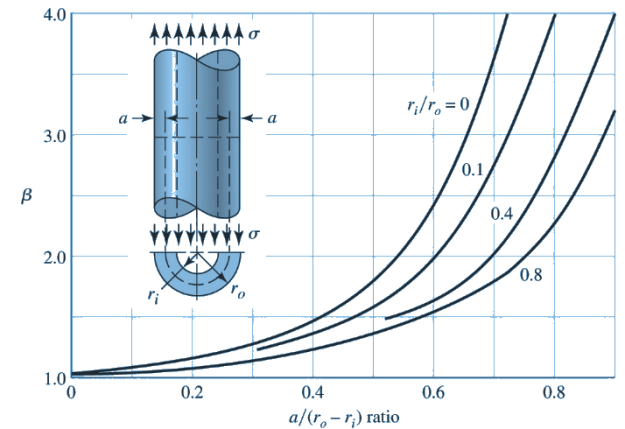
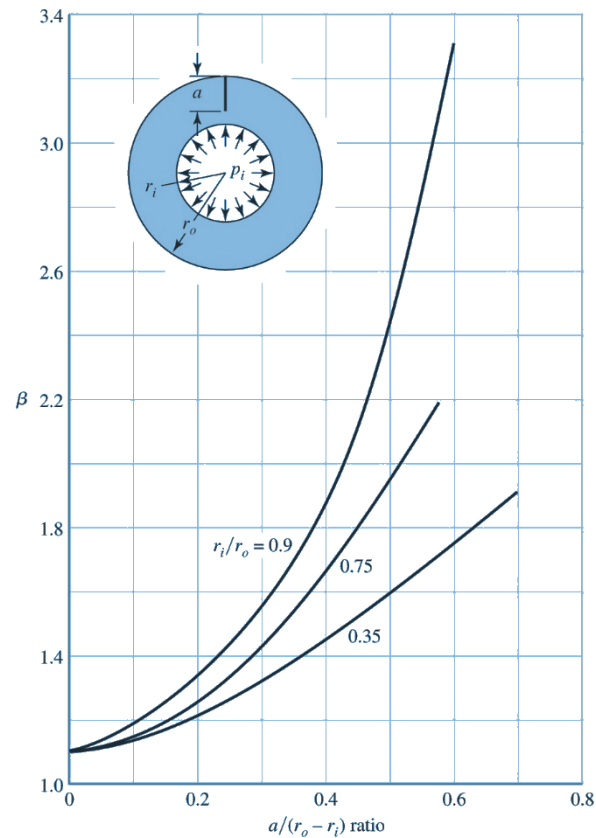
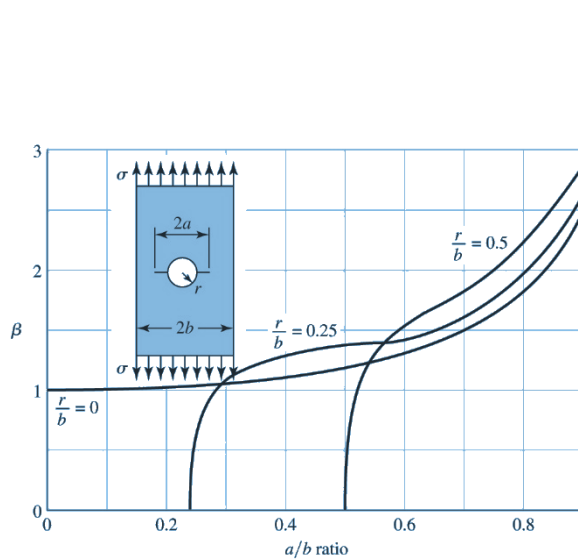
Stress Intensity Factor $K_I = \beta \cdot \sigma \cdot \sqrt{\pi \cdot a}$



Linear-Elastic Fracture Mechanics:

β Dependent on Geometry

Stress Intensity Factor $K_I = \beta \cdot \sigma \cdot \sqrt{\pi \cdot a}$



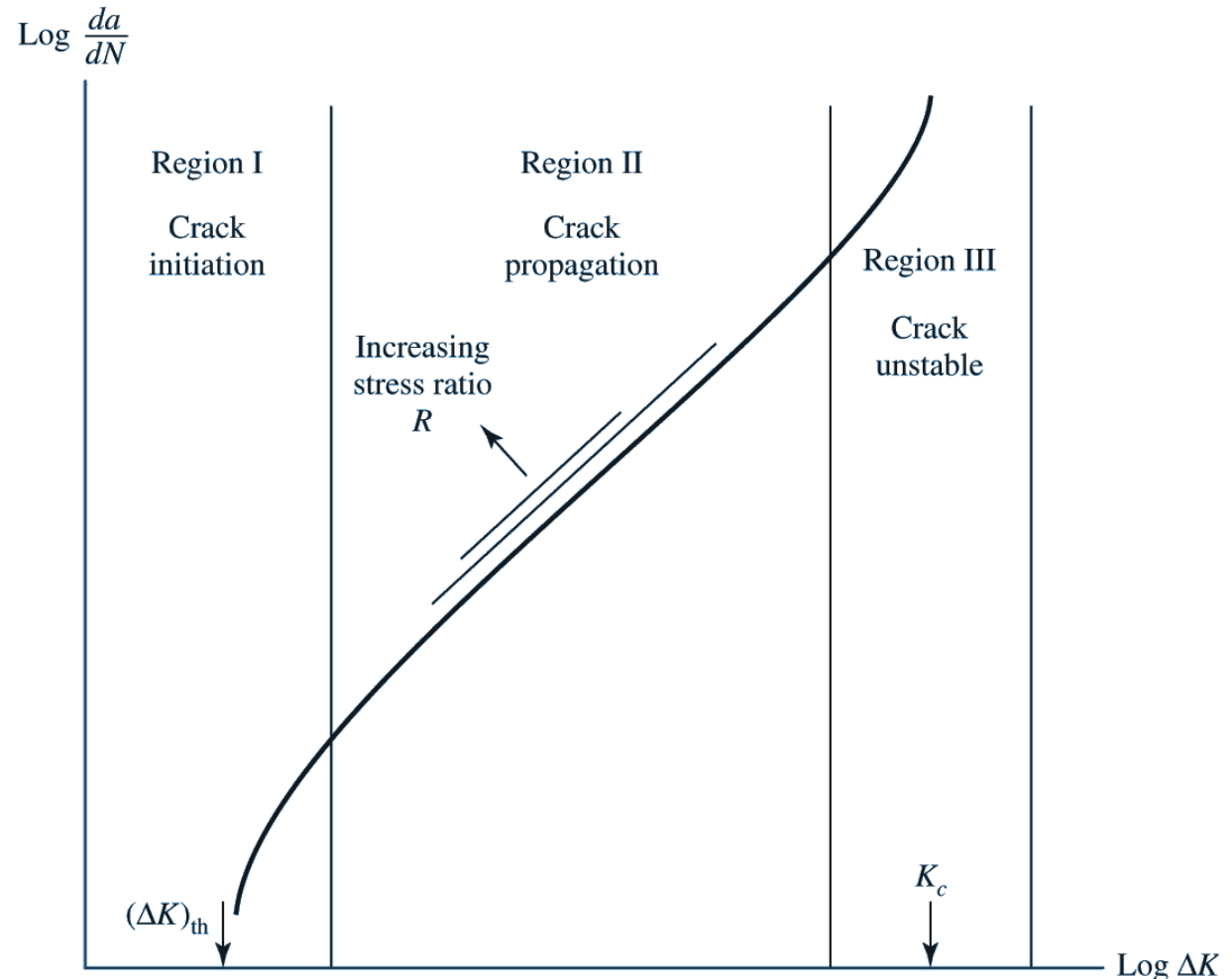
Linear-Elastic Fracture Mechanics: Three Stages of Crack Growth

Stress Ratio

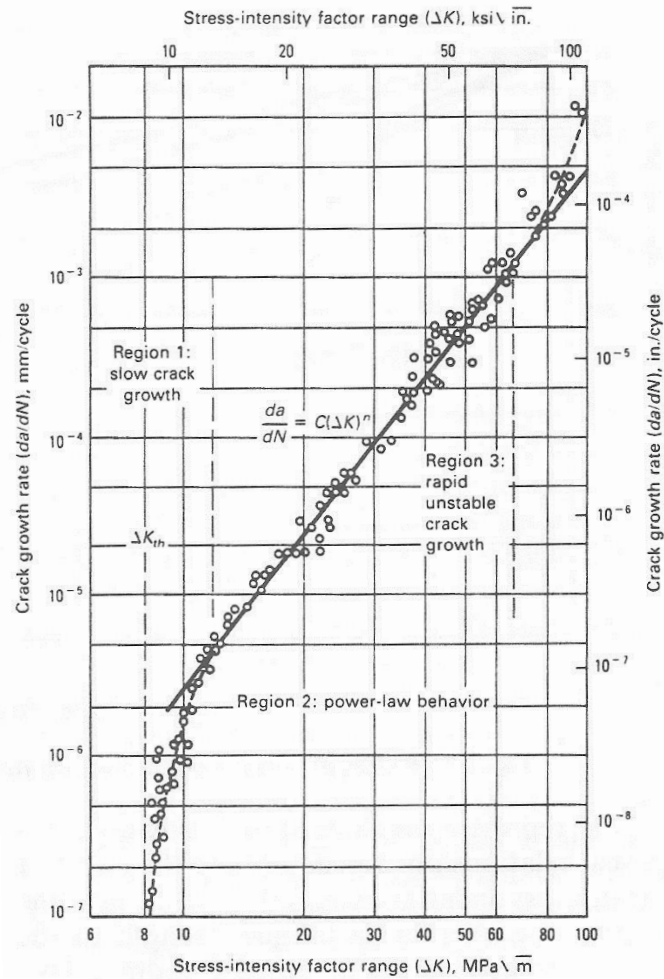
$$R = \frac{\sigma_{\min}}{\sigma_{\max}}$$

**Stage II is Linear
on Log-Log scale.**

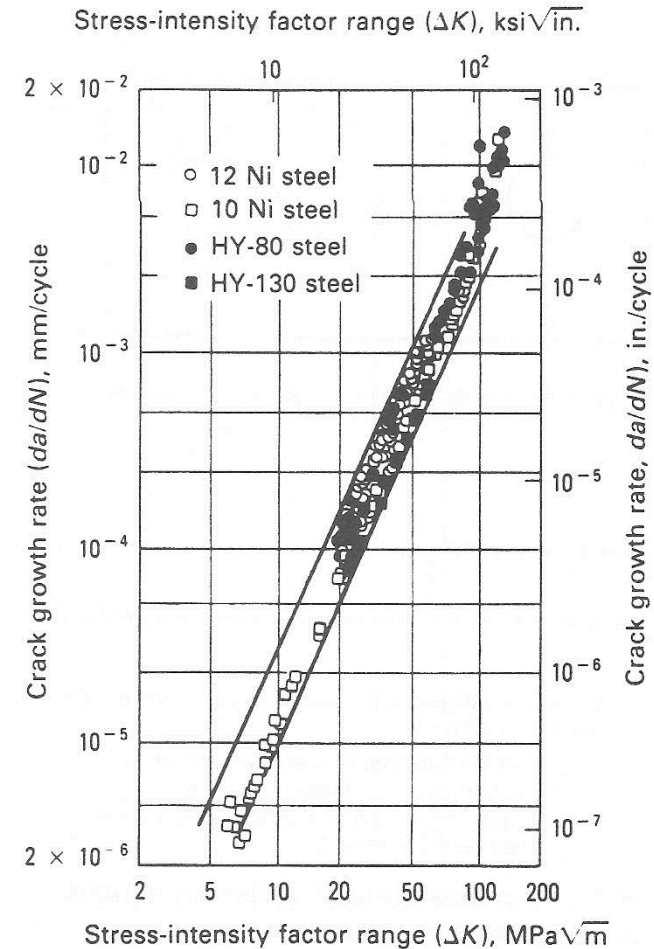
**Stage II Referred
to as Linear
Elastic Fracture
Mechanics
(LEFM)**



Linear-Elastic Fracture Mechanics: Typical Fracture Curves



ASTM A533 B1 Steel



Martensitic Steels

Linear-Elastic Fracture Mechanics:

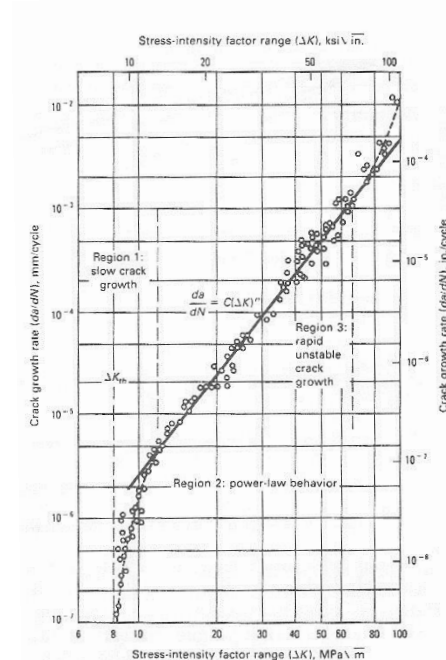
Growth in Region II follows Paris' Law

Paris' Equation

$$\frac{da}{dN} = C \cdot (\Delta K_I)^m$$

C and m are empirical constants

Typical values shown below for $R = \sigma_{\min} / \sigma_{\max} = 0$



Material	$C, \frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$	$C, \frac{\text{in/cycle}}{(\text{ksi}\sqrt{\text{in}})^m}$	m
Ferritic-pearlitic steels	$6.89(10^{-12})$	$3.60(10^{-10})$	3.00
Martensitic steels	$1.36(10^{-10})$	$6.60(10^{-9})$	2.25
Austenitic stainless steels	$5.61(10^{-12})$	$3.00(10^{-10})$	3.25

