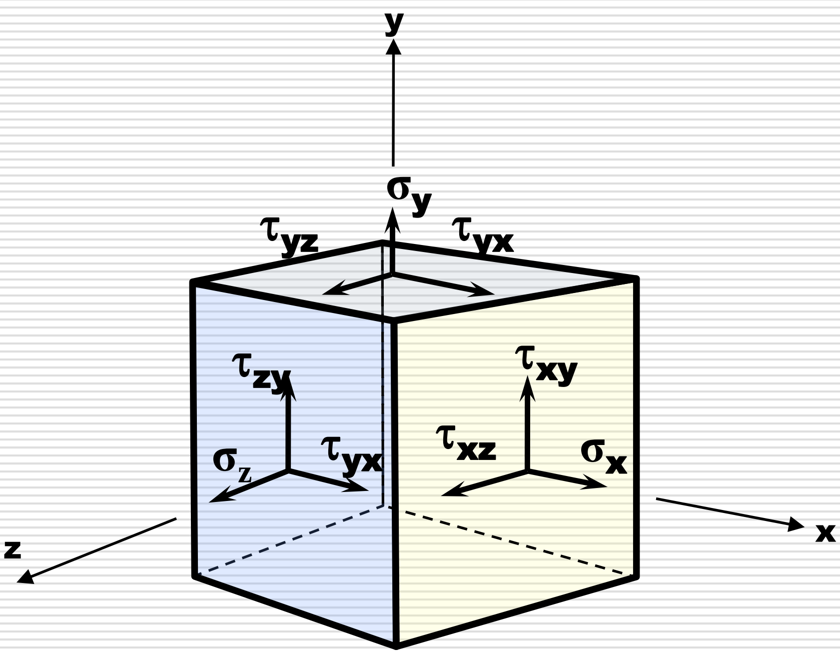
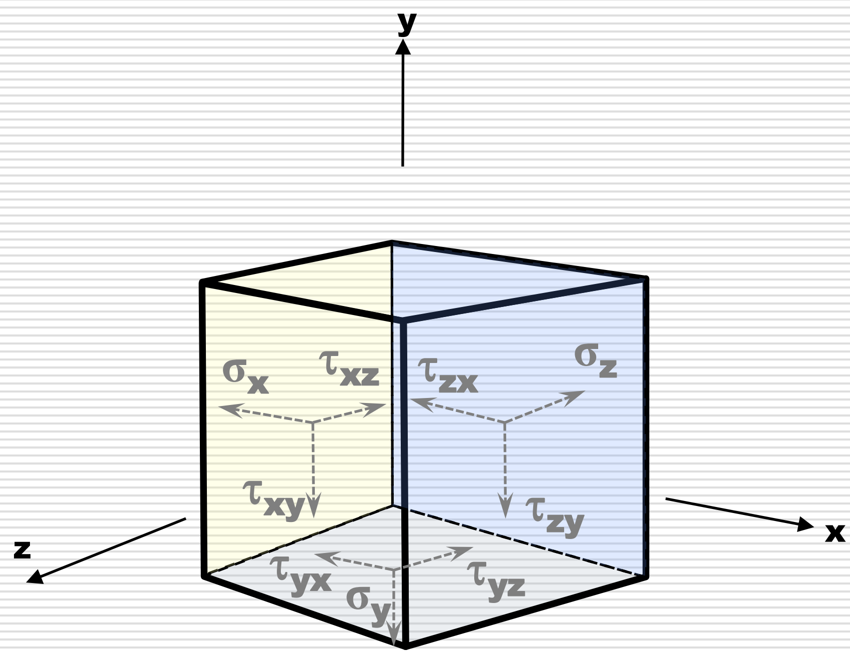
PE-CE: Lecture 2
Review of Elasticity

LECTURE OUTLINE

- Stress Tensor
- Equilibrium
- Stress Transformations
- Principal Stress
- Mohr's Circle For Stress
- Strain – Displacement Relations

Stress at a Point
Shown in the Tensile (+) Direction





Surfaces with a Positive Directed Area Normal

Surfaces with a Negative Directed Area Normal

Stress Tensor

$$[\sigma] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} = \sigma_{ij} = \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix} = \tau_{ij}$$

Equilibrium Equations
Sum of the Moments

$$\tau_{xy} = \tau_{yx}$$

$$\tau_{yz} = \tau_{zy}$$

$$\tau_{xz} = \tau_{zx}$$

Stress Tensor

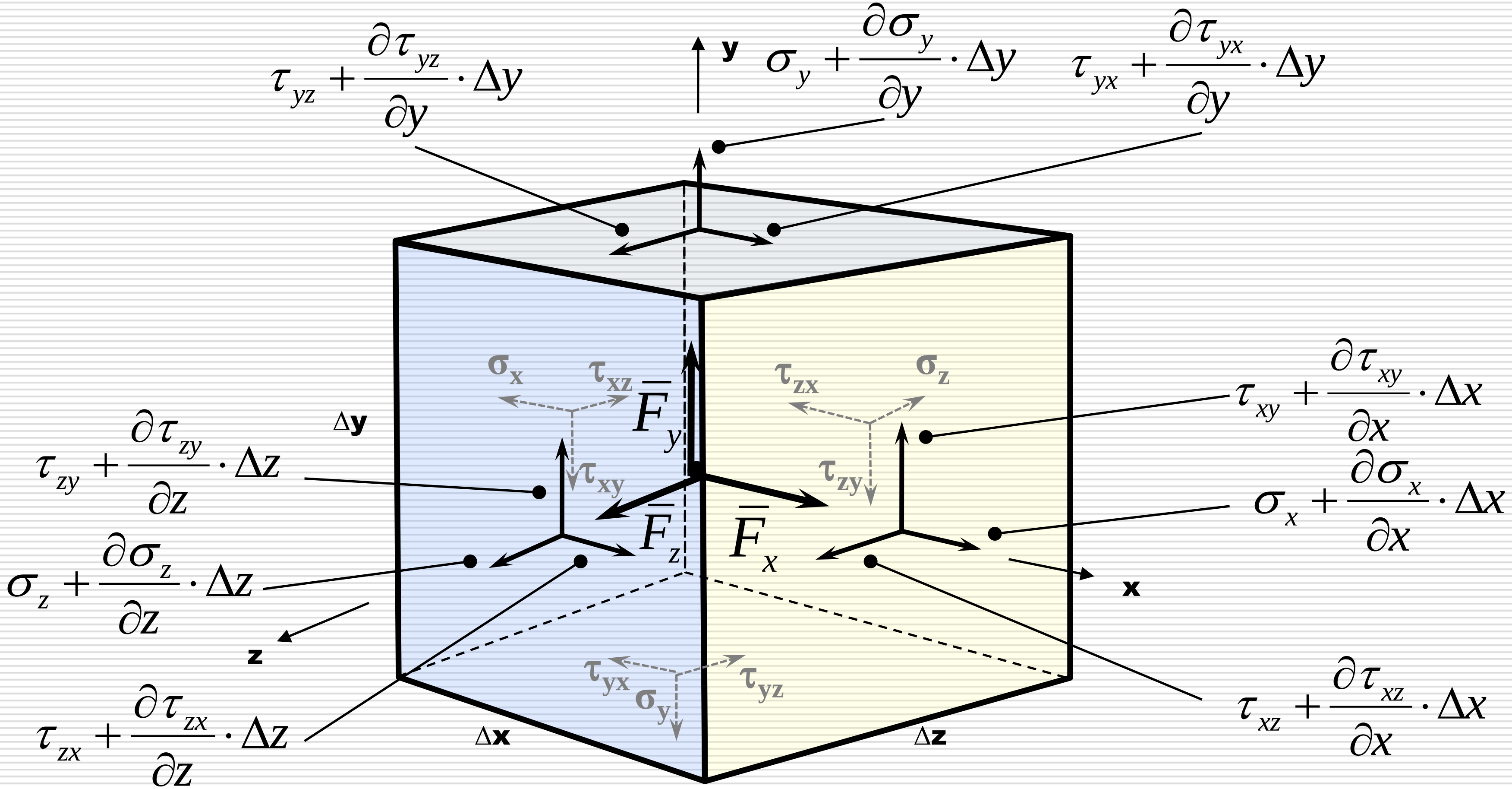
$$[\sigma] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} = \sigma_{ij}$$

$$= \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix} = \tau_{ij}$$

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xz} \\ \tau_{yz} \\ \tau_{xy} \end{Bmatrix}$$

Element with Finite Dimensions



Equilibrium Equations
Sum of the Forces

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + \bar{F}_x = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + \bar{F}_y = 0$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + \bar{F}_z = 0$$

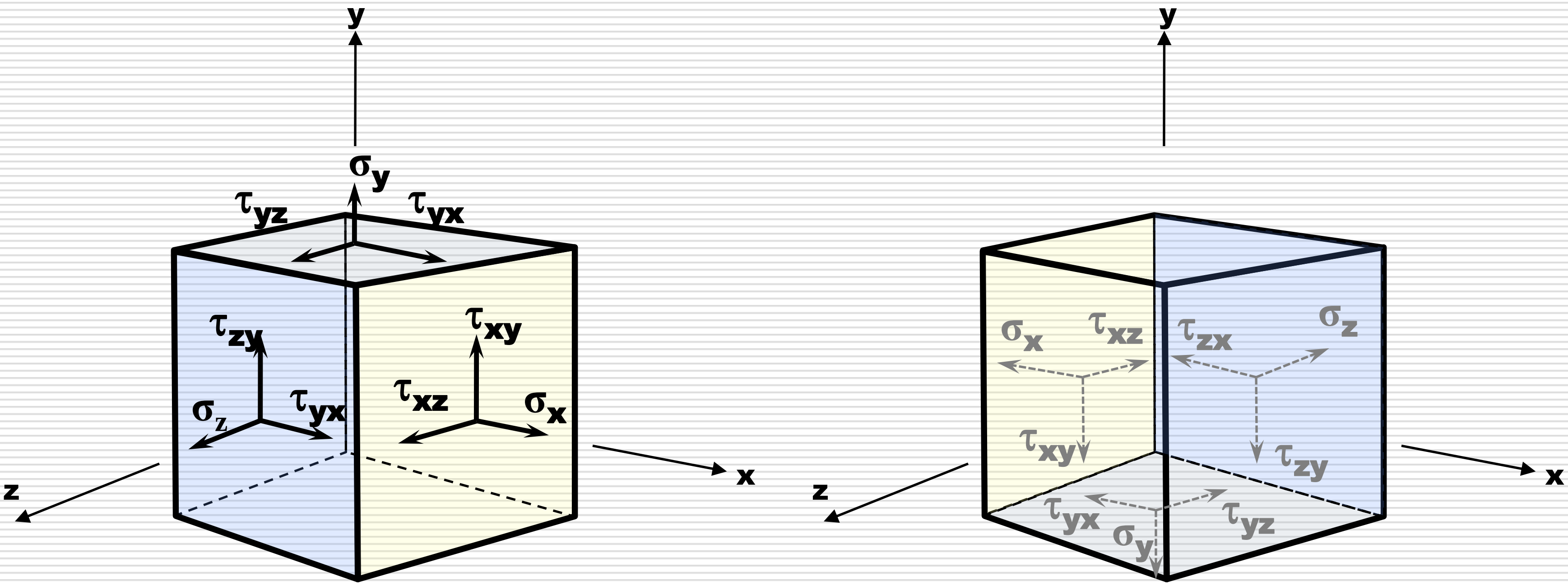
Example

The stress field within an elastic structural member is expressed as follows:

$$\sigma_x = -x^3 + y^2,\quad \tau_{xy} = 5z + 2y^2,\quad \tau_{xz} = xz^3 + x^2 y$$
$$\sigma_y = 2x^3 + .5y^2,\quad \tau_{yz} = 0,\quad \sigma_z = 4y^2 - z^3$$

Determine the body force distribution required for equilibrium.

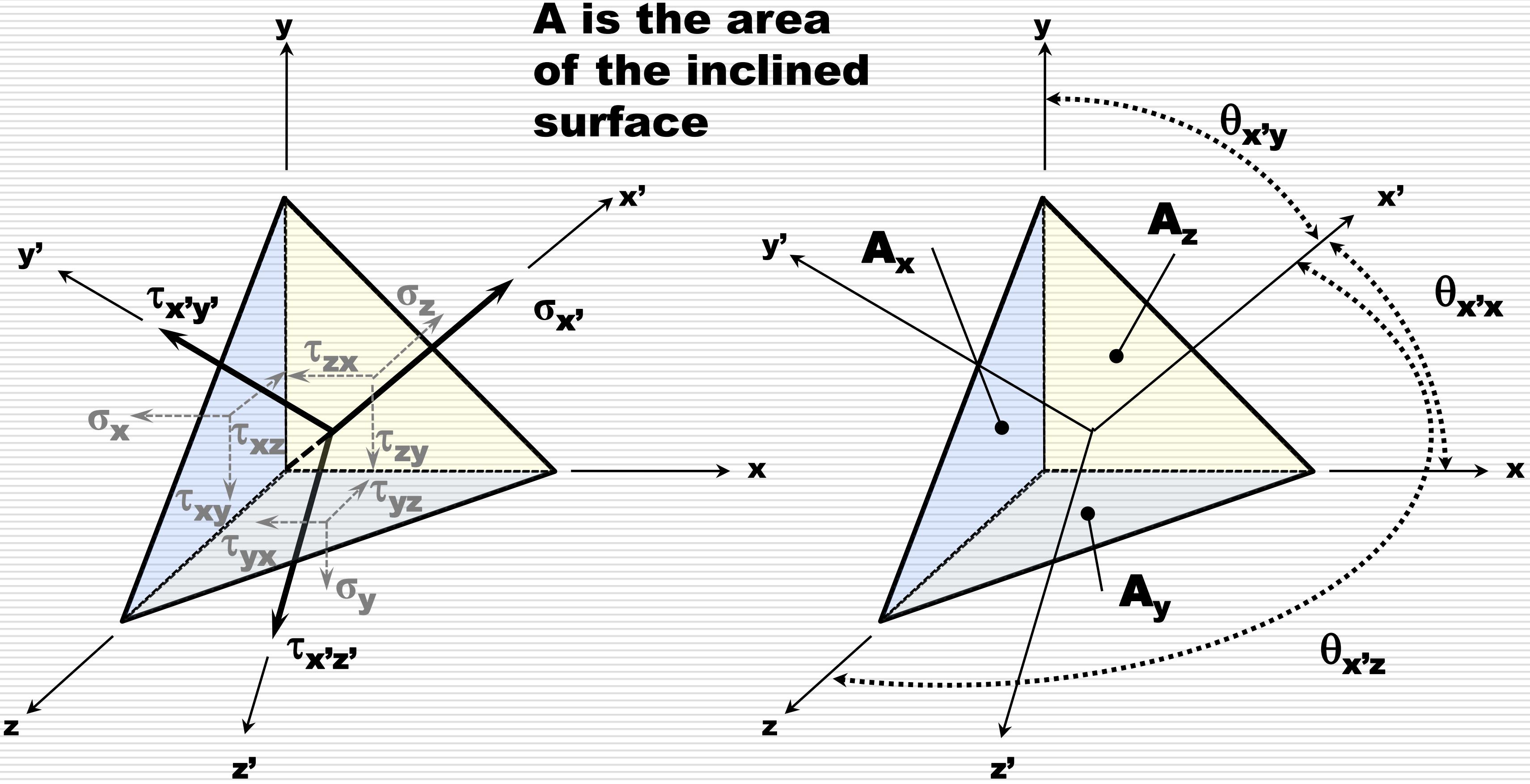
Stress at a Point Shown in the Tensile (+) Direction



Surfaces with a Positive Directed Area Normal

Surfaces with a Negative Directed Area Normal

Transforming Stress in Three Dimensions



Transformation Equations

$$\sigma_{x'} = \sigma_x \cdot n_{x'x}^2 + \sigma_y \cdot n_{x'y}^2 + \sigma_z \cdot n_{x'z}^2 + 2 \cdot \tau_{xy} \cdot n_{x'x} \cdot n_{x'y} + 2 \cdot \tau_{yz} \cdot n_{x'y} \cdot n_{x'z} + 2 \cdot \tau_{zx} \cdot n_{x'z} \cdot n_{x'x}$$

$$\sigma_{y'} = \sigma_x \cdot n_{y'x}^2 + \sigma_y \cdot n_{y'y}^2 + \sigma_z \cdot n_{y'z}^2 + 2 \cdot \tau_{xy} \cdot n_{y'x} \cdot n_{y'y} + 2 \cdot \tau_{yz} \cdot n_{y'y} \cdot n_{y'z} + 2 \cdot \tau_{zx} \cdot n_{y'z} \cdot n_{y'x}$$

$$\sigma_{z'} = \sigma_x \cdot n_{z'x}^2 + \sigma_y \cdot n_{z'y}^2 + \sigma_z \cdot n_{z'z}^2 + 2 \cdot \tau_{xy} \cdot n_{z'x} \cdot n_{z'y} + 2 \cdot \tau_{yz} \cdot n_{z'y} \cdot n_{z'z} + 2 \cdot \tau_{zx} \cdot n_{z'z} \cdot n_{z'x}$$

$$\tau_{x'y'} = \sigma_x \cdot n_{x'x} \cdot n_{y'x} + \sigma_y \cdot n_{x'y} \cdot n_{y'y} + \sigma_z \cdot n_{x'z} \cdot n_{y'z} + \tau_{xy} \cdot (n_{x'x} \cdot n_{y'y} + n_{x'y} \cdot n_{y'x})$$
$$+ \tau_{yz} \cdot (n_{x'y} \cdot n_{y'z} + n_{x'z} \cdot n_{y'y}) + \tau_{zx} \cdot (n_{x'x} \cdot n_{y'z} + n_{x'z} \cdot n_{y'x})$$

$$\tau_{z'x'} = \sigma_x \cdot n_{x'x} \cdot n_{z'x} + \sigma_y \cdot n_{x'y} \cdot n_{z'y} + \sigma_z \cdot n_{x'z} \cdot n_{z'z} + \tau_{xy} \cdot (n_{x'x} \cdot n_{z'y} + n_{x'y} \cdot n_{z'x})$$
$$+ \tau_{yz} \cdot (n_{x'y} \cdot n_{z'z} + n_{x'z} \cdot n_{z'y}) + \tau_{zx} \cdot (n_{x'x} \cdot n_{z'z} + n_{x'z} \cdot n_{z'x})$$

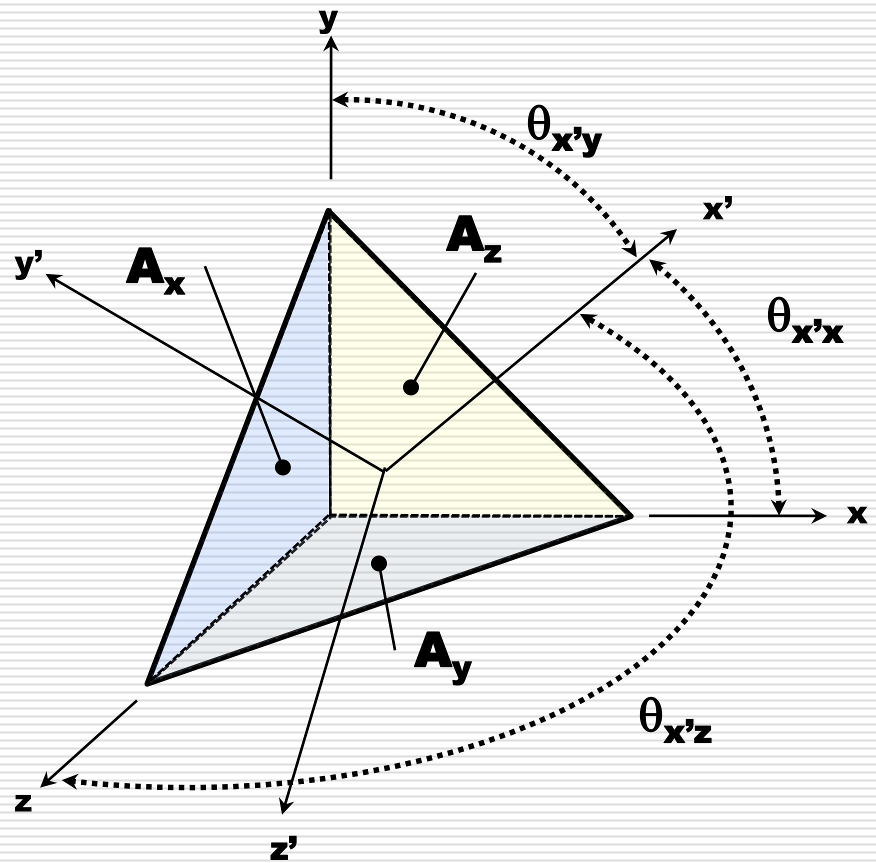
$$\tau_{y'z'} = \sigma_x \cdot n_{y'x} \cdot n_{z'x} + \sigma_y \cdot n_{y'y} \cdot n_{z'y} + \sigma_z \cdot n_{y'z} \cdot n_{z'z} + \tau_{xy} \cdot (n_{y'x} \cdot n_{z'y} + n_{y'y} \cdot n_{z'x})$$
$$+ \tau_{yz} \cdot (n_{y'y} \cdot n_{z'z} + n_{y'z} \cdot n_{z'y}) + \tau_{zx} \cdot (n_{y'x} \cdot n_{z'z} + n_{y'z} \cdot n_{z'x})$$

Inverting the Stress Tensor

$$[\sigma]_{x'y'z'} = [T] \cdot [\sigma]_{xyz} \cdot [T]^T$$

$$T = \begin{bmatrix} n_{x',x} & n_{x',y} & n_{x',z} \\ n_{y',x} & n_{y',y} & n_{y',z} \\ n_{z',x} & n_{z',y} & n_{z',z} \end{bmatrix}$$

$$n_{i'j} = \cos(\theta_{i'j})$$



Example 1:

Write the transformation Matrix for the following:
- First a positive 45° about z axis
- Second a positive 30° about the new x' axis

$$T1 = \begin{bmatrix} 0.7017 & 0.7017 & 0 \\ -0.7017 & 0.7017 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad T2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.866 & 0.5 \\ 0 & -0.5 & 0.866 \end{bmatrix}$$

$$T2 * T1 = \begin{bmatrix} 0.7071 & 0.7071 & 0 \\ -0.6124 & 0.6124 & 0.5 \\ 0.3536 & -0.3536 & 0.866 \end{bmatrix}$$

Solution to Example 1:

Write the transformation Matrix for the following:
-First a positive 45° about z axis

$$T1 = \begin{bmatrix} 0.7017 & 0.7017 & 0 \\ -0.7017 & 0.7017 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Second a positive 30° about the new x' axis

$$T2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.866 & 0.5 \\ 0 & -0.5 & 0.866 \end{bmatrix} \qquad T2 * T1 = \begin{bmatrix} 0.7071 & 0.7071 & 0 \\ -0.6124 & 0.6124 & 0.5 \\ 0.3536 & -0.3536 & 0.866 \end{bmatrix}$$

Example 2:

The stress tensor at a point in a machine element with respect to the inertial coordinate system is

$$[\sigma] = \begin{bmatrix} 50 & 10 & 0 \\ 10 & 20 & 40 \\ 0 & 40 & 30 \end{bmatrix} MPa$$

Determine the state of stress if the stress element is rotated 45^{o} counterclockwise about the z axis followed by 30° about the new x' axis.