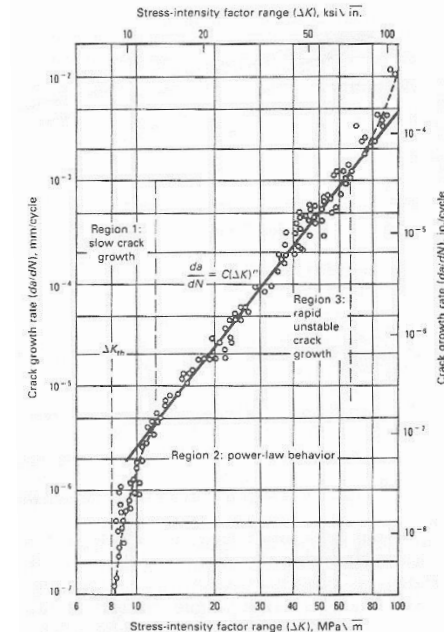
Linear-Elastic Fracture Mechanics:

Growth in Region II follows Paris' Law

Paris' Equation can be Integrated

$$\int_0^{N_f} dN = N_f = \frac{1}{C} \cdot \int_{a_i}^{a_f} \frac{da}{\left(\beta \cdot \Delta \sigma \cdot \sqrt{\pi \cdot a} \right)^m}$$

Typical values shown below for $R = \sigma_{\min} / \sigma_{\max} = 0$



Material	$C, \frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$	$C, \frac{\text{in/cycle}}{(\text{kpsi}\sqrt{\text{in}})^m}$	m
Ferritic-pearlitic steels	$6.89(10^{-12})$	$3.60(10^{-10})$	3.00
Martensitic steels	$1.36(10^{-10})$	$6.60(10^{-9})$	2.25
Austenitic stainless steels	$5.61(10^{-12})$	$3.00(10^{-10})$	3.25

Linear-Elastic Fracture Mechanics: Numerical Methods Applied

Paris' Equation in Discrete Form

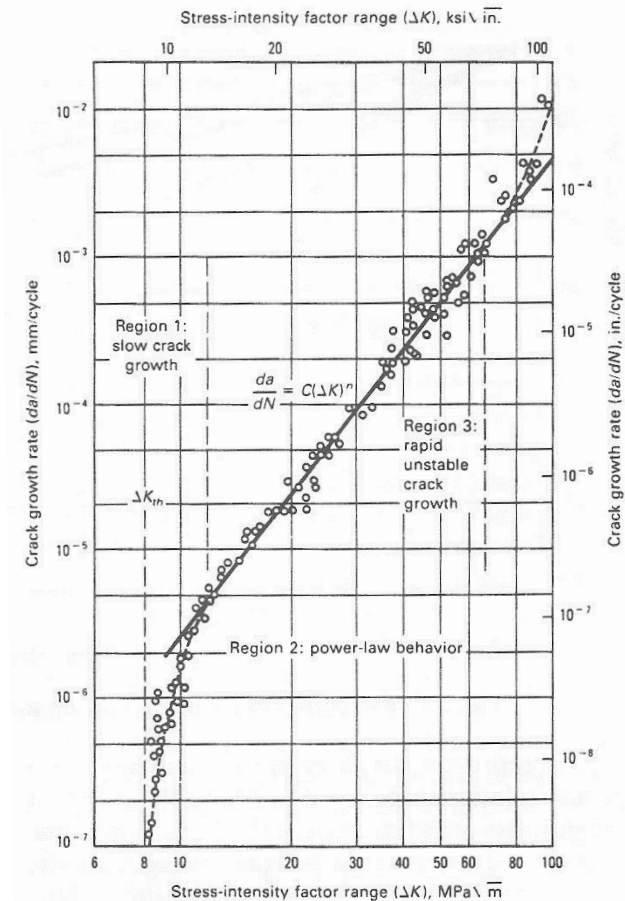
$$\delta a_j = C \cdot (\Delta K_I)_j^m \cdot (\delta N)_j$$

$$a_{j+1} = a_j + \delta a_j$$

$$N_{j+1} = N_j + \delta N_j$$

$$N_f = \sum \delta N_j$$

NASA/FLAWGROW



Fatigue and Fracture

- Introduction to Fatigue
- Fatigue-Life Methods
- **Fatigue Strength and Endurance Limits**
- Endurance Limit Modifying Factors
- Stress Concentrations and Notch Sensitivity
- Fluctuating Stresses
- Combination of Loading Modes
- Varying, Fluctuating Stresses; Cumulative Fatigue Damage

Endurance Limit and Strength

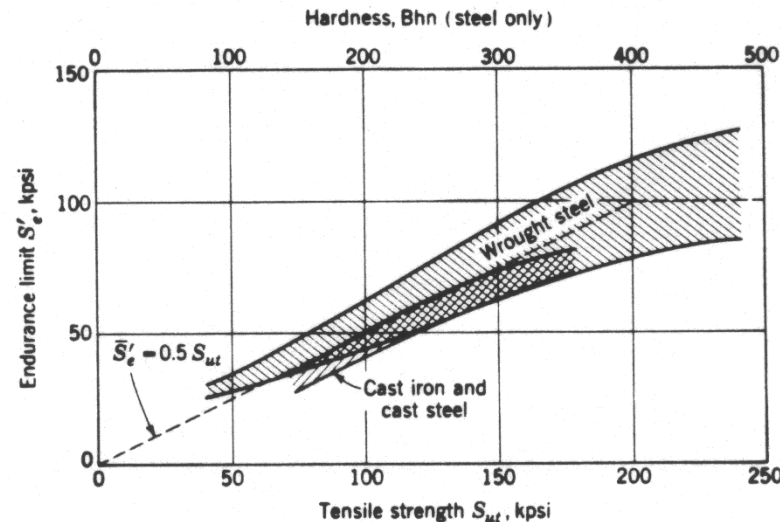
$S_{UT} \equiv$ *Ultimate Tensile Strength*

$\bar{S}'_e \equiv$ *Mean Endurance Limit of Test Specimen*

$\bar{S}_e \equiv$ *Mean Endurance Limit of Structural Element*

$S'_f \equiv$ *Fatigue Strength of Test Specimen*

$S_f \equiv$ *Fatigue Strength of Structural Element*



Rules of Thumb for STEEL

- English Units

$$\bar{S}'_e = 0.5 \cdot S_{UT} \quad \text{when} \quad S_{UT} \leq 200 \text{ksi}$$

$$\bar{S}'_e = 100 \text{ksi} \quad \text{when} \quad S_{UT} > 200 \text{ksi}$$

- Metric Units

$$\bar{S}'_e = 0.5 \cdot S_{UT} \quad \text{when} \quad S_{UT} \leq 1400 \text{MPa}$$

$$\bar{S}'_e = 700 \text{MPa} \quad \text{when} \quad S_{UT} > 1400 \text{MPa}$$

Some Additional Rules of Thumb

- **Cast Iron**

$$\bar{S}'_e = 0.4 \cdot S_{UT}$$

- **Aluminum and Magnesium**

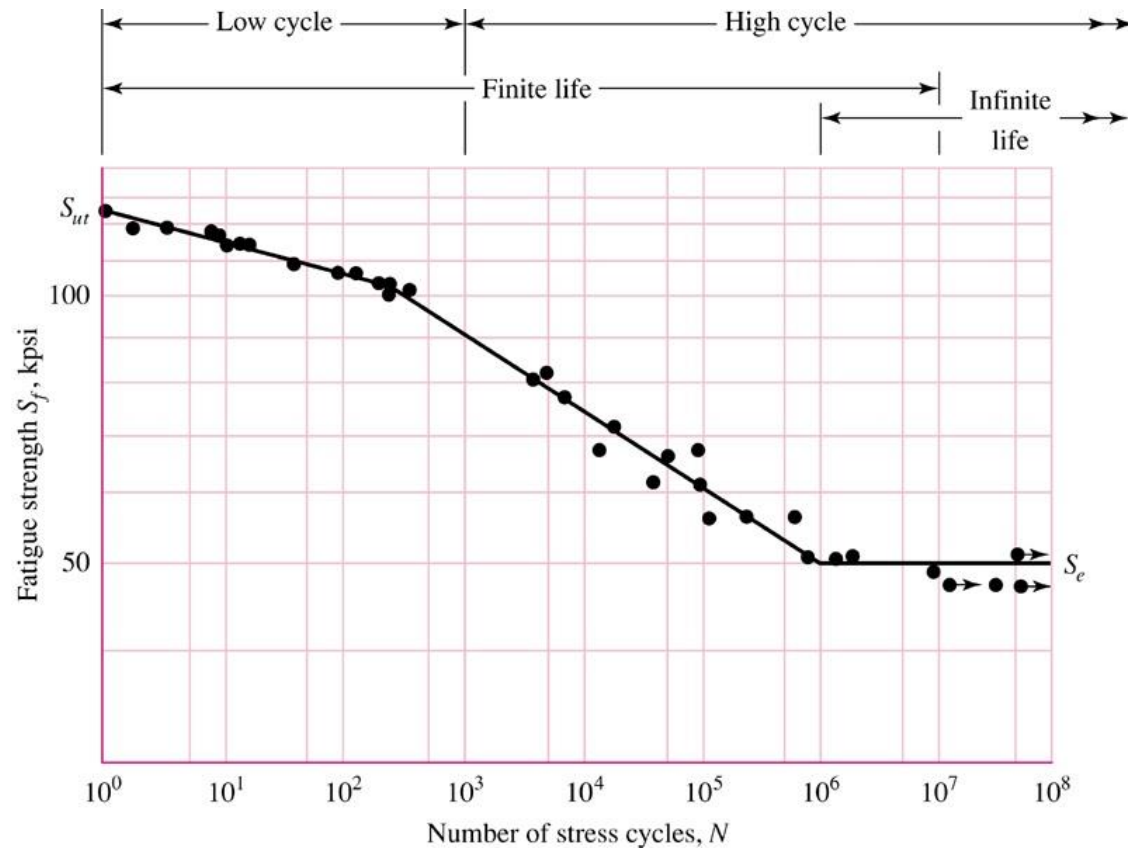
$$\bar{S}'_e \approx 0.3 \text{ to } 0.4 \cdot S_{UT}$$

Values

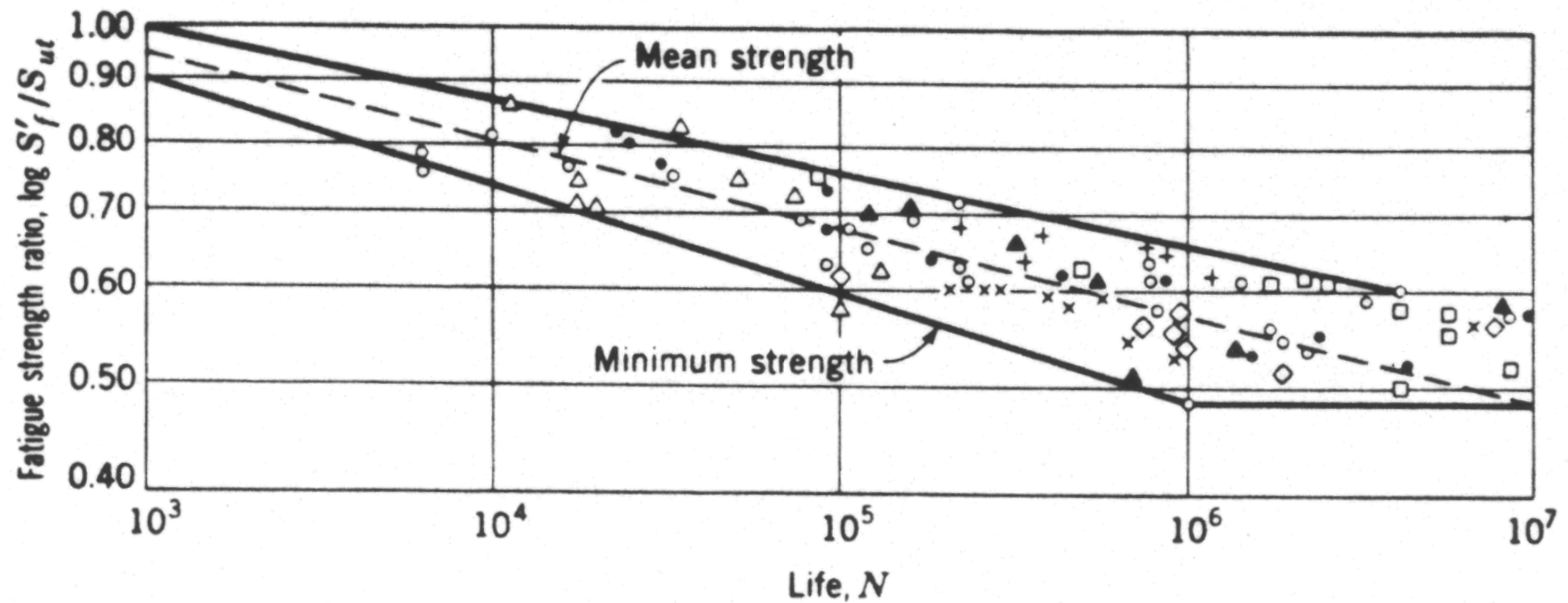
(Reported by F.B. Stulen, H.N. Cummings, and W.C. Schulte, Preventing Fatigue Failures, Part 5, *Machine Design*, vol. 33, P. 161, 22 June 1961)