Solution to Example 2:

Stress Transformation

$$[\sigma]_{x'y'z'} = [T]\cdot[\sigma]_{xyz}\cdot[T]^T = \begin{bmatrix} 45.0 & 1.15 & 32.0 \\ 1.15 & 50.7 & 16.3 \\ 32.0 & 16.3 & 4.25 \end{bmatrix} MPa$$

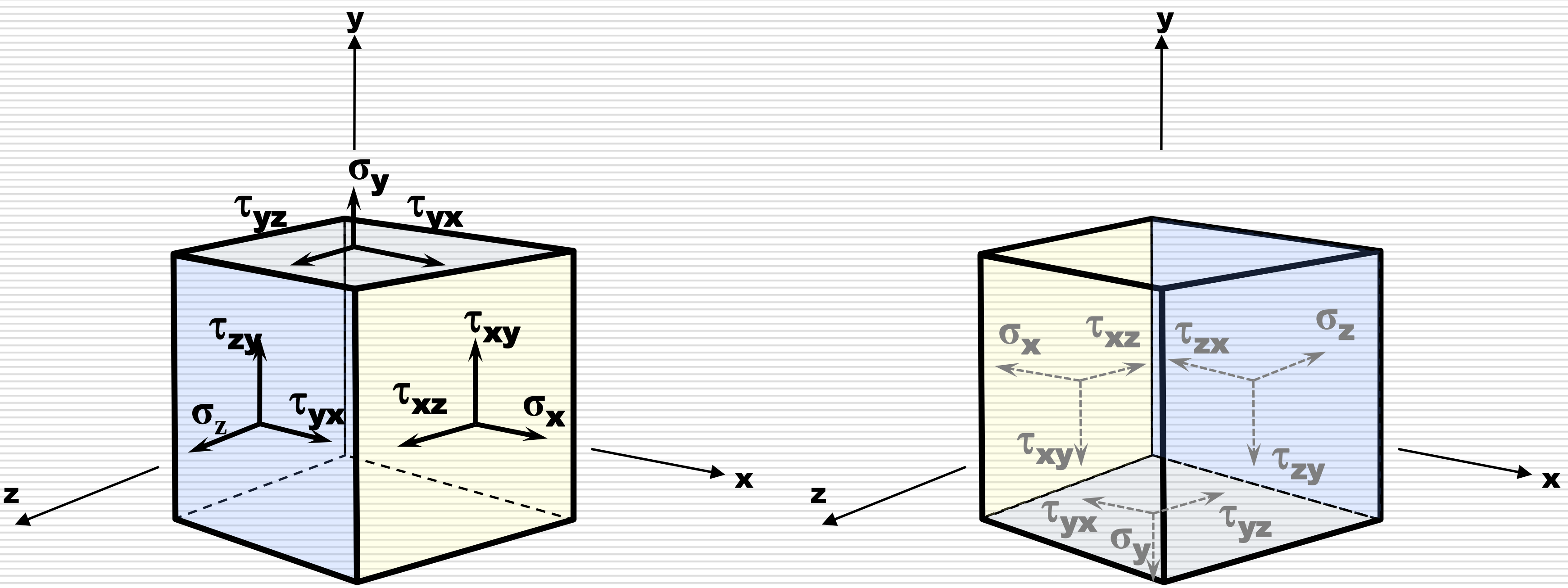
Transformation Matrix

$$T = T2 * T1 = \begin{bmatrix} 0.7071 & 0.7071 & 0 \\ -0.6124 & 0.6124 & 0.5 \\ 0.3536 & -0.3536 & 0.866 \end{bmatrix}$$

Transpose of Transformation Matrix

$$T' = \begin{bmatrix} 0.7071 & -0.6124 & 0.3536 \\ 0.7071 & 0.6124 & -0.3536 \\ 0 & 0.5 & 0.866 \end{bmatrix}$$

Stress at a Point
Shown in the Tensile (+) Direction



Surfaces with a Positive Directed Area Normal

Surfaces with a Negative Directed Area Normal

Principal Stresses

- $\sigma_1, \sigma_2, \sigma_3$
- Eigenvalues of Stress Tensor
- Eigenvectors are the direction cosines for the Principal Stresses

Two Dimensional/Plane Stress

- **Transformations**

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cdot \cos 2\theta + \tau_{xy} \cdot \sin 2\theta$$

$$\sigma_{y_1} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cdot \cos 2\theta - \tau_{xy} \cdot \sin 2\theta$$

$$\sigma_{x_1} = -\frac{\sigma_x - \sigma_y}{2} \cdot \sin 2\theta + \tau_{xy} \cdot \cos 2\theta$$

- **Principal Stresses**

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad \tan 2\theta_p = \frac{2 \cdot \tau_{xy}}{\sigma_x - \sigma_y}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Three Dimensional Stress Invariants

$$\begin{vmatrix} \sigma_x - \sigma_p & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y - \sigma_p & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z - \sigma_p \end{vmatrix} = 0$$

$$\begin{aligned} \sigma_p^3 &- (\sigma_x + \sigma_y + \sigma_z)\cdot\sigma_p^2 \\ &+ (\sigma_x\cdot\sigma_y + \sigma_y\cdot\sigma_z + \sigma_x\cdot\sigma_z - \tau_{yz}^2 - \tau_{zx}^2 - \tau_{xy}^2)\cdot\sigma_p \\ &- (\sigma_x\cdot\sigma_y\cdot\sigma_z + 2\cdot\tau_{yz}\cdot\tau_{xz}\cdot\tau_{xy} - \sigma_x\cdot\tau_{yz}^2 - \sigma_y\cdot\tau_{zx}^2 - \sigma_z\cdot\tau_{xy}^2) = 0 \end{aligned}$$

$$\sigma_p^3 - I_1\cdot\sigma_p^2 + I_2\cdot\sigma_p - I_3 = 0$$

Example 1

Determine the principal stresses and their directions for the tensor shown.

$$[\sigma] = \begin{bmatrix} 50 & 10 & 0 \\ 10 & 20 & 40 \\ 0 & 40 & 30 \end{bmatrix} MPa$$

Solution to Example 1

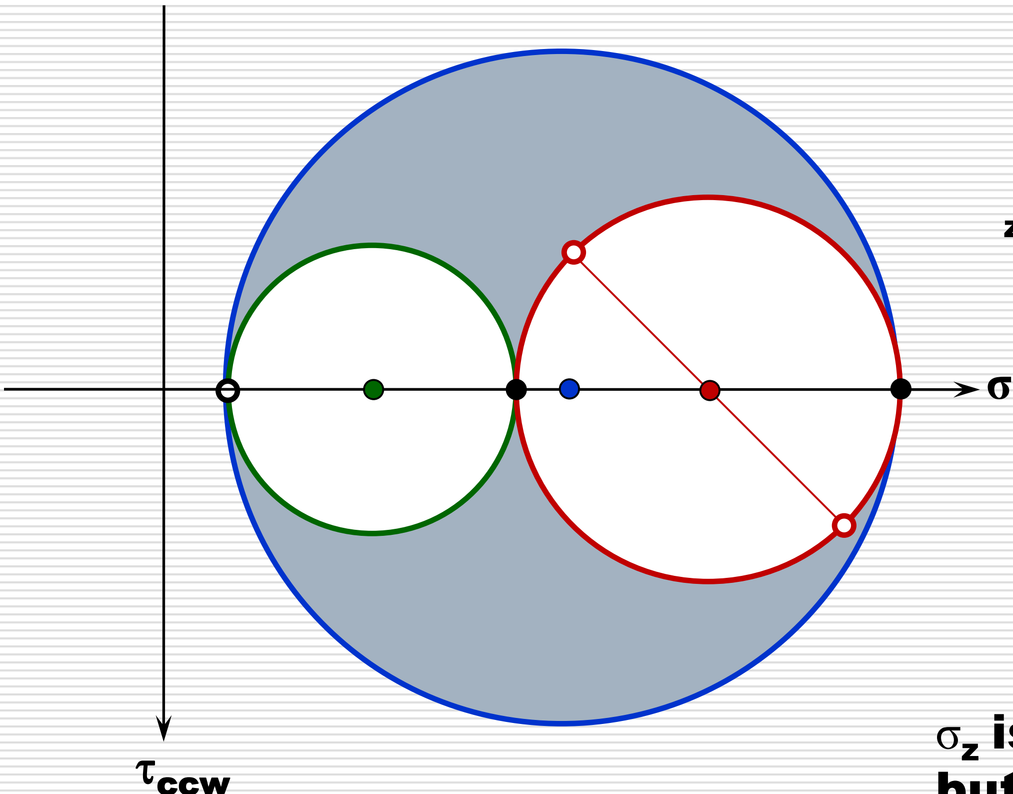
- Principal Stress

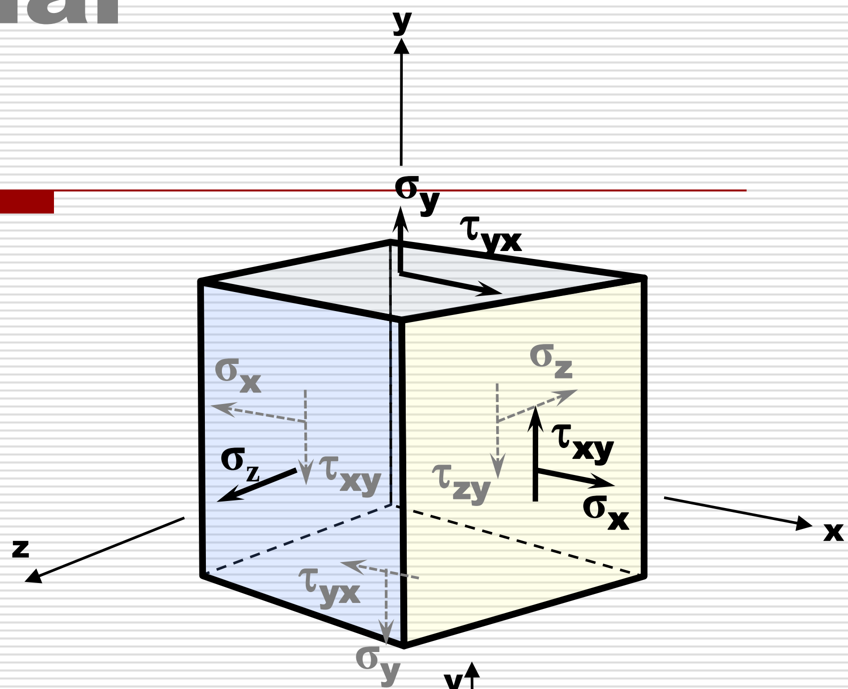
$$\left[\sigma_p\right] = \begin{bmatrix} -16.17 & 0 & 0 \\ 0 & 48.3 & 0 \\ 0 & 0 & 67.9 \end{bmatrix} MPa$$

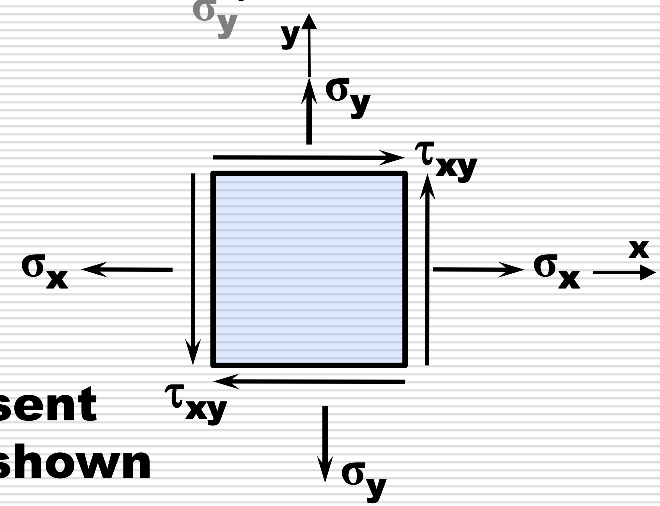
- Direction Cosines

$$[T] = \begin{bmatrix} -0.1135 & 0.7509 & -0.6506 \\ 0.9263 & -0.1568 & -0.3425 \\ 0.3592 & 0.6415 & 0.6778 \end{bmatrix}$$

Three Dimensional Mohr's Circle







σ_z is present but not shown

Example 2

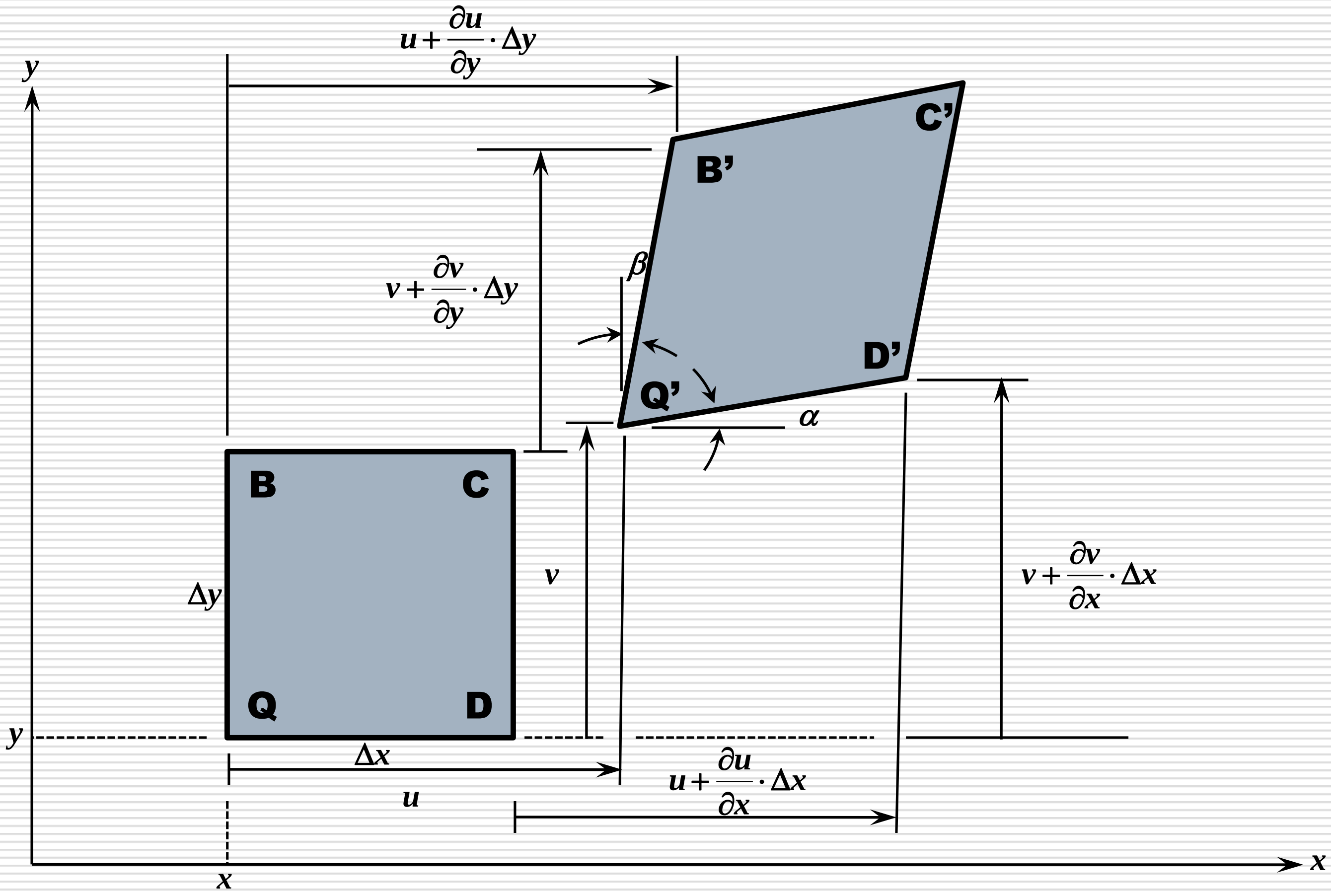
A structural member is found to have an axial stress of 150MPa and a transverse stress of 100MPa. The stress orthogonal to these stresses is zero. Calculate the maximum shear stress in this member at this point.

Example 3

Determine the principal stresses and the maximum shear stress for the following state of stress.

$$[\sigma] = \begin{bmatrix} 12 & 4 & 0 \\ 4 & -8 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

Strain-Displacement Relationships



Normal Strain - Displacements

$$\varepsilon_x = \partial u / \partial x$$

$$\varepsilon_y = \partial v / \partial y$$

$$\varepsilon_x = \partial w / \partial z$$

Shear Strain - Displacements

$$\gamma_{xy} = \partial v / \partial x + \partial u / \partial y$$

$$\gamma_{xz} = \partial w / \partial x + \partial u / \partial z$$

$$\gamma_{zy} = \partial v / \partial z + \partial w / \partial y$$

Curvature - Displacements

$$\Theta_{xy} = \frac{1}{2}\left(\partial v \big/ \partial x - \partial u \big/ \partial y \right)$$

$$\Theta_{xz} = \frac{1}{2}\left(\partial u \big/ \partial z - \partial w \big/ \partial x \right)$$

$$\Theta_{zy} = \frac{1}{2}\left(\partial w \big/ \partial y - \partial v \big/ \partial z \right)$$

Strain-Displacement Relations
Cylindrical System

$$\varepsilon_\rho = \frac{\partial u_r}{\partial r} \quad ; \quad \gamma_{\theta z} = \frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial w}{\partial \theta}$$

$$\varepsilon_\theta = \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r} \quad ; \quad \gamma_{rz} = \frac{\partial w}{\partial r} + \frac{\partial u_r}{\partial z}$$

$$\varepsilon_z = \frac{\partial w}{\partial z} \quad ; \quad \gamma_{r\theta} = \frac{1}{r}\cdot\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}$$

Compatibility

- Displacements match boundary conditions
- Displacements are single valued
- Displacements are continuous functions of position
- The body must be pieced together; no voids are created in the deformed body

Compatibility

$$\frac{\partial^2 \gamma_{xy}}{\partial x \cdot \partial y} = \frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2}$$

$$\frac{\partial^2 \gamma_{yz}}{\partial y \cdot \partial z} = \frac{\partial^2 \varepsilon_y}{\partial z^2} + \frac{\partial^2 \varepsilon_z}{\partial y^2}$$

$$\frac{\partial^2 \gamma_{xz}}{\partial x \cdot \partial z} = \frac{\partial^2 \varepsilon_x}{\partial z^2} + \frac{\partial^2 \varepsilon_z}{\partial x^2}$$

Compatibility Continued

$$2\cdot\frac{\partial^2\varepsilon_x}{\partial y\cdot\partial z}=\frac{\partial}{\partial x}\cdot\left(-\frac{\partial\gamma_{yz}}{\partial x}+\frac{\partial\gamma_{zx}}{\partial y}+\frac{\partial\gamma_{xy}}{\partial z}\right)$$

$$2\cdot\frac{\partial^2\varepsilon_y}{\partial z\cdot\partial x}=\frac{\partial}{\partial y}\cdot\left(\frac{\partial\gamma_{yz}}{\partial x}-\frac{\partial\gamma_{zx}}{\partial y}+\frac{\partial\gamma_{xy}}{\partial z}\right)$$

$$2\cdot\frac{\partial^2\varepsilon_z}{\partial x\cdot\partial y}=\frac{\partial}{\partial z}\cdot\left(\frac{\partial\gamma_{yx}}{\partial x}+\frac{\partial\gamma_{zx}}{\partial y}-\frac{\partial\gamma_{xy}}{\partial z}\right)$$

Example

The following displacement field is applied to a certain body where $k=10^{-4}$.

$u=k(2x+y^2)$, $v=k(x^2-3y^2)$, $w=0$

(a) Show the distorted configuration of a two-dimensional element with sides dx and dy and its lower left corner (point A) initially at the point (2,1,0), i.e., determine the new length and angular position of each side.