Material UNS No. (Alloys are heat treated, hot worked, specimens smooth, subjected to long life rotating beam test)	Tensile Strength, S_{ut}		Endurance Limit, S'_E		Standard Deviation	
	MPa	ksi	MPa	ksi	ksi	%
G43400 Steel	965	140	489	71	3.5	4.9
	1310	190	586	85	6.7	7.8
	1580	230	620	90	5.3	5.9
	1790	260	668	97	6.3	6.5
G43500 Steel	2070	300	689	100	4.4	4.4
R50001-series Titanium Alloy	1000	145	579	84	5.4	6.4
A97076 Aluminum Alloy	524	76	186	27	1.6	6.0
C63000 Aluminum Bronze	806	117	331	48	4.5	9.4
C17200 Beryllium Copper	1210	175	248	36	2.7	7.5

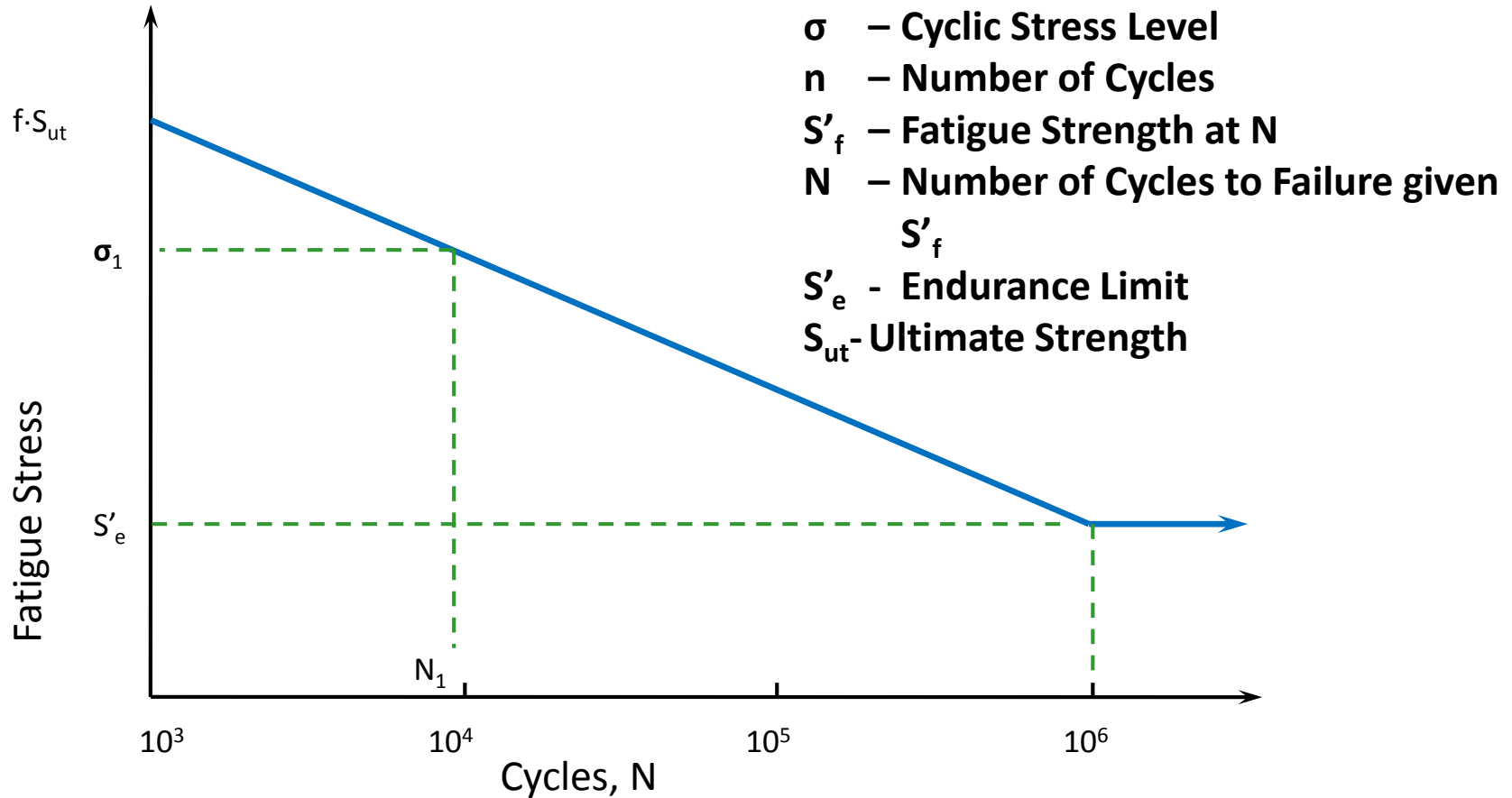
Finite Life, S'_f , The S-N Curve: High Cycle Fatigue



Finite Life, S'_f , The S-N Curve



Constructing the Log-Log Plot



Equation for the S-N Curve

S-N Curve Equation

$$\log S'_f = -m \cdot \log n + b$$

where

$$m = \frac{1}{3} \cdot \log \left(\frac{f \cdot S_{ut}}{S'_e} \right)$$

$$b = \log \left(\frac{(f \cdot S_{ut})^2}{S'_e} \right)$$

f is often taken as 0.9

S-N Curve Calculations

In the range $10^3 \leq y \leq 10^6$

Given N , S'_f is

$$S' = \frac{10^b}{N^m}$$

Given S'_f , N is

$$N = \frac{10^{b/m}}{S'^{1/m}_f}$$

Example

The endurance limit of a steel member is 112MPa and the tensile strength is 385MPa. What fatigue strength corresponds to a life of $70(10^3)$ cycles.

Fatigue and Fracture

- Introduction to Fatigue
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- **Endurance Limit Modifying Factors**
- Stress Concentrations and Notch Sensitivity
- Fluctuating Stresses
- Combination of Loading Modes
- Varying, Fluctuating Stresses; Cumulative Fatigue Damage

Endurance Limit Modifying Factors

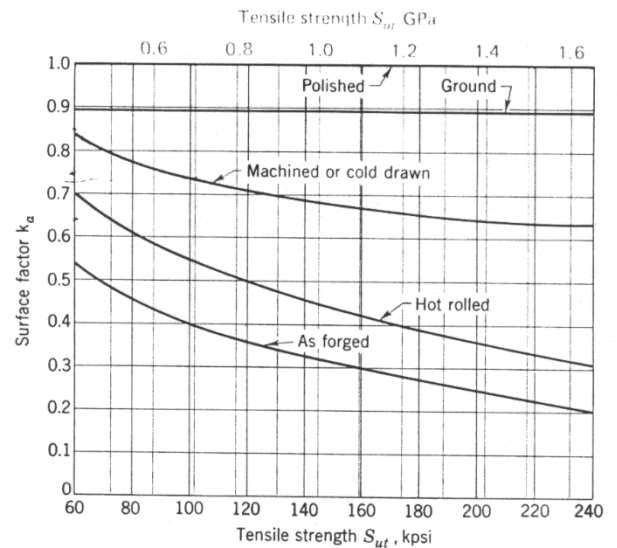
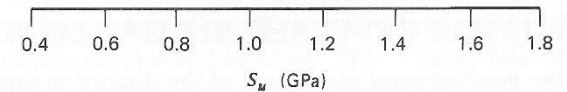
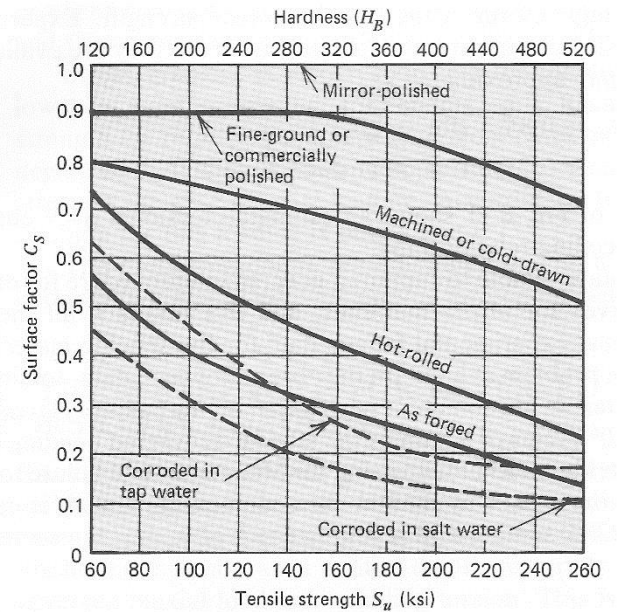
- Unrealistic to expect Structural or Mechanical Component to Match Laboratory Specimen
- $S_e = k_a k_b k_c k_d k_e k_f S'_e$
 - k_a -surface factor
 - k_b -size factor
 - k_c -load factor
 - k_d -temperature factor
 - k_e -reliability factor
 - k_f -miscellaneous effects factor

k_a – Surface factor

$$k_a = a \cdot S_{ut}^b$$

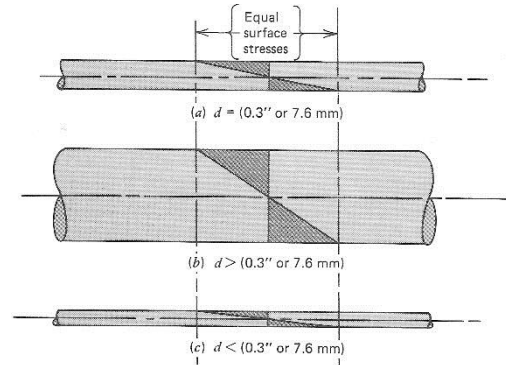
Surface Finish	Factor a		Exponent b
	S_{ut} , kpsi	S_{ut} , MPa	
Ground	1.34	1.58	-0.085
Machined or cold-drawn	2.70	4.51	-0.265
Hot-rolled	14.4	57.7	-0.718
As-forged	39.9	272.	-0.995

k_a for Ordinary Gray Cast Iron $\cong 1$



k_b – Size Effect, Circular X-Sections

- Rotating beam test specimen is 0.3 in in diameter



- For rotating bending and torsion, k_b

$$k_b = \begin{cases} \left(\frac{d}{0.3}\right)^{-0.107} = 0.879 \cdot d^{-0.107} & 0.11in \leq d \leq 2in \\ 0.91 \cdot d^{-0.157} & 2in \leq d \leq 10in \\ \left(\frac{d}{7.62}\right)^{-0.107} = 1.24 \cdot d^{-0.107} & 2.79mm \leq d \leq 51mm \\ 1.51 \cdot d^{-0.157} & 51mm \leq d \leq 254mm \end{cases}$$

- For Axial loading, $k_b=1$

k_b – Size Effect, Non-Circular X-Sections

- k_b for NOT round and NOT rotating parts
- Equivalent round rotating diameter is (d_e) obtained
- Equating volume of material stressed AT and ABOVE 95% of the maximum rotating-beam stress
- Lengths cancel → Equate Areas
- ROTATING ROUND SECTION 95% STRESS AREA

$$A_{0.95\sigma} = \frac{\pi}{4} \cdot \left[d^2 - (0.95 \cdot d)^2 \right] = 0.0766 \cdot d^2$$