Strain Tensor

$$[\varepsilon] = \begin{bmatrix} \varepsilon_x & \dfrac{\gamma_{xy}}{2} & \dfrac{\gamma_{xz}}{2} \\ \dfrac{\gamma_{yx}}{2} & \varepsilon_y & \dfrac{\gamma_{yz}}{2} \\ \dfrac{\gamma_{zx}}{2} & \dfrac{\gamma_{zy}}{2} & \varepsilon_z \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} = \varepsilon_{ij}$$

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \dfrac{\gamma_{xz}}{2} \\ \dfrac{\gamma_{yz}}{2} \\ \dfrac{\gamma_{xy}}{2} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \varepsilon_{xz} \\ \varepsilon_{yz} \\ \varepsilon_{xy} \end{Bmatrix}$$

Strain Transformations

$$T = \begin{bmatrix} n_{x',x} & n_{x',y} & n_{x',z} \\ n_{y',x} & n_{y',y} & n_{y',z} \\ n_{z',x} & n_{z',y} & n_{z',z} \end{bmatrix}$$

$$[\varepsilon]_{x'y'z'} = [T]\cdot[\varepsilon]_{xyz}\cdot[T]^T$$

Two Dimensional/Plane Strain Transformations

- **General Transformation Equations**

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cdot \cos 2\theta + \frac{\gamma_{xy}}{2} \cdot \sin 2\theta$$

$$\frac{\gamma_{x_1 y_2}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2} \cdot \sin 2\theta + \frac{\gamma_{xy}}{2} \cdot \sin 2\theta \cos 2\theta$$

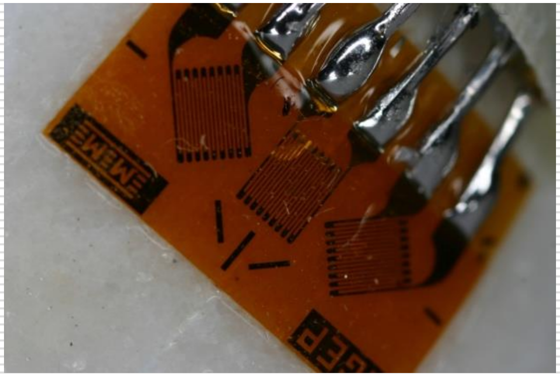
- **Principal Strains**

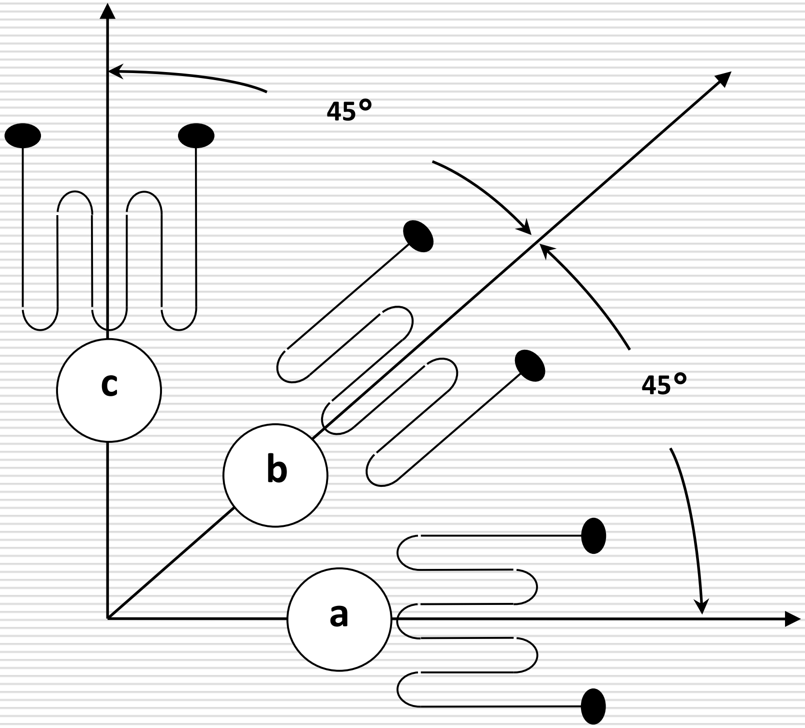
$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

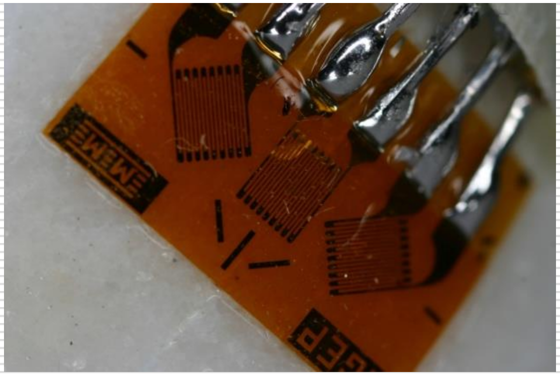
$$\frac{\gamma_{\max}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

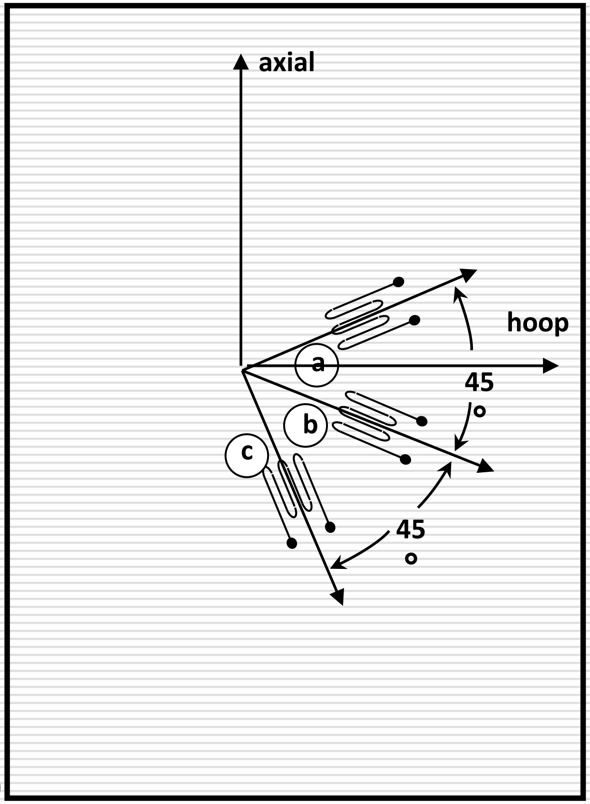
Typical Strain Gage Rosettes





Typical Strain Gage Rosettes





$|\varepsilon_{a*}| = \varepsilon_a = 1237(10^{-6})$ in/in $= 1237\mu\varepsilon$ (12)
$|\varepsilon_{b*}| = \varepsilon_b = 1270(10^{-6})$ in/in $= 1270\mu\varepsilon$ (13)
$|\varepsilon_{c*}| = \varepsilon_c = 402(10^{-6})$ in/in $= 402\mu\varepsilon$ (14)

(1)

Transverse Sensitivity

(2)

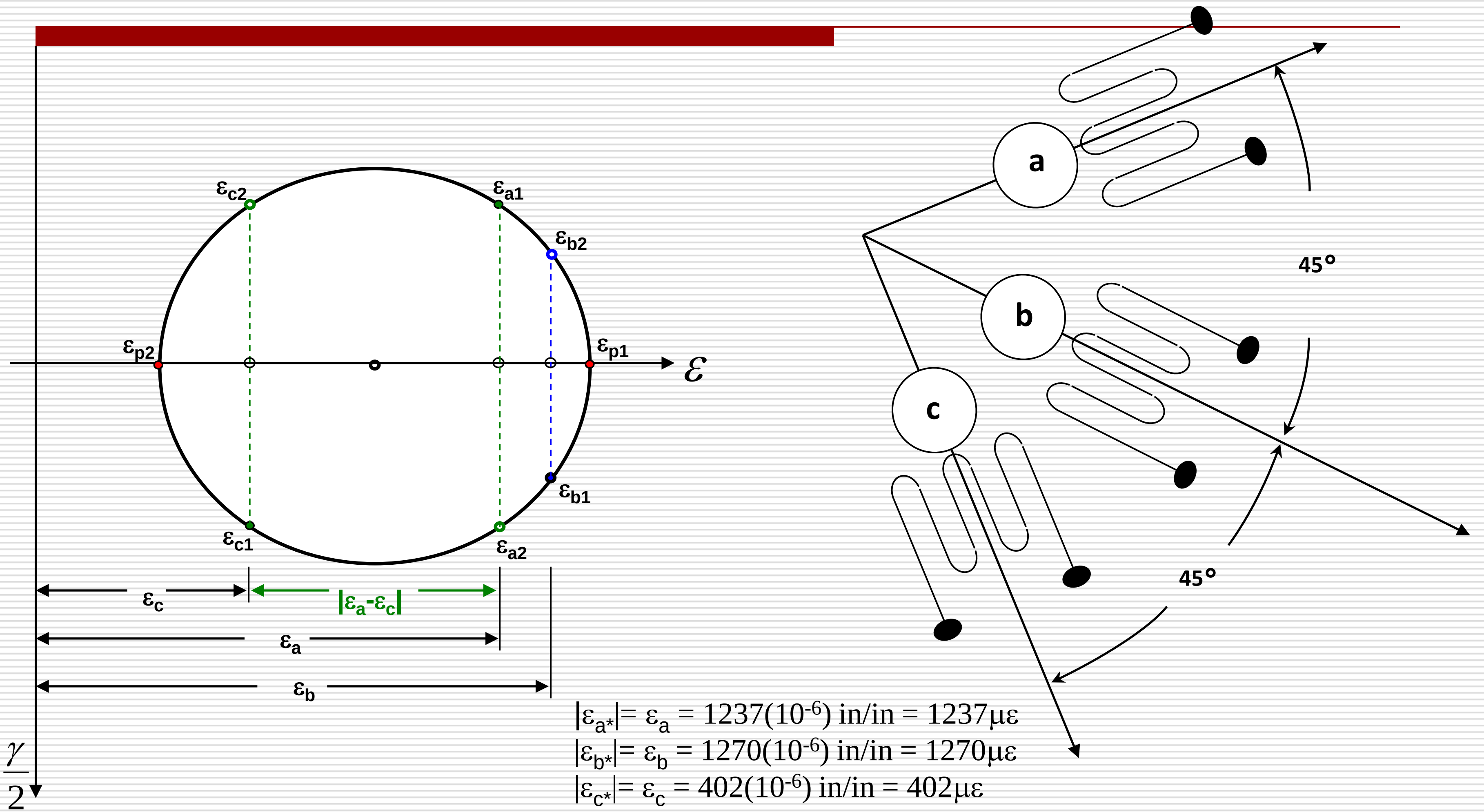
(3)

$$\varepsilon_a = \frac{\hat{\varepsilon}_a \cdot (1 - \nu_0 \cdot K_a) - K_a \cdot \hat{\varepsilon}_c \cdot (1 - \nu_0 \cdot K_c)}{1 - K_a \cdot K_c}$$

$$\varepsilon_b = \frac{\hat{\varepsilon}_b \cdot (1 - \nu_0 \cdot K_b)}{1 - K_b} - \frac{K_b \cdot [\hat{\varepsilon}_a \cdot (1 - \nu_0 \cdot K_a) \cdot (1 - K_c) + \hat{\varepsilon}_c \cdot (1 - \nu_0 \cdot K_c) \cdot (1 - K_a)]}{(1 - K_a \cdot K_c) \cdot (1 - K_b)}$$

$$\varepsilon_c = \frac{\hat{\varepsilon}_c \cdot (1 - \nu_0 \cdot K_c) - K_c \cdot \hat{\varepsilon}_a \cdot (1 - \nu_0 \cdot K_a)}{1 - K_a \cdot K_c}$$