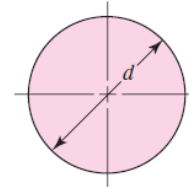
- $d = d_e$

k_b – Size Effect, Non-Circular X-Sections

- For Non-Rotating Round

$$A_{0.95\sigma} = 0.0104 \cdot d^2$$

$$d_e = 0.370 \cdot d$$

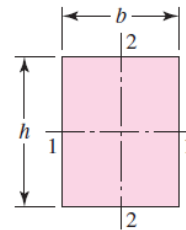


$$A_{0.95\sigma} = 0.01046d^2$$

$$d_e = 0.370d$$

- Non-Rotating Rectangular Section

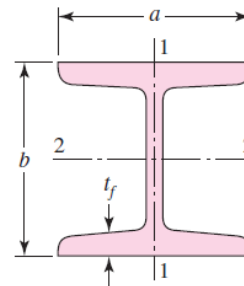
$$d_e = 0.808 \cdot (b \cdot h)^{1/2}$$



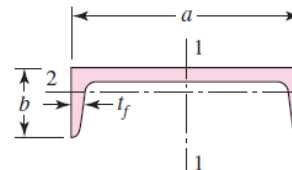
$$A_{0.95\sigma} = 0.05hb$$

$$d_e = 0.808\sqrt{hb}$$

- Other Non-Rotating Sections in Table



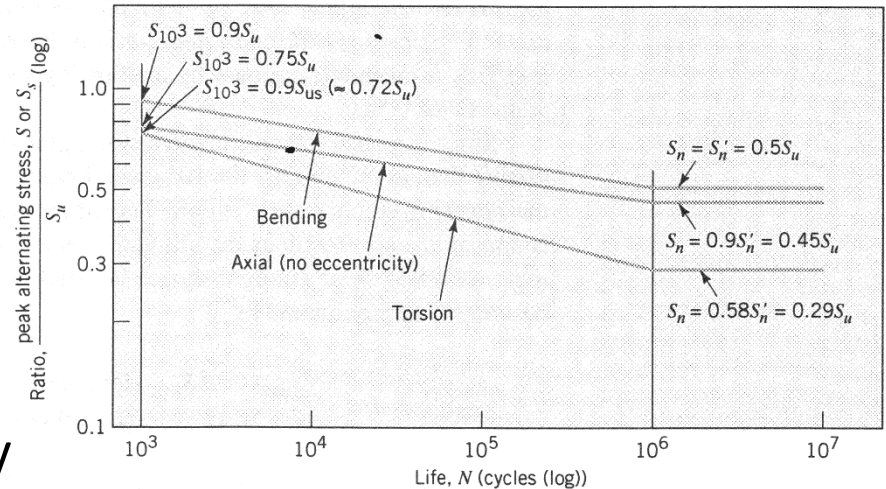
$$A_{0.95\sigma} = \begin{cases} 0.10at_f & \text{axis 1-1} \\ 0.05ba & \text{axis 2-2} \end{cases} \quad t_f > 0.025a$$



$$A_{0.95\sigma} = \begin{cases} 0.05ab & \text{axis 1-1} \\ 0.052xa + 0.1t_f(b - x) & \text{axis 2-2} \end{cases}$$

k_c - Loading Factor

- Bending $k_c=1$
- Axial $k_c=0.85$
- Torsion $k_c=0.59$
 - Distortional Energy Theory
 - $S_{us}=0.8 \cdot S_u$ for Steel
 - $S_{us}=0.7 \cdot S_u$ for Other Ductile Metals
 - Only for pure torsion
 - Combined loading
 - $k_c=1$
 - Use Equivalent Von Mises



k_d - Temperature

For Carbon and Alloy Steels, $70^\circ\text{F} \leq T_F \leq 1000^\circ\text{F}$

$$k_d = \frac{S_T}{S_{RT}} \equiv \frac{\text{Tensile Strength at Operating Temperature}}{\text{Tensile Strength at Room Temperature}}$$

$$k_d = 0.975 + 0.432(10^{-3}) \cdot T_F - 0.115(10^{-5}) \cdot T_F^2 \\ + 0.104(10^{-8}) \cdot T_F^3 - 0.595(10^{-12}) \cdot T_F^4$$

Temperature, °C	S_T/S_{RT}	Temperature, °F	S_T/S_{RT}
20	1.000	70	1.000
50	1.010	100	1.008
100	1.020	200	1.020
150	1.025	300	1.024
200	1.020	400	1.018
250	1.000	500	0.995
300	0.975	600	0.963
350	0.943	700	0.927
400	0.900	800	0.872
450	0.843	900	0.797
500	0.768	1000	0.698
550	0.672	1100	0.567
600	0.549		

k_e - Reliability

$$k_e = 1 - 0.08 \cdot z_a$$

Reliability, R	Standardized Variable, z_R	k_c
0.50	0	1.000
0.90	1.288	0.897
0.95	1.645	0.868
0.99	2.326	0.814
0.999	3.091	0.753
0.999 9	3.719	0.702
0.999 99	4.265	0.659
0.999 999	4.753	0.620
0.999 999 9	5.199	0.584
0.999 999 99	5.612	0.551
0.999 999 999	5.997	0.520

k_f – Miscellaneous – Effects Factors

- Residual Stresses

- Compressive surface residual stresses improves endurance limit

- Corrosion

- Corrosion and stressing occur at the same time
- Try to minimize factors that affect fatigue: mean or static stress, alternating stress, electrolyte concentration, dissolved oxygen in electrolyte, material properties and composition, temperature, cyclic frequency, fluid flow rate around specimen, and local crevices

- Electrolytic Plating

- Chromium plating, nickel plating, and cadmium plating REDUCE the endurance limit
- Zinc plating does not affect the fatigue strength
- Anodic oxidation of light alloys reduces bending endurance limit, but not torsional

- Metal Spraying

- Cyclic Frequency

- When corrosion or high temperatures are encountered, cyclic frequency impact crack growth rate

- Fretting Corrosion

Influence of Loading Configuration on 10^3 Cycle Strength ($S_{f,k}$)

- Bending Loads: $S_{f,k} = 0.9 \cdot k_d \cdot S_u$
- Axial Loads: $S_{f,k} = 0.75 \cdot k_d \cdot S_u$
- Torsional Loads: $S_{f,k} = 0.9 \cdot k_d \cdot S_{us}$

Fatigue and Fracture

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Stress Concentration

- K_t – theoretical or geometric static stress concentration factor
- K_f – fatigue strength reduction factor
 - (endurance limit of notch free specimen)/(endurance limit of notched specimens)
 - $K_f = 1 + q(K_t - 1)$, q -notch sensitivity
 - $k_e = 1/K_f$

Stress Concentration

- K_t – theoretical or geometric static stress concentration factor

- K_f – fatigue strength reduction factor

$$K_f = 1 + q \cdot (K_t - 1)$$

- Notch Sensitivity Factor

$$q = \frac{K_f - 1}{K_t - 1} = \frac{1}{1 + \left(\frac{\sqrt{a}}{\sqrt{r}} \right)}$$

- Neuber Constant

- Bending and Axial Loading

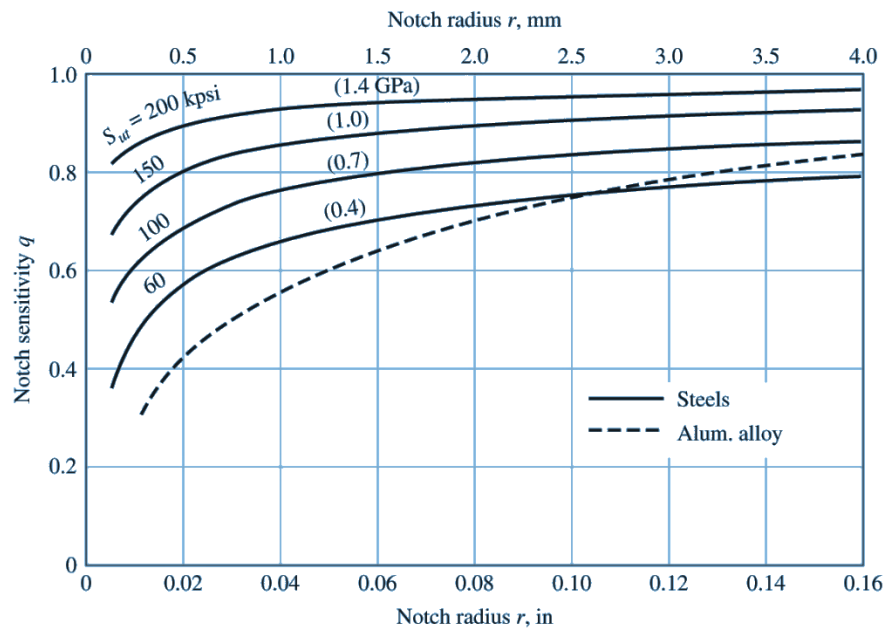
$$\sqrt{a} = 0.246 - 3.08(10^{-3}) \cdot S_{ut} + 1.51(10^{-5}) \cdot S_{ut}^2 - 2.67(10^{-8}) \cdot S_{ut}^3$$

- Torsion

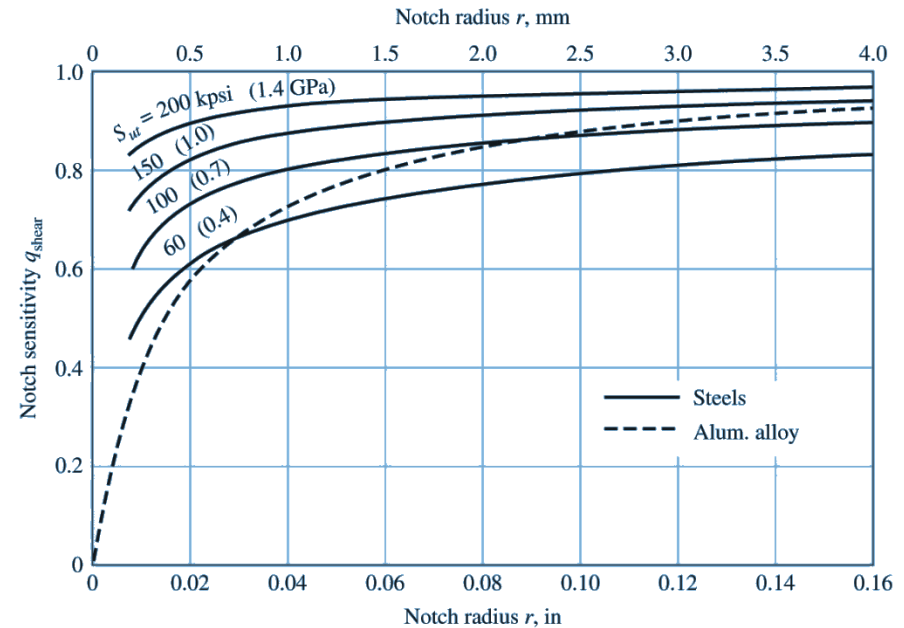
$$\sqrt{a} = 0.190 - 2.51(10^{-3}) \cdot S_{ut} + 1.35(10^{-5}) \cdot S_{ut}^2 - 2.67(10^{-8}) \cdot S_{ut}^3$$

Notch Sensitivity Curves

Bending and Axial Loading



Torsional Loading



$q=0.2$ recommended for CAST IRON

Example

A rotating shaft is supported by ball bearings at A and D and loaded by the non-rotating force F. Estimate the life of this part.