Mohr's Circle for Strain

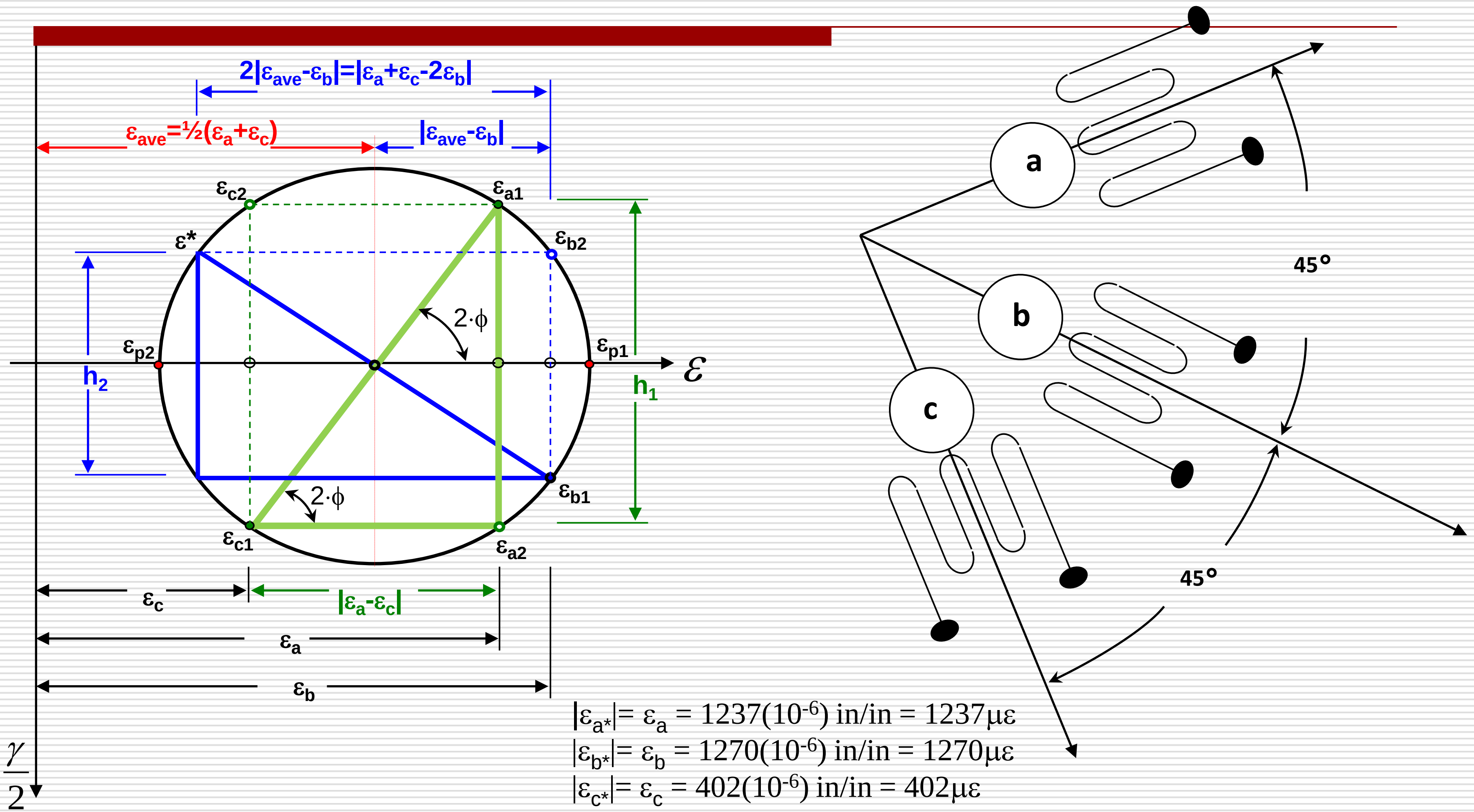


$|\varepsilon_{a*}| = \varepsilon_a = 1237(10^{-6})$ in/in $= 1237\mu\varepsilon$

$|\varepsilon_{b*}| = \varepsilon_b = 1270(10^{-6})$ in/in $= 1270\mu\varepsilon$

$|\varepsilon_{c*}| = \varepsilon_c = 402(10^{-6})$ in/in $= 402\mu\varepsilon$

Mohr's Circle for Strain



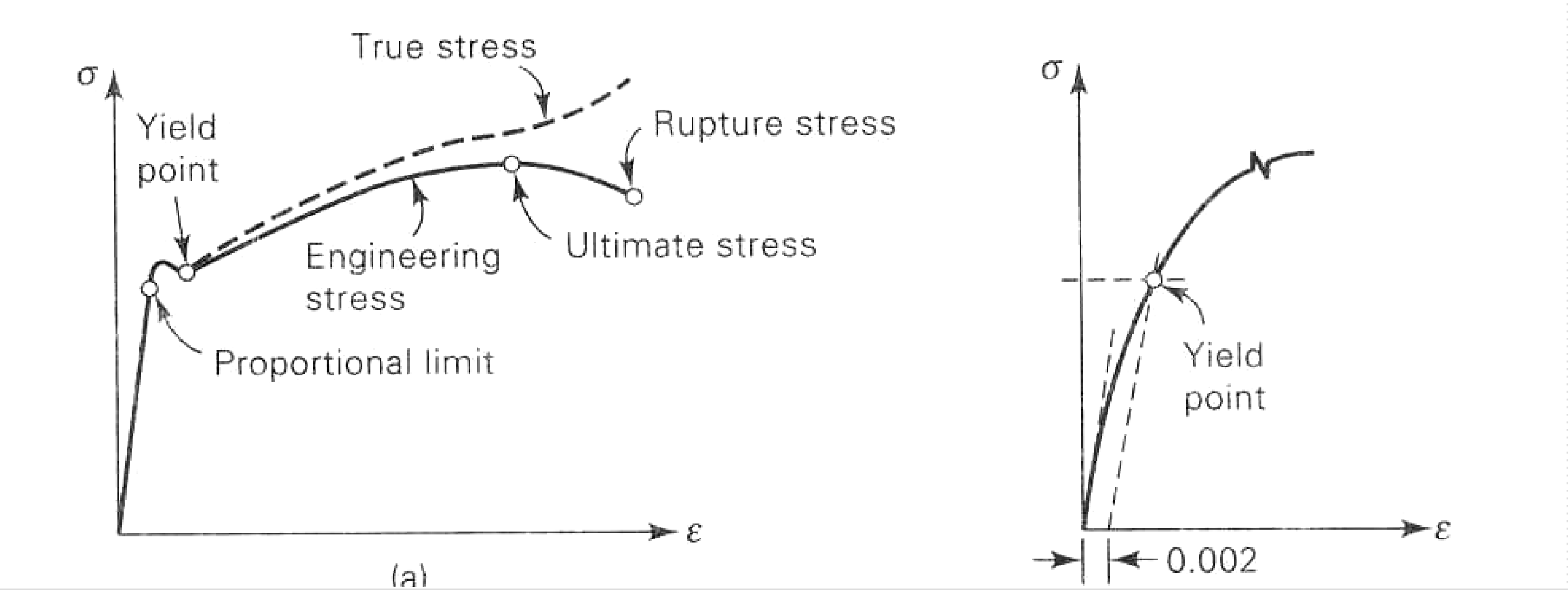
$|\varepsilon_{a*}| = \varepsilon_a = 1237(10^{-6})$ in/in $= 1237\mu\varepsilon$

$|\varepsilon_{b*}| = \varepsilon_b = 1270(10^{-6})$ in/in $= 1270\mu\varepsilon$

$|\varepsilon_{c*}| = \varepsilon_c = 402(10^{-6})$ in/in $= 402\mu\varepsilon$

Stress-Strain Curve

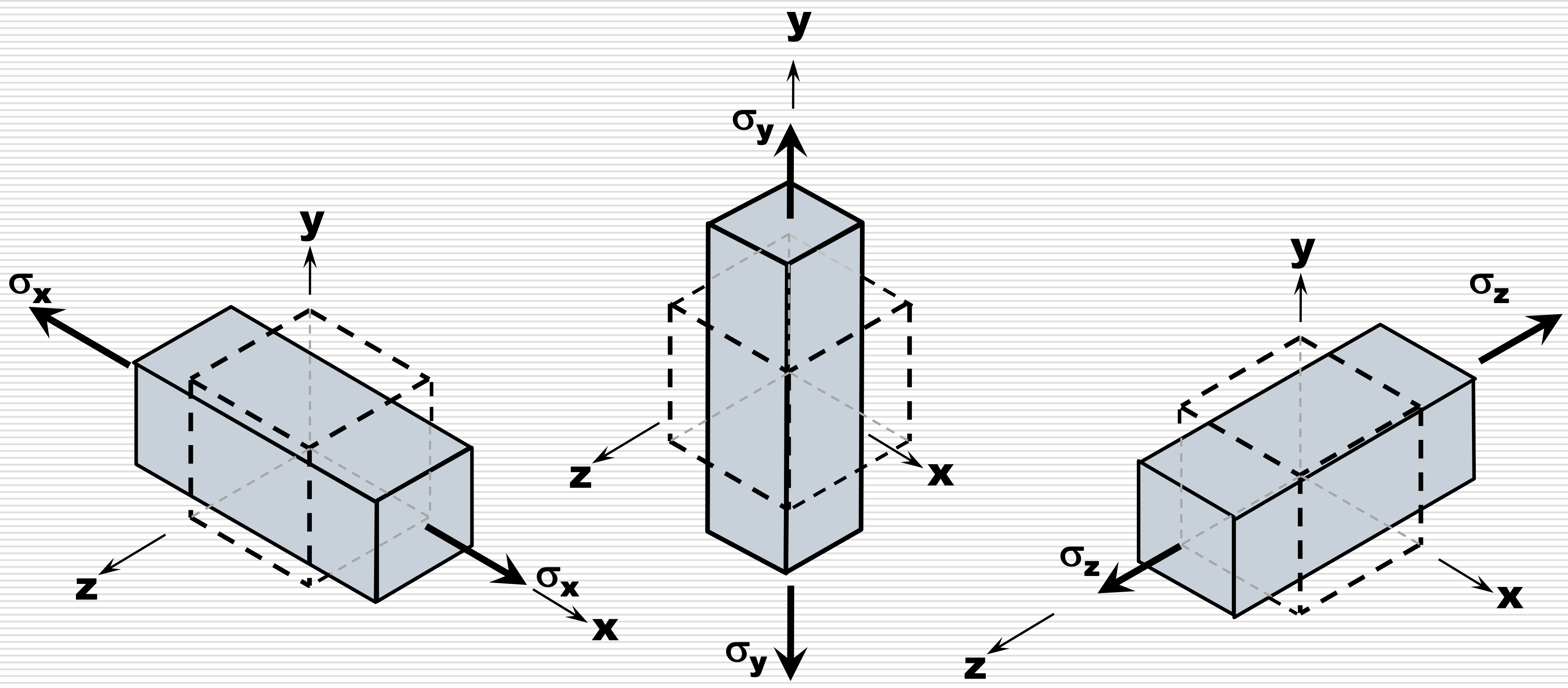
True Stress-Strain versus Engineering Stress-Strain



$$\sigma = k \cdot \frac{P}{A}$$

$$\varepsilon = \int_{L_0}^{L} \frac{dl}{l} = \ln \frac{L}{L_0} = \ln\left(1 + \varepsilon_0\right)$$

Relationship Between Stress and Strain



Stress-Strain Relations

$$\varepsilon_x = \frac{1}{E}\cdot\left[\sigma_x - \nu\cdot\left(\sigma_y + \sigma_z\right)\right] \qquad \gamma_{yz} = \frac{\tau_{yz}}{G}$$

$$\varepsilon_y = \frac{1}{E}\cdot\left[\sigma_y - \nu\cdot\left(\sigma_x + \sigma_z\right)\right] \qquad \gamma_{xz} = \frac{\tau_{xz}}{G}$$

$$\varepsilon_z = \frac{1}{E}\cdot\left[\sigma_z - \nu\cdot\left(\sigma_y + \sigma_x\right)\right] \qquad \gamma_{xy} = \frac{\tau_{xy}}{G}$$