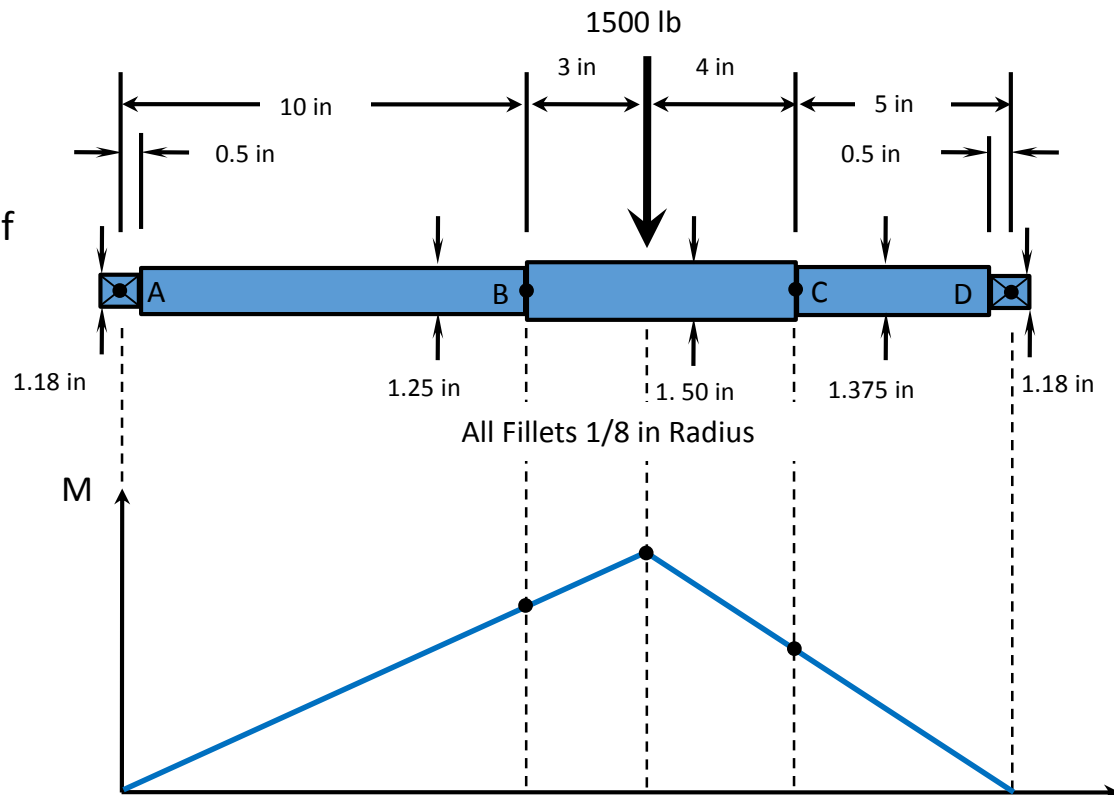
$$S_{ut}=103\text{ksi}$$

$$S_{yt}=72\text{ksi}$$

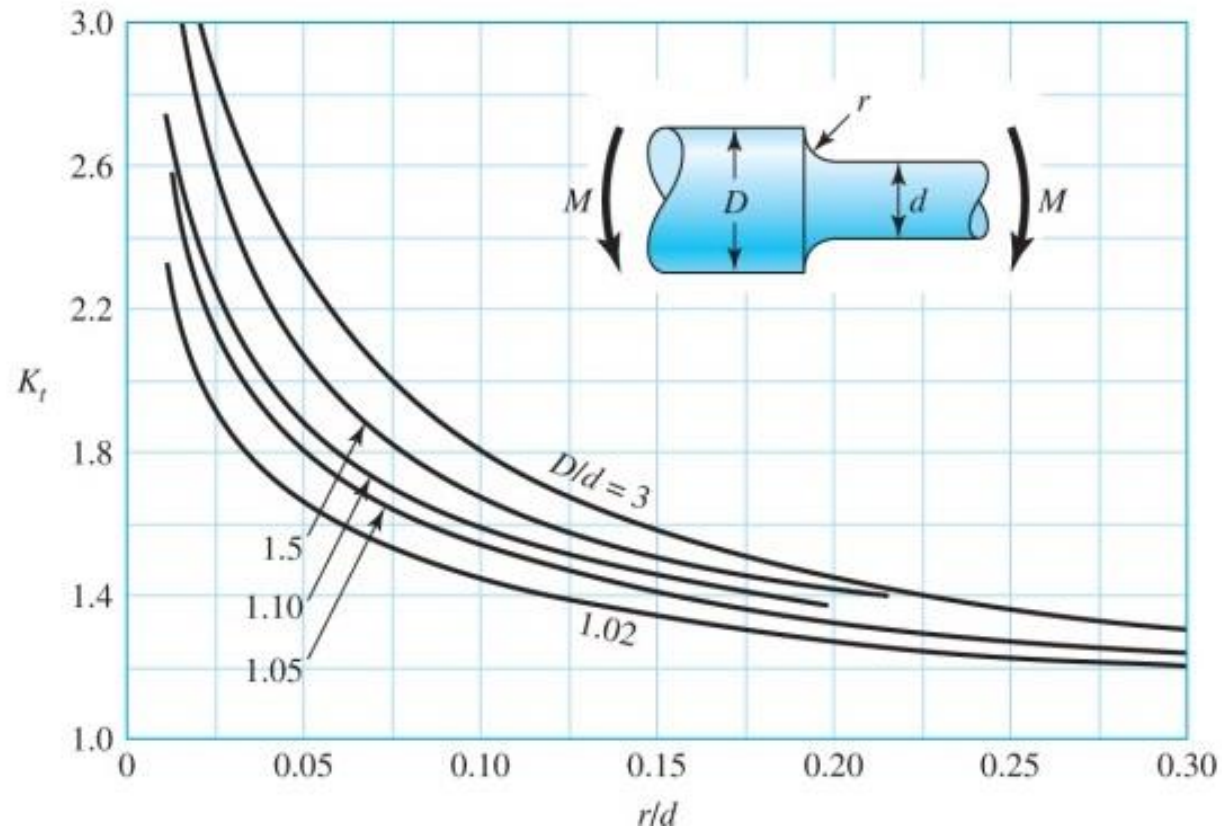
Shaft is drawn

UNS G10350 steel, drawn at 1000 C

Machined finish



Static Stress Concentration Factor



Fatigue and Fracture

- Introduction to Fatigue
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- Stress Concentrations and Notch Sensitivity
- **Fluctuating Stresses**
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- Varying, Fluctuating Stresses; Cumulative Fatigue Damage

Fatigue Terminology

- Stress Ratio

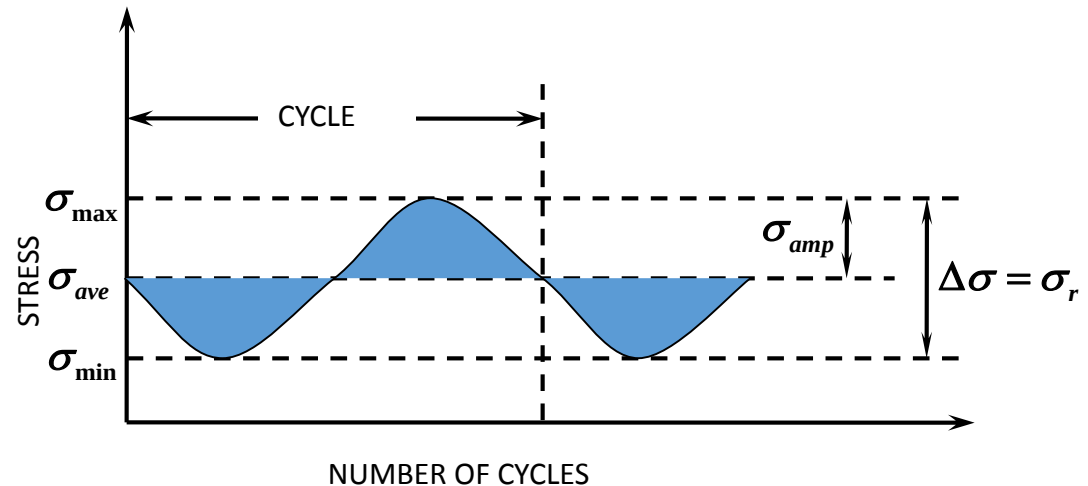
- $A = \sigma_{\text{amp}} / \sigma_{\text{ave}}$

- $R = \sigma_{\text{min}} / \sigma_{\text{max}}$

- Definitions

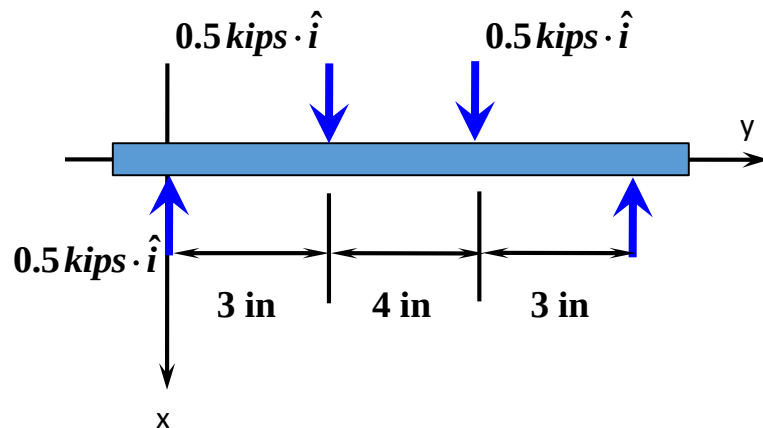
- $\sigma_{\text{ave}} = (\sigma_{\text{max}} + \sigma_{\text{min}}) / 2$

- $\sigma_{\text{amp}} = |\sigma_{\text{max}} - \sigma_{\text{min}}| / 2$



Example

- For the beam below determine
 - Stress ratio
 - Mean stress
 - Stress range
 - Stress amplitude



$$r = 0.564 \text{ in.}$$

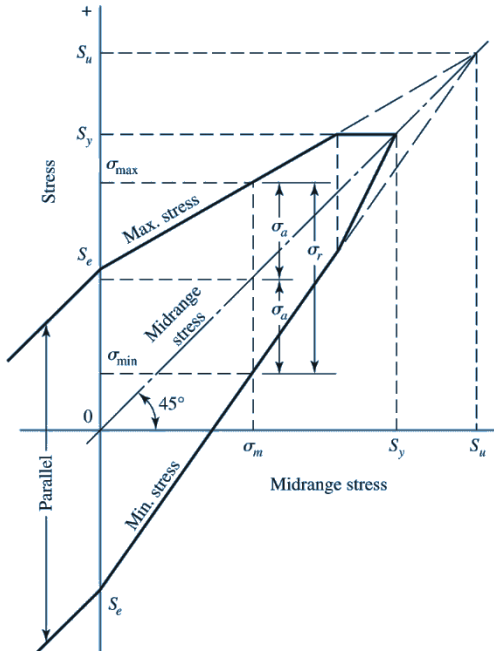
$$I = \frac{\pi \cdot d^4}{64}$$

Application of K_f for Fluctuating Stress

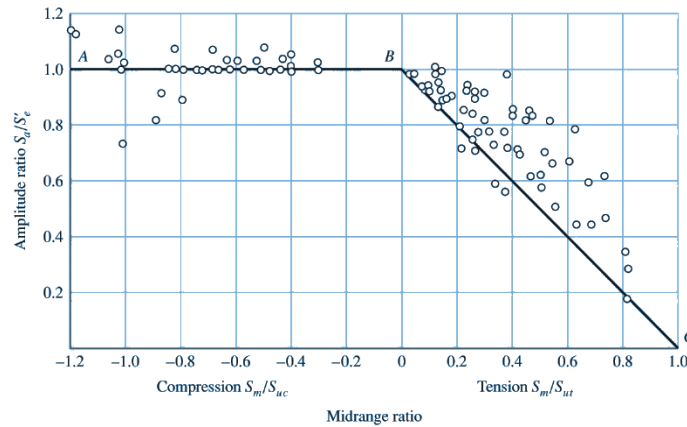
- At points with stress concentrations, best to AVOID localized plastic strains
- At points of stress concentration, K_f applied to both alternating and midrange stress components
- When localized plastic strain does occur
 - Nominal Mean Stress Method and Residual Stress Method
→ Apply K_f only to alternating stress
 - Dowling Method
→ Apply K_f to the alternating stress and K_{fm} to the mid-range stress
→ Will not be discussed

Plotting Components of Fluctuating Stress

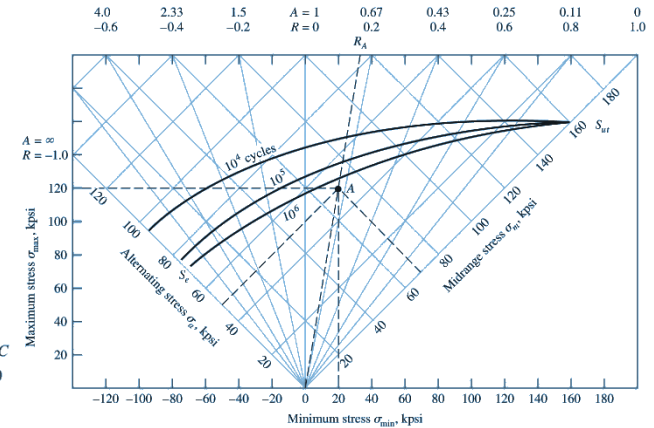
Modified Goodman Diagram



S_a-S_m Diagram

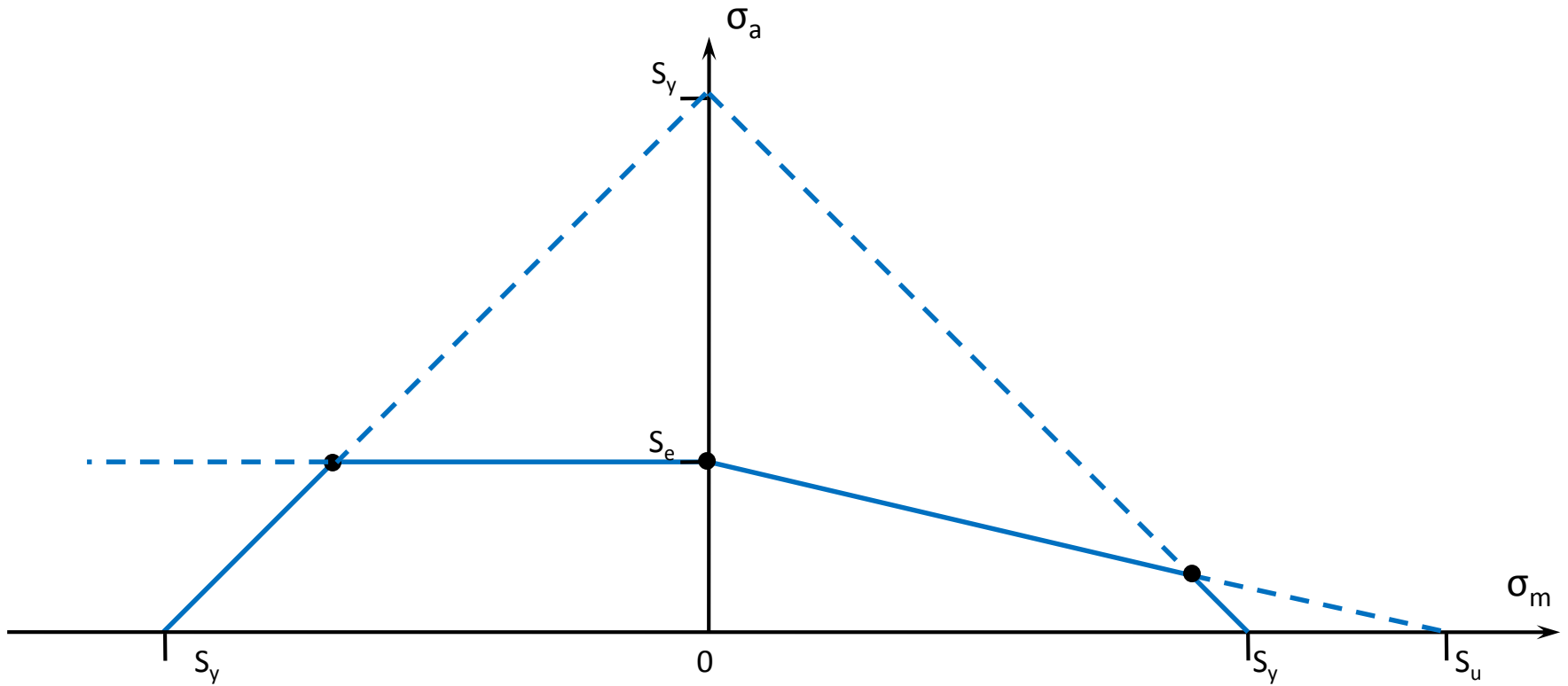


$S_{\max}$ - $S_{\min}$ Diagram



σ_m - σ_a Diagrams, A Closer Look

Modified Goodman Diagram



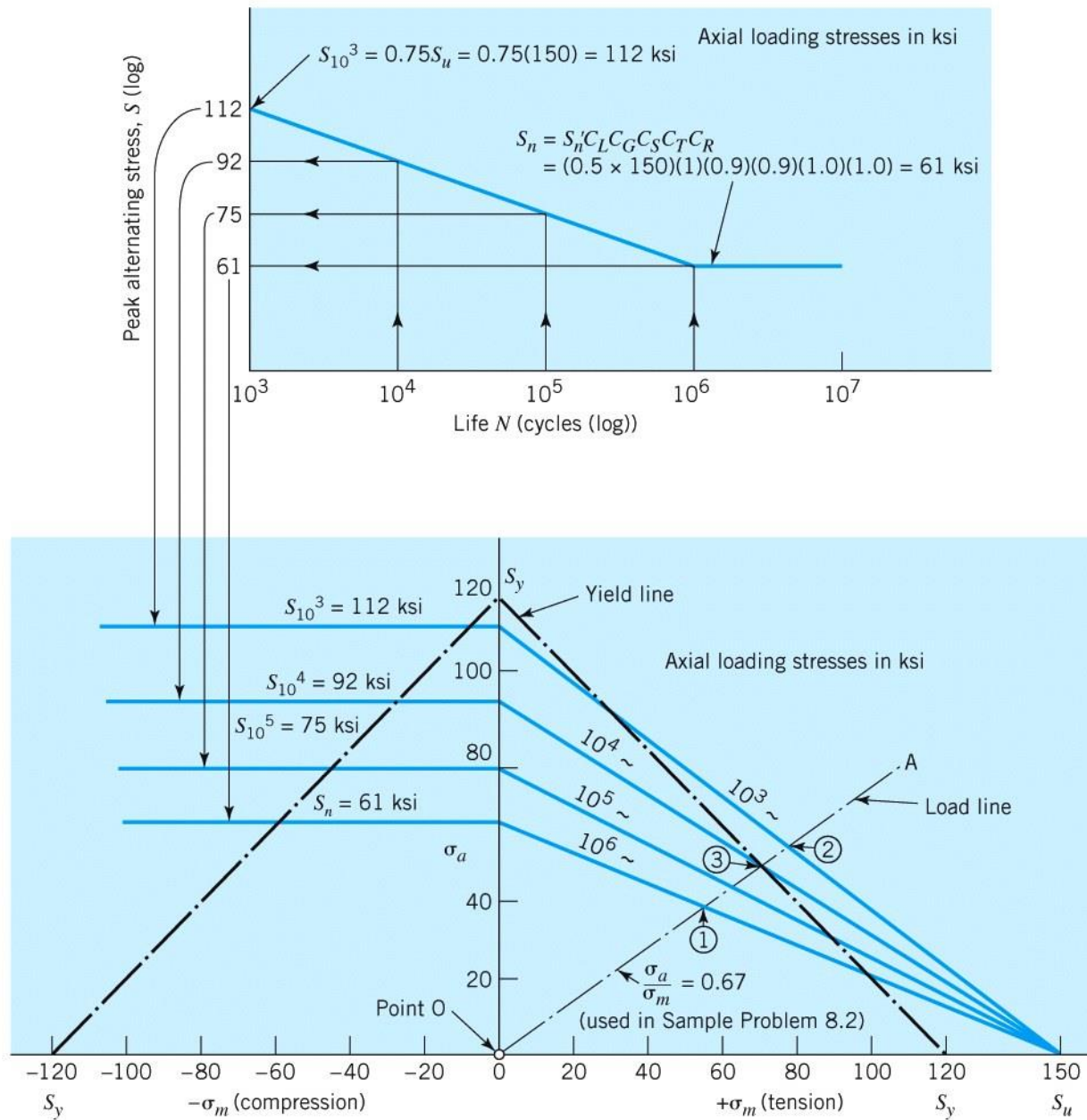
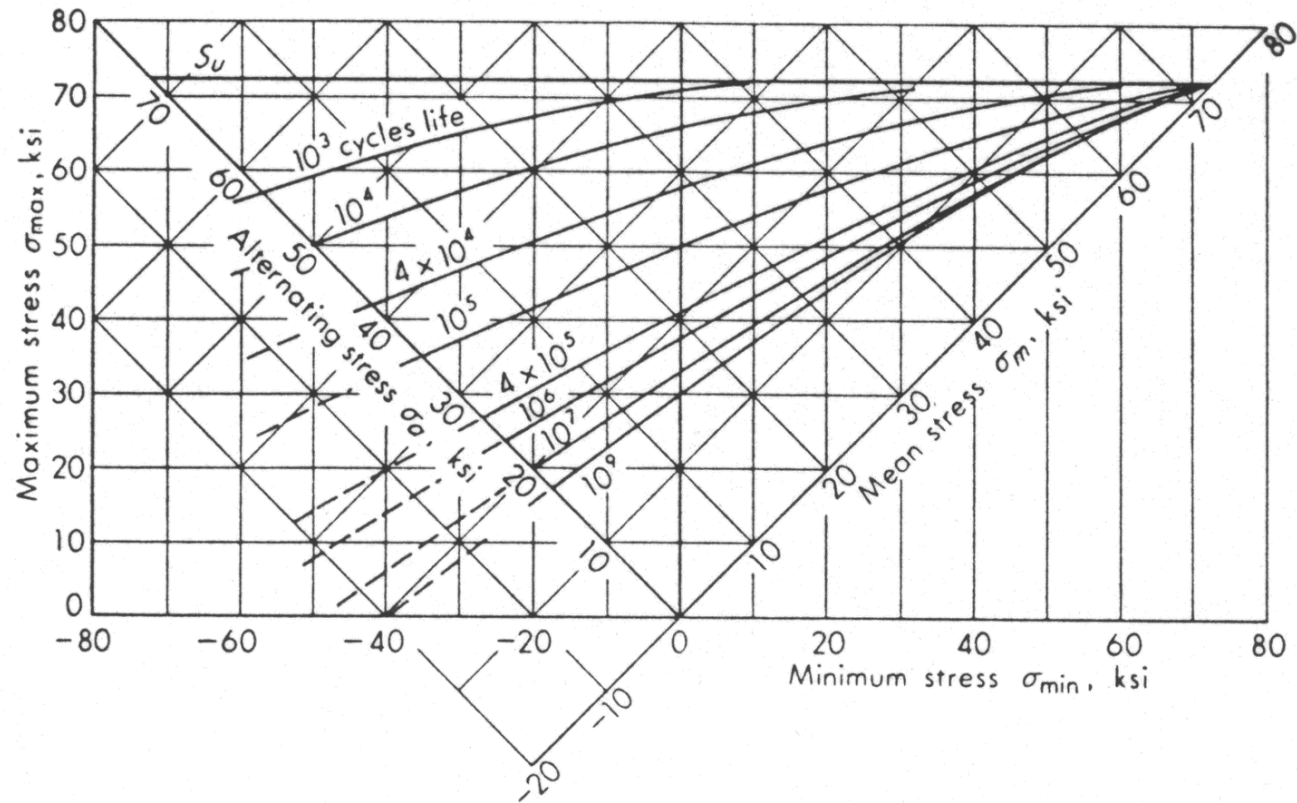
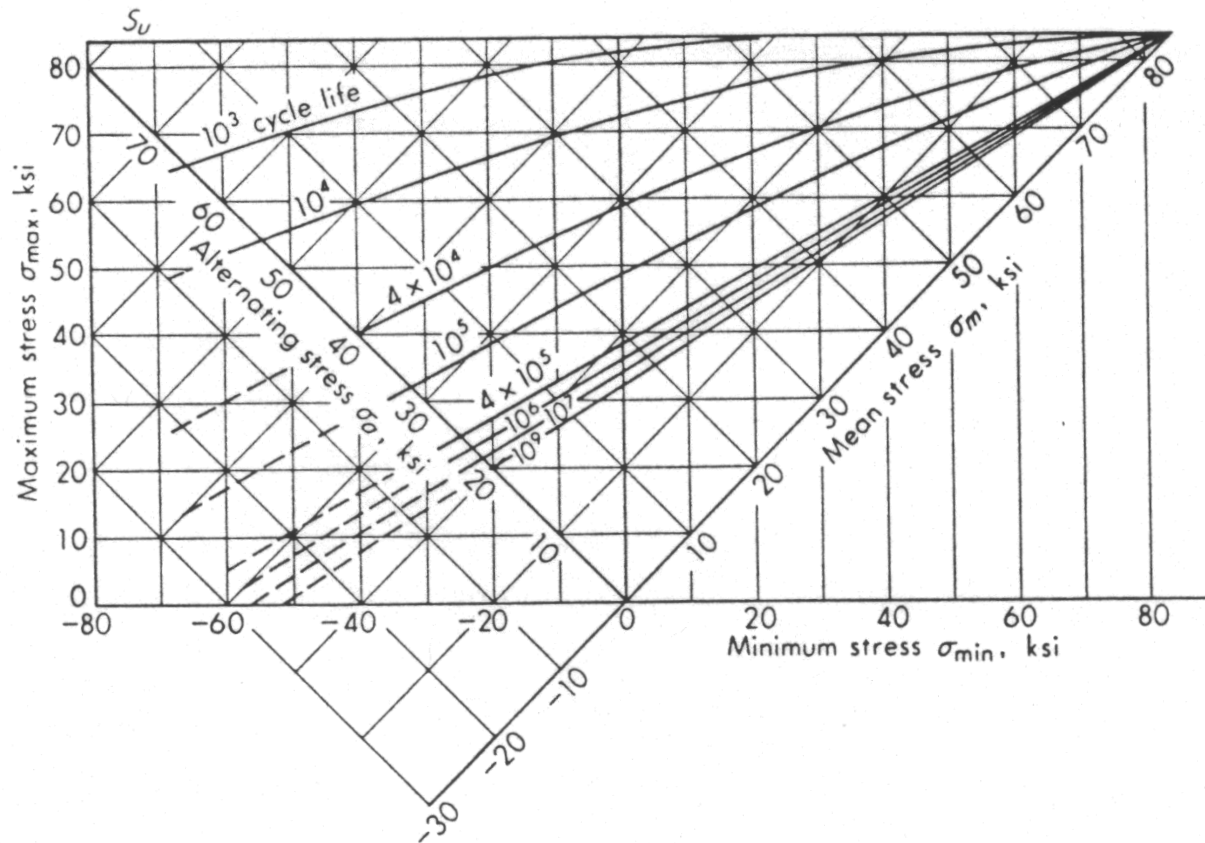