Matrix Form of Stress-Strain Relations

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix} \cdot \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix}$$

Matrix Form of Stress-Strain Relations

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2\cdot(1+\nu)}{E} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2\cdot(1+\nu)}{E} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2\cdot(1+\nu)}{E} \end{bmatrix} \cdot \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix}$$

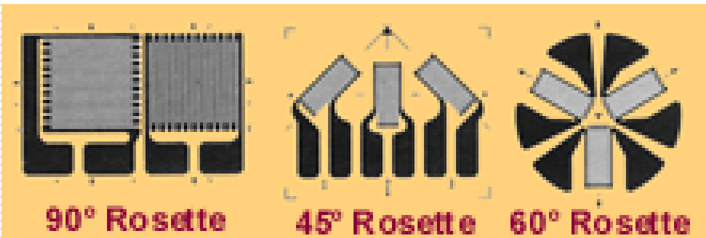
Strain-Stress Relations

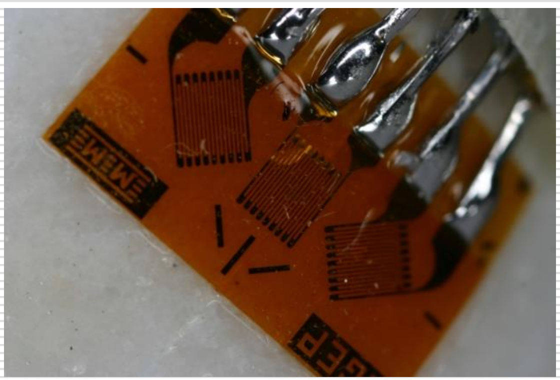
$$\sigma_x = \frac{E}{(1+\nu)\cdot(1-2\cdot\nu)}\left[(1-\nu)\cdot\varepsilon_x + \nu\cdot\left(\varepsilon_y + \varepsilon_z\right)\right]$$

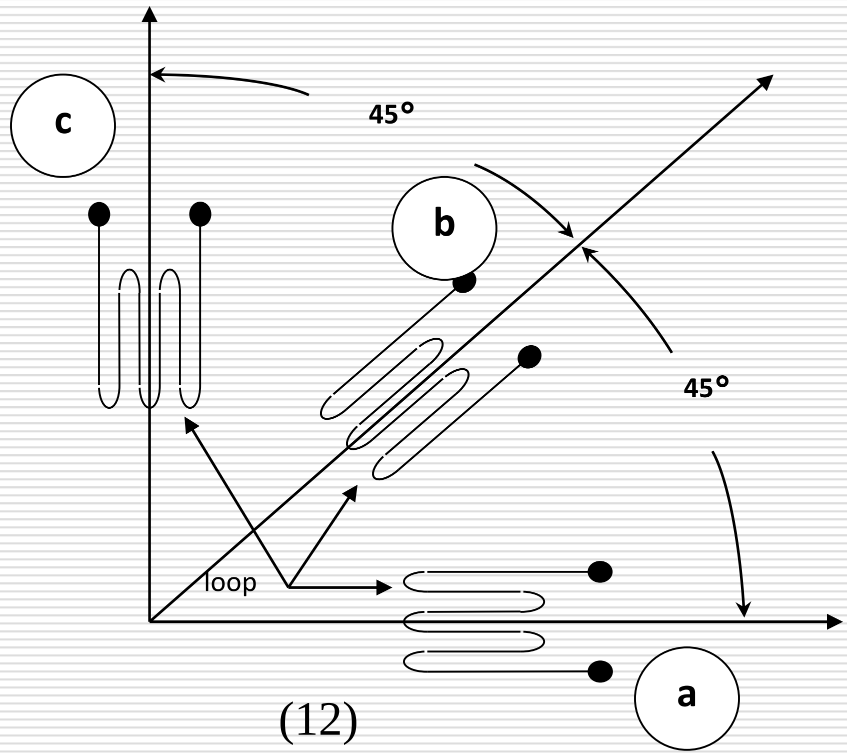
$$\sigma_y = \frac{E}{(1+\nu)\cdot(1-2\cdot\nu)}\left[(1-\nu)\cdot\varepsilon_y + \nu\cdot\left(\varepsilon_x + \varepsilon_z\right)\right]$$

$$\sigma_z = \frac{E}{(1+\nu)\cdot(1-2\cdot\nu)}\left[(1-\nu)\cdot\varepsilon_z + \nu\cdot\left(\varepsilon_x + \varepsilon_y\right)\right]$$

Typical Strain Gage Rosettes







$|\varepsilon_{a*}| = \varepsilon_a = 1237(10^{-6})$ in/in $= 1237\mu\varepsilon$ (12)

$|\varepsilon_{b*}| = \varepsilon_b = 1270(10^{-6})$ in/in $= 1270\mu\varepsilon$ (13)

$|\varepsilon_{c*}| = \varepsilon_c = 402(10^{-6})$ in/in $= 402\mu\varepsilon$ (14)

Anisotropic Materials

- **ISOTROPIC** - material properties are the same in all directions
- **ANISOTROPIC** - material properties change with direction
- **HOMOGENEOUS** - material of uniform composition throughout and whose properties are constant at every point
- **HETEROGENEOUS** - material uniformity within a body consisting of dissimilar constituents separately identifiable

Assumptions

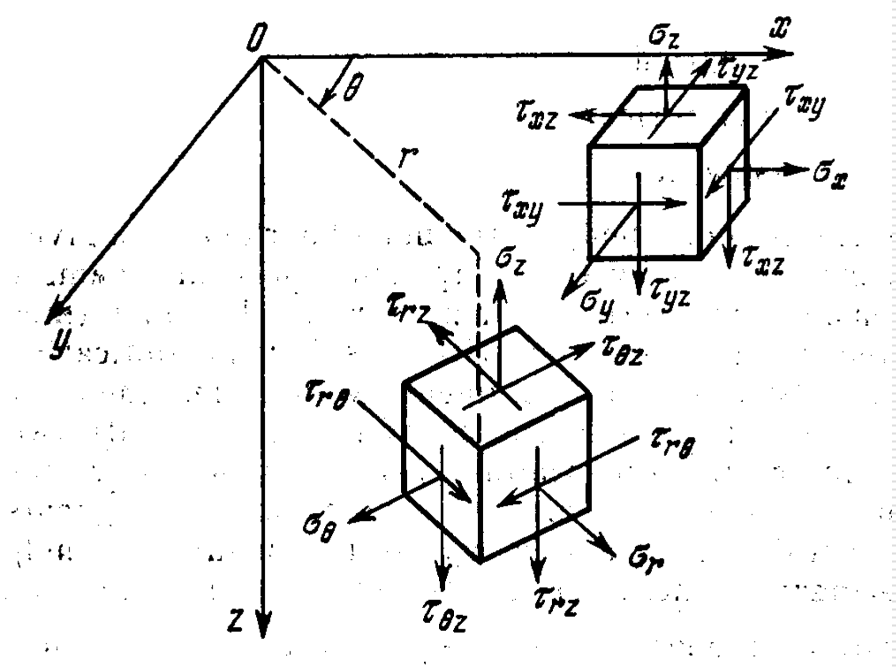
- The elastic body is a continuous medium
  - the body is a solid
- The relation between the components of strain and the projections of displacement and their first derivatives with respect to the coordinates is linear
  - only small strains are considered

Assumptions

- The stress-strain relations are linear
  - the material follows the generalized Hooke's law
  - the coefficients in these linear relations may be either constant (homogeneous body) or variable - functions of position, continuous or discontinuous (non-homogeneous body)
- Theory is based on classical linear theory of homogeneous or non-homogeneous elastic bodies

Major Notation

- Coordinate Systems
  - Cartesian - x, y, z
  - Cylindrical - r, θ, z
  - Spherical - ρ, θ, ϕ
- Stresses acting on planes normal to the co-ordinate directions
  - one normal = normal Stress
  - two tangential = shearing stresses



Stress-Strain Relations
36 (81) Constants

Stiffness

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \tau_{23}\\ \tau_{13}\\ \tau_{12}\end{Bmatrix} = \begin{Bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16}\\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26}\\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{63}\\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{64}\\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{65}\\ C_{61} & C_{62} & C_{63} & C_{64} & C_{54} & C_{66}\end{Bmatrix} \cdot \begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \gamma_{23}\\ \gamma_{13}\\ \gamma_{12}\end{Bmatrix}$$

Compliance

$$\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \gamma_{23}\\ \gamma_{13}\\ \gamma_{12}\end{Bmatrix} = \begin{Bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16}\\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26}\\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36}\\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46}\\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56}\\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66}\end{Bmatrix} \cdot \begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \tau_{23}\\ \tau_{13}\\ \tau_{12}\end{Bmatrix}$$

Symmetry of the Stiffness Matrix

- Elastic Potential/Strain Energy Density
  - Incremental work per unit volune
  - $dW=\sigma_i d\varepsilon_i$
- Using the Stress-Strain Relations
  - $dW=C_{ij}\varepsilon_j d\varepsilon_j$
- Work per Unit Volume
  - $W=1/2\ C_{ij}\varepsilon_i\varepsilon_j$
- $dW/d\varepsilon_i=C_{ij}\varepsilon_j$ or $dW^2/d\varepsilon_i\, d\varepsilon_j=C_{ij}$ thus $C_{ij}=C_{ji}$

Stiffness and Compliance down from 36 to 21 Constants

Stiffness

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix} = \begin{Bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{63} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{64} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{65} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{Bmatrix} \cdot \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix}$$

Compliance

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix} = \begin{Bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{12} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{13} & S_{23} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{14} & S_{24} & S_{34} & S_{44} & S_{45} & S_{46} \\ S_{15} & S_{25} & S_{35} & S_{45} & S_{55} & S_{56} \\ S_{16} & S_{26} & S_{36} & S_{46} & S_{56} & S_{66} \end{Bmatrix} \cdot \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix}$$

One Plane of Elastic Symmetry

Monoclinic

13 Independent Constants

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26} \\ C_{13} & C_{23} & C_{33} & 0 & 0 & C_{63} \\ 0 & 0 & 0 & C_{44} & C_{45} & 0 \\ 0 & 0 & 0 & C_{45} & C_{55} & 0 \\ C_{16} & C_{26} & C_{36} & 0 & 0 & C_{66} \end{bmatrix} \cdot \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix}$$

Three Planes of Elastic Symmetry

Orthotropic Body
9 Independent Constants

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix} = \begin{Bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{Bmatrix} \cdot \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix}$$