Figure 8.22
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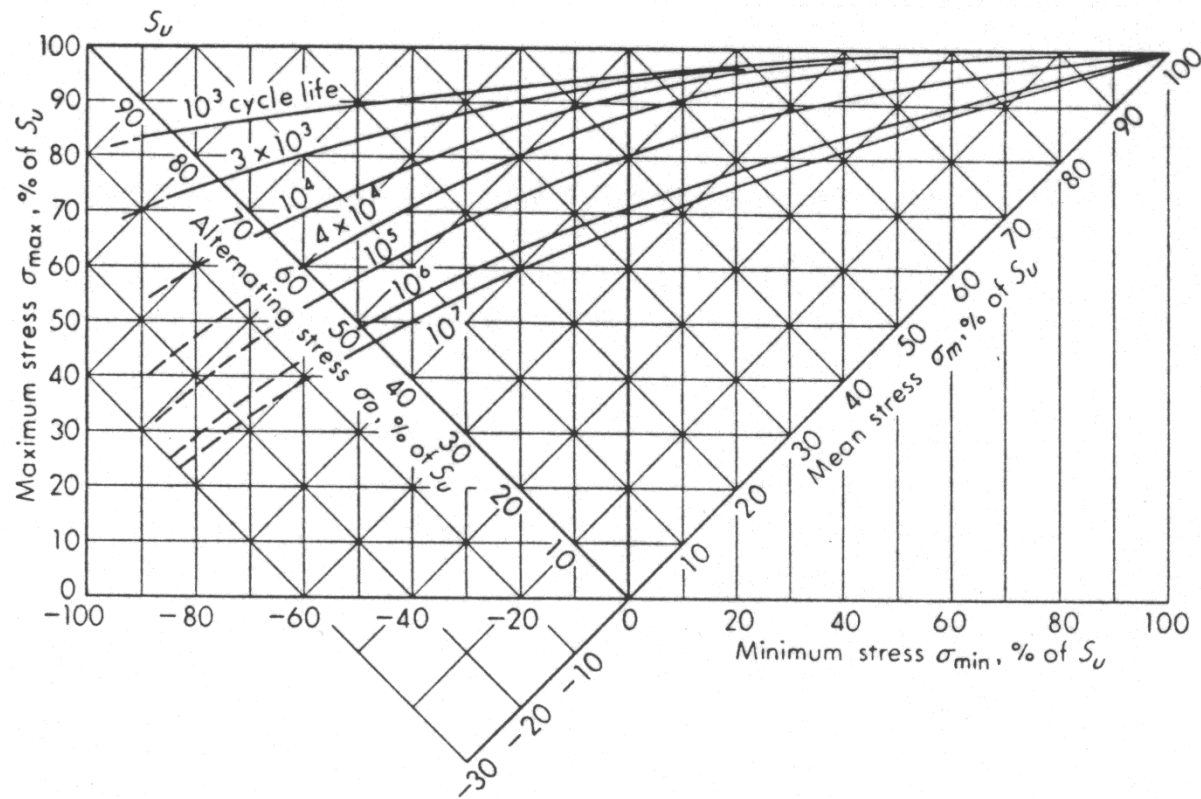
2024 AI



7075 Al



Steel



Commonly Used Failure Criteria

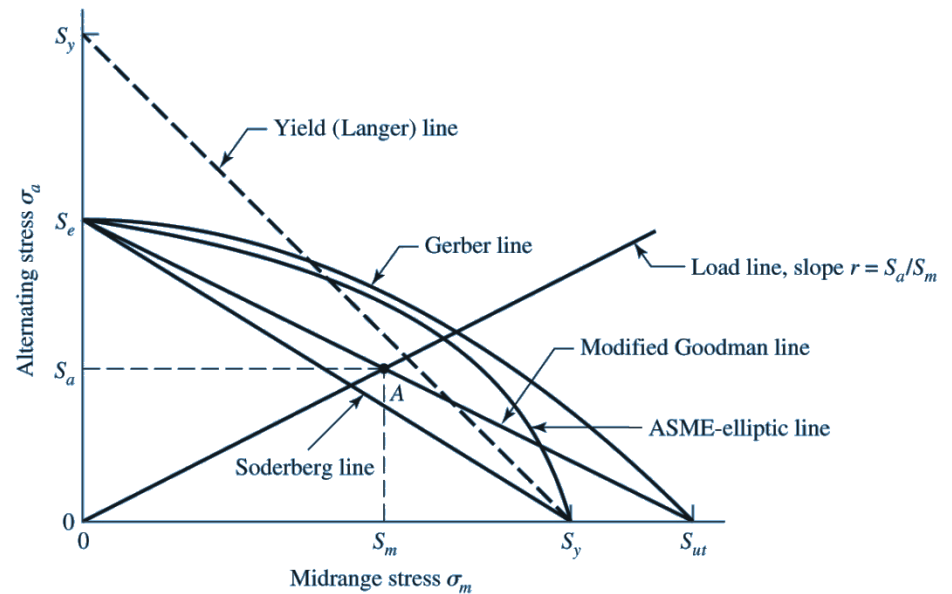
Soderberg
$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n}$$

Modified Goodman
$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$$

Gerber
$$\frac{n \cdot \sigma_a}{S_e} + \left(\frac{n \cdot \sigma_m}{S_{ut}} \right)^2 = 1$$

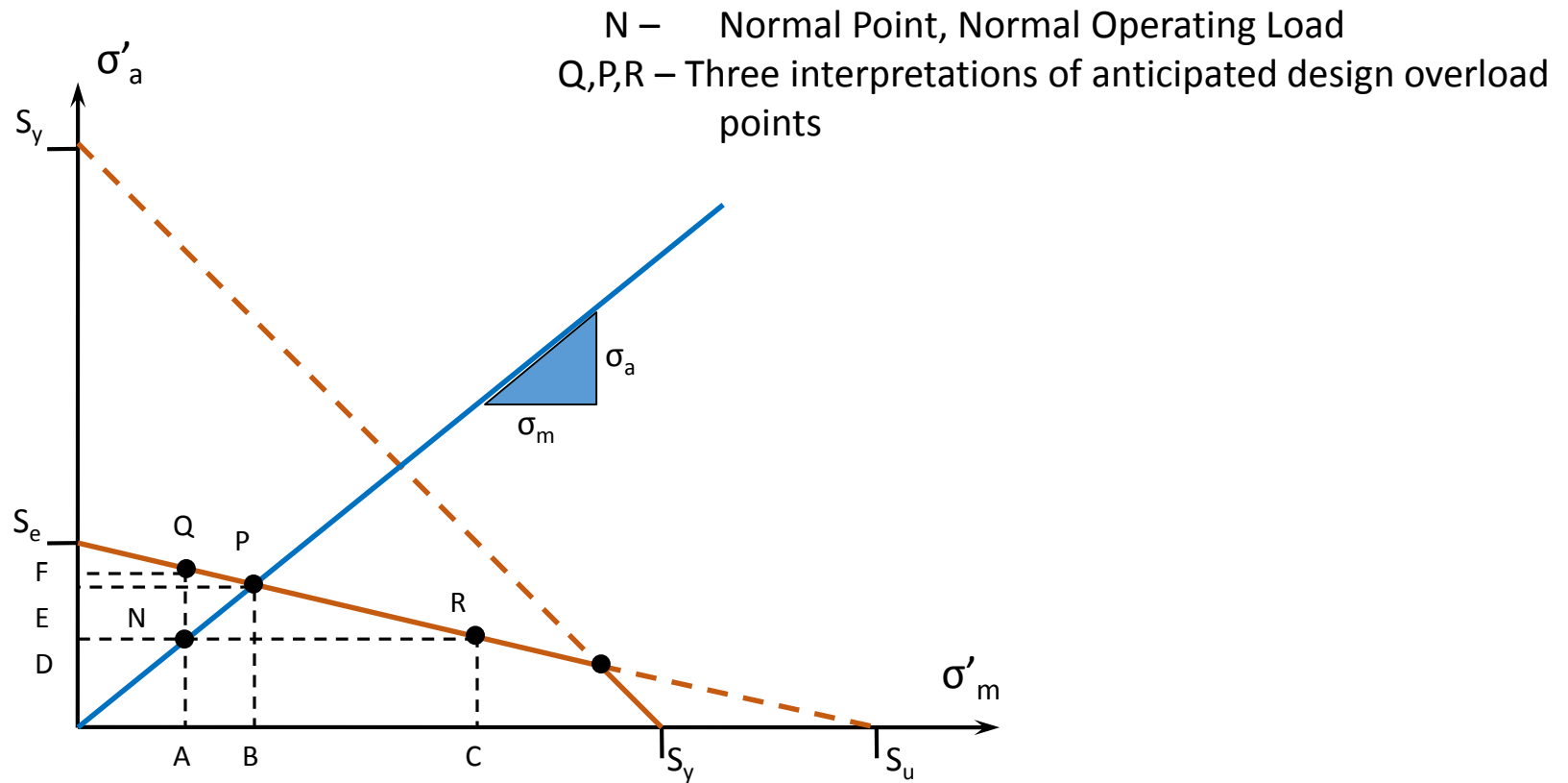
ASME Elliptic
$$\left(\frac{n \cdot \sigma_a}{S_e} \right)^2 + \left(\frac{n \cdot \sigma_m}{S_y} \right)^2 = 1$$

n=Design Factor



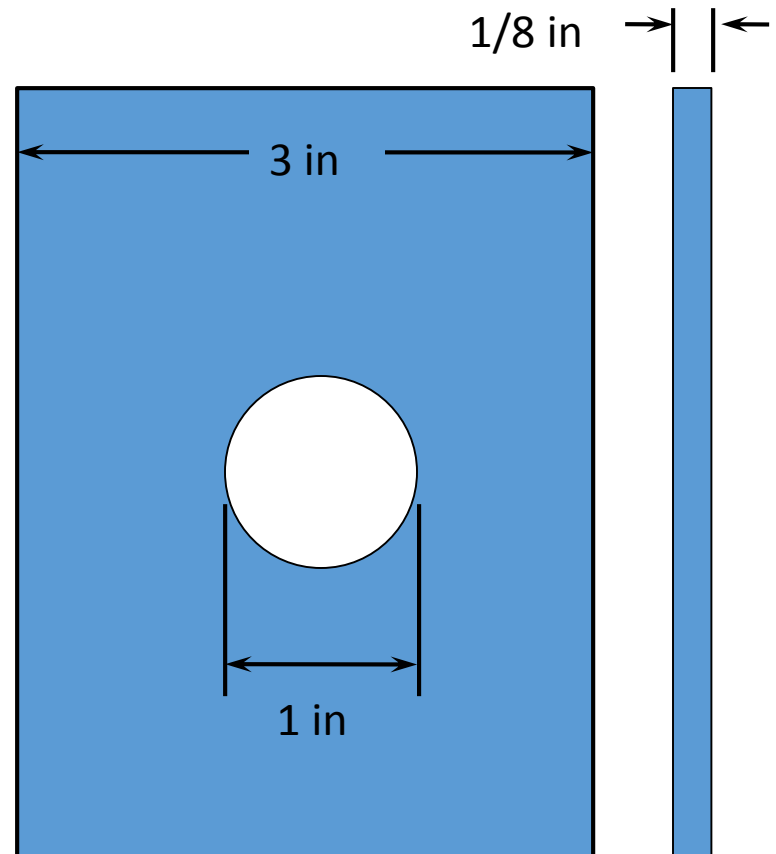
Factor of Safety:

Three Interpretations

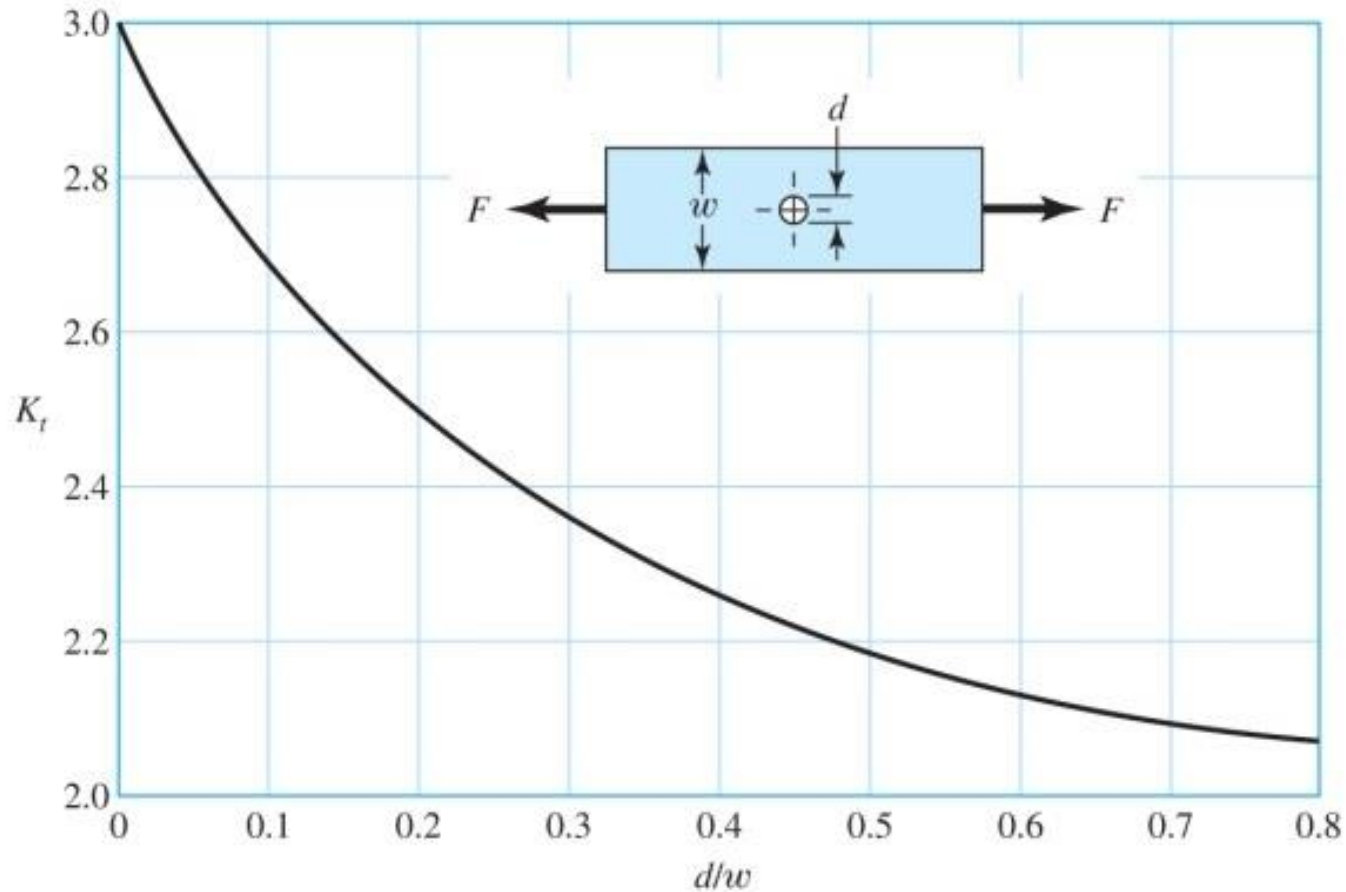


Example

A part made of 1/8in thick 7075-T6 aluminum alloy. It is subjected to a tensile load that fluctuates between 1000 and 5000lb. Determine its estimated life.



Static Stress Concentration Factor



Example

It is desired to determine the size of a UNS G10500 cold drawn steel bar to withstand a tensile preload of 8kips and a fluctuating tensile load varying from 0 to 16kips. Owing to the design of the ends, the bar will have a geometric stress concentration factor of 2.02 corresponding to a fillet whose radius is $3/16$ in. Determine a suitable diameter for an infinite life and a factor of safety of at least 2.0.

Fatigue and Fracture

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- Varying, Fluctuating Stresses; Cumulative Fatigue Damage

Fatigue Failure Due to Combined Stresses

- Distortional Energy Method

- Mean von Mises Stresses

$$\sigma_{vM,m} = \sqrt{\frac{1}{2} \cdot \left[\left(\sigma_{1,m} - \sigma_{2,m} \right)^2 + \left(\sigma_{2,m} - \sigma_{3,m} \right)^2 + \left(\sigma_{3,m} - \sigma_{1,m} \right)^2 \right]}$$

- Alternating von Mises Stresses

$$\sigma_{vM,a} = \sqrt{\frac{1}{2} \cdot \left[\left(\sigma_{1,a} - \sigma_{2,a} \right)^2 + \left(\sigma_{2,a} - \sigma_{3,a} \right)^2 + \left(\sigma_{3,a} - \sigma_{1,a} \right)^2 \right]}$$

More than One Type of Loading

- Distortional Energy Method
 - Mean von Mises Stresses

$$\sigma'_a = \sqrt{\left[(K_f)_{bending} \cdot (\sigma_a)_{bending} + (K_f)_{axial} \cdot \frac{(\sigma_a)_{axial}}{0.85} \right]^2 + 3 \cdot \left[(K_{fs})_{torsion} \cdot (\tau_a)_{torsion} \right]^2}$$

- Alternating von Mises Stresses

$$\sigma'_m = \sqrt{\left[(K_f)_{bending} \cdot (\sigma_m)_{bending} + (K_f)_{axial} \cdot \frac{(\sigma_m)_{axial}}{0.85} \right]^2 + 3 \cdot \left[(K_{fs})_{torsion} \cdot (\tau_m)_{torsion} \right]^2}$$

Example

A bar of steel has $S_u=700\text{MPa}$, $S_y=500\text{MPa}$, and a fully corrected endurance limit of $S_3=200\text{MPa}$. For each case below find the factor of safety which guards against static and fatigue failures.

1. $\tau_m = 140\text{MPa}$

2. $\tau_m = 140\text{MPa}, \quad \tau_a = 70\text{MPa}$

3. $\tau_{xy,m} = 100\text{MPa}, \quad \sigma_{x,a} = 80\text{MPa}$

4. $\sigma_{x,m} = 60\text{MPa}, \quad \sigma_{x,a} = 80\text{MPa}$

$\tau_{xy,m} = 70\text{MPa}, \quad \tau_{xy,a} = 35\text{MPa}$

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Cumulative Damage

- Multiple Stress Levels

- σ_1 for n_1 cycles
- σ_2 for n_2 cycles
- Etc.

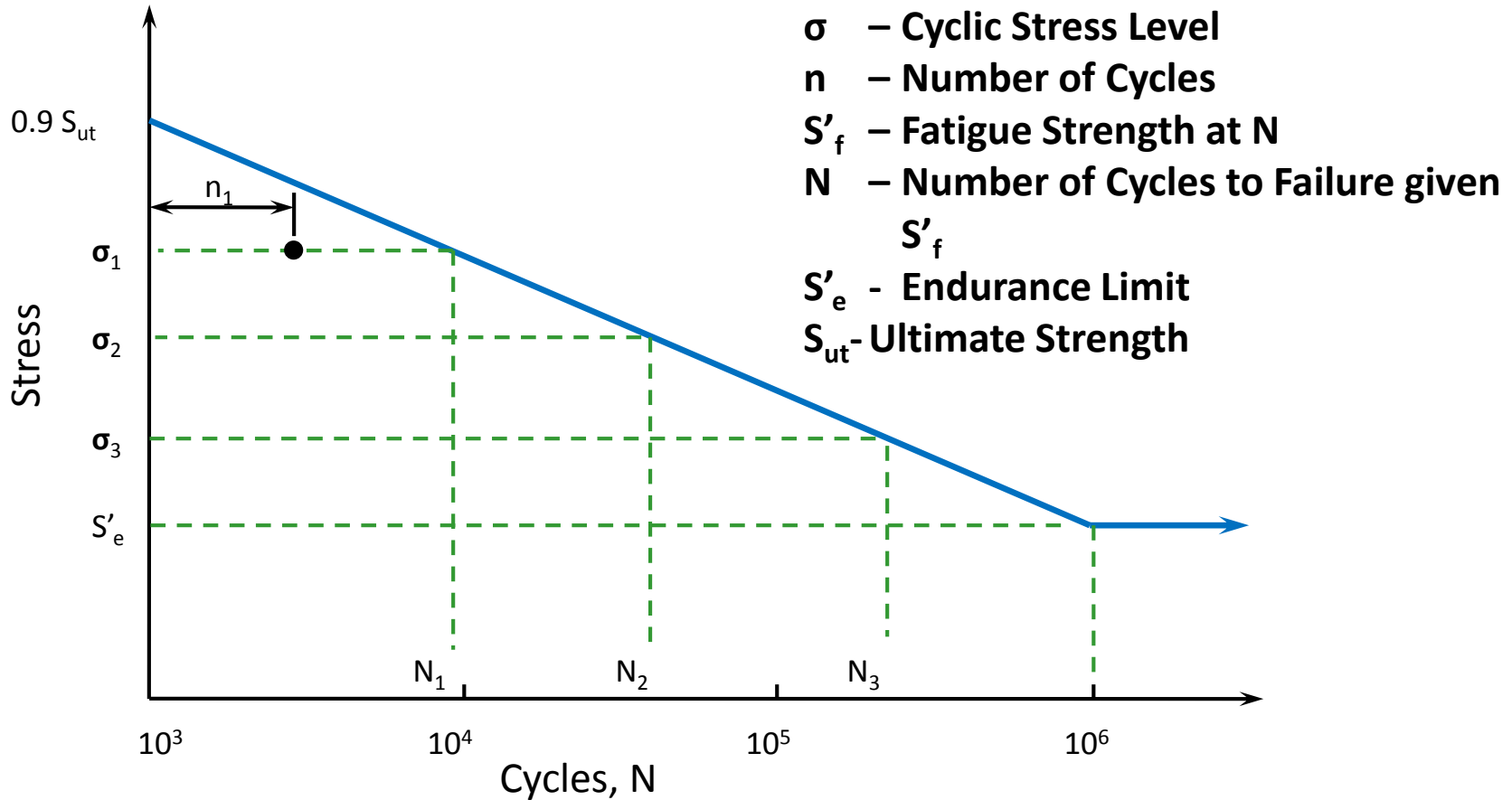
- Palmgren-Miner's Rule

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \dots + \frac{n_n}{N_n} = C \Rightarrow \sum \frac{n_i}{N_i} = 1$$

- n-number of cycles of stress σ applied
- N-life corresponding to σ
- C-determined experimentally
 - $0.7 < C < 2.2$
 - Typically $C=1$

Cumulative Damage

Log-Log Plot For Test Sample



Equation for the S-N Curve

S-N Curve Equation

$$\log S'_f = -m \cdot \log n + b$$

where

$$m = \frac{1}{3} \cdot \log \left(\frac{f \cdot S_{ut}}{S'_e} \right)$$

$$b = \log \left(\frac{(f \cdot S_{ut})^2}{S'_e} \right)$$

f is often taken as 0.9

S-N Curve Calculations

In the range $10^3 \leq y \leq 10^6$

Given N , S'_f is

$$S' = \frac{10^b}{N^m}$$

Given S'_f , N is

$$N = \frac{10^{b/m}}{S'^{1/m}_f}$$

Consider the Following

$S_{ut}=560$ MPa

$f=0.9$

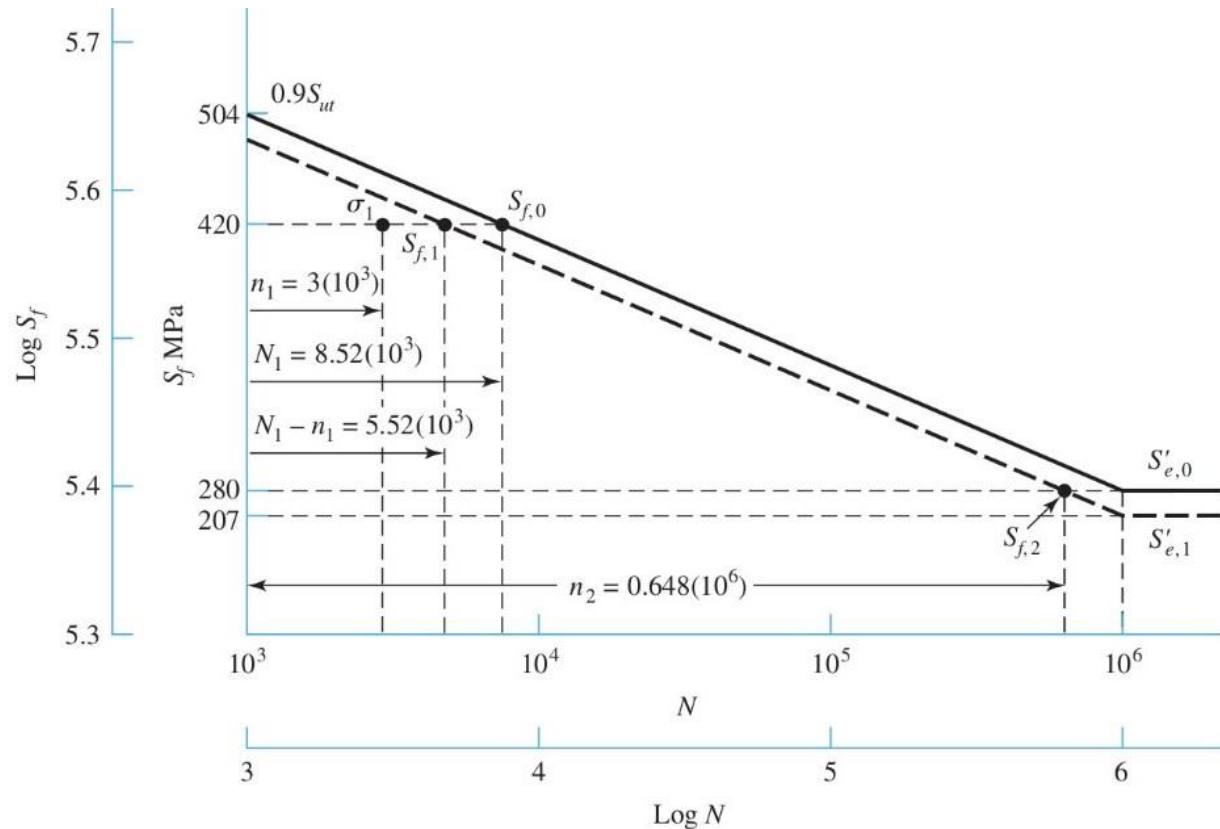
$S'_e=280$ MPa

First Cycle

$\sigma_1=420$ MPa

$N_1=3000$ cycles

How many cycles left
at 280 MPa



Example

Steel, $S_{ut}=80\text{ksi}$
3000 cycles at 60ksi
How many cycles remain at
50ksi and 40ksi?