Stiffness Matrix
Orthotropic Body

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \tau_{23}\\ \tau_{13}\\ \tau_{12}\end{Bmatrix}=\begin{bmatrix}C_{11} & C_{12} & C_{13} & 0 & 0 & 0\\ C_{21} & C_{22} & C_{23} & 0 & 0 & 0\\ C_{31} & C_{32} & C_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & C_{66}\end{bmatrix}\cdot\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \gamma_{23}\\ \gamma_{13}\\ \gamma_{12}\end{Bmatrix}$$

$$C_{11}=\frac{(1-\upsilon_{23}\cdot\upsilon_{32})\cdot E_1}{1-\upsilon}$$

$$C_{12}=\frac{(\upsilon_{21}-\upsilon_{31}\cdot\upsilon_{23})\cdot E_1}{1-\upsilon}=\frac{(\upsilon_{12}-\upsilon_{32}\cdot\upsilon_{13})\cdot E_2}{1-\upsilon}$$

$$C_{13}=\frac{(\upsilon_{31}-\upsilon_{21}\cdot\upsilon_{32})\cdot E_1}{1-\upsilon}=\frac{(\upsilon_{13}-\upsilon_{12}\cdot\upsilon_{23})\cdot E_3}{1-\upsilon}$$

$$C_{22}=\frac{(1-\upsilon_{13}\cdot\upsilon_{31})\cdot E_2}{1-\upsilon}$$

$$C_{23}=\frac{(\upsilon_{32}-\upsilon_{12}\cdot\upsilon_{31})\cdot E_2}{1-\upsilon}=\frac{(\upsilon_{23}-\upsilon_{21}\cdot\upsilon_{13})\cdot E_3}{1-\upsilon}$$

$$C_{33}=\frac{(1-\upsilon_{12}\cdot\upsilon_{21})\cdot E_3}{1-\upsilon}$$

$$C_{44}=G_{23}\quad C_{55}=G_{13}\quad C_{66}=G_{12}$$

$$\upsilon=\upsilon_{12}\cdot\upsilon_{21}+\upsilon_{23}\cdot\upsilon_{32}+\upsilon_{31}\cdot\upsilon_{13}$$

$$C_{11}=\frac{S_{22}\cdot S_{33}-S_{23}\cdot S_{23}}{S}\qquad C_{12}=\frac{S_{13}\cdot S_{23}-S_{12}\cdot S_{33}}{S}\qquad C_{44}=\frac{1}{S_{44}}$$

$$C_{11}=\frac{S_{33}\cdot S_{11}-S_{13}\cdot S_{13}}{S}\qquad C_{13}=\frac{S_{12}\cdot S_{23}-S_{13}\cdot S_{22}}{S}\qquad C_{55}=\frac{1}{S_{55}}$$

$$C_{11}=\frac{S_{11}\cdot S_{22}-S_{12}\cdot S_{12}}{S}\qquad C_{23}=\frac{S_{12}\cdot S_{13}-S_{23}\cdot S_{11}}{S}\qquad C_{66}=\frac{1}{S_{66}}$$

$$S=S_{11}\cdot S_{22}\cdot S_{33}+S_{11}\cdot S_{23}\cdot S_{23}+S_{22}\cdot S_{13}\cdot S_{13}+S_{33}\cdot S_{12}\cdot S_{12}+2\cdot S_{12}\cdot S_{23}\cdot S_{13}$$

Compliance Matrix
Orthotropic Body

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{21} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{31} & S_{32} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \cdot \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix}$$

$$S_{11} = \frac{1}{E_1} \qquad S_{12} = -\frac{\upsilon_{21}}{E_2} \qquad S_{13} = -\frac{\upsilon_{31}}{E_3}$$

$$S_{21} = -\frac{\upsilon_{12}}{E_1} \qquad S_{22} = \frac{1}{E_2} \qquad S_{23} = -\frac{\upsilon_{32}}{E_3}$$

$$S_{31} = -\frac{\upsilon_{13}}{E_1} \qquad S_{32} = -\frac{\upsilon_{23}}{E_2} \qquad S_{33} = \frac{1}{E_3}$$

$$S_{44} = \frac{1}{G_{23}} \qquad S_{55} = \frac{1}{G_{13}} \qquad S_{66} = \frac{1}{G_{12}}$$

Relationship Between S and C

$$C_{11} = \frac{S_{22} \cdot S_{33} - S_{23}^2}{S} \quad ; \quad C_{44} = \frac{1}{S_{44}} \quad ; \quad C_{12} = \frac{S_{13}S_{23} - S_{12}S_{33}}{S}$$

$$C_{22} = \frac{S_{33} \cdot S_{11} - S_{13}^2}{S} \quad ; \quad C_{55} = \frac{1}{S_{55}} \quad ; \quad C_{13} = \frac{S_{12}S_{23} - S_{13}S_{22}}{S}$$

$$C_{33} = \frac{S_{11} \cdot S_{22} - S_{12}^2}{S} \quad ; \quad C_{66} = \frac{1}{S_{66}} \quad ; \quad C_{23} = \frac{S_{12}S_{13} - S_{23}S_{11}}{S}$$

$$S = S_{11}S_{22}S_{33} - S_{11}S_{23}^2 - S_{22}S_{13}^2 - S_{33}S_{12}^2 + 2S_{12}S_{23}S_{13}$$

One Plane in which the Mechanical Properties are Equal

Transversely Isotropic
6 Independent Constants

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(C_{11} - C_{12}) \end{bmatrix} \cdot \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix}$$

Material Properties Equal in all Directions

Isotropic
2 Independent Constants

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(C_{11}-C_{12}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(C_{11}-C_{12}) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(C_{11}-C_{12}) \end{bmatrix} \cdot \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix}$$

Matrix Form of Stress-Strain Relations

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix} \cdot \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix}$$

Matrix Form of Stress-Strain Relations

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2\cdot(1+\nu)}{E} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2\cdot(1+\nu)}{E} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2\cdot(1+\nu)}{E} \end{bmatrix} \cdot \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix}$$

Example 1

Given the following state of stress, determine the state of strain. E=200Gpa, υ=0.3

$$[\sigma] = \begin{bmatrix} 12 & 6 & 9 \\ 6 & 10 & 3 \\ 9 & 3 & 14 \end{bmatrix} MPa$$

Example 2

Given the following state of strain, determine the state of stress. E=200GPa, υ=0.3

$$[\varepsilon] = \begin{bmatrix} 3 & -1 & 2 \\ -1 & 0 & -4 \\ 2 & -4 & 5 \end{bmatrix} \times 10^{-4}$$

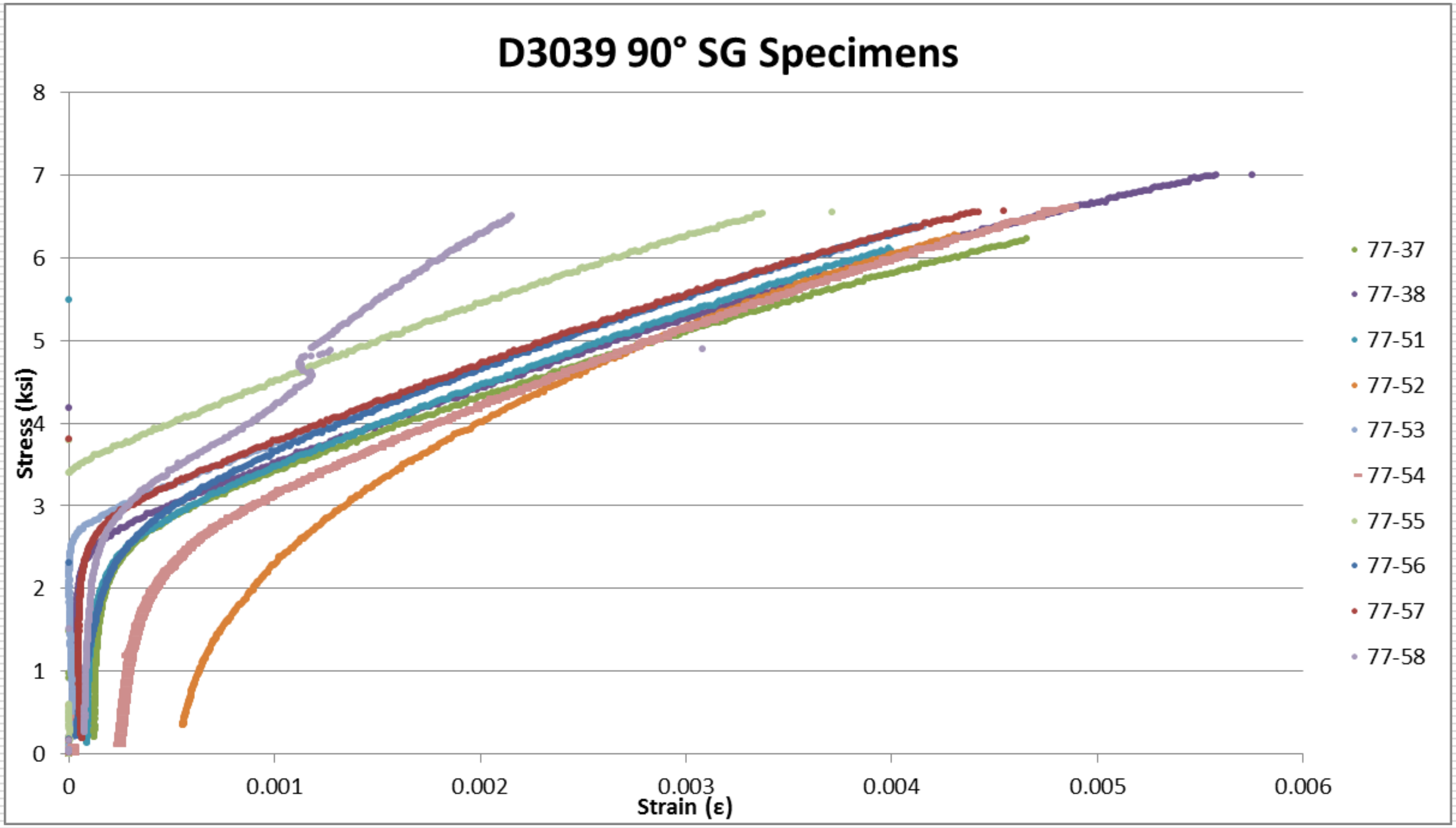
Typical Constituent Properties

	Graphite AS4 & T300	Graphite HMS & VS0032	Kevlar 49	E-Glass	S-Glass
E_L (GPa)	220	370	124	72.3	85.4
E_T (GPa)	22	7.6	6.9	72.3	85.4
ν_L	0.30	0.41	0.33	0.2	0.22
ν_T	0.35	0.45	0.33	0.2	0.22
G_L (GPa)	22	15.2	2.8	30.1	35.1
G_T (GPa)	8.3	2.8	2.8	30.1	35.1
σ^{tu}_L (GPa)	2.8	1.2	2.8	3.4	4.5
ε^{tu}_L (GPa)	1.3	0.3	2.5	4.8	5.4
α_L ($10^{-6}/^oC$)	-1.3	-0.7	-1.8	12.0	12.0
α_T ($10^{-6}/^oC$)	7.0	9.7	54	12.0	12.0
K_L ($cal/s \cdot cm \cdot ^oC$)	$49(10^{-3})$	$20(10^{-3})$	6.9	$2.3(10^{-3})$	$2.3(10^{-3})$
K_L ($cal/s \cdot cm \cdot ^oC$)	$3.5(10^{-3})$	$4.2(10^{-3})$	1.0	$2.3(10^{-3})$	$2.3(10^{-3})$
ρ (g/cm^3)	1.72	1.99	1.44	2.55	2.49

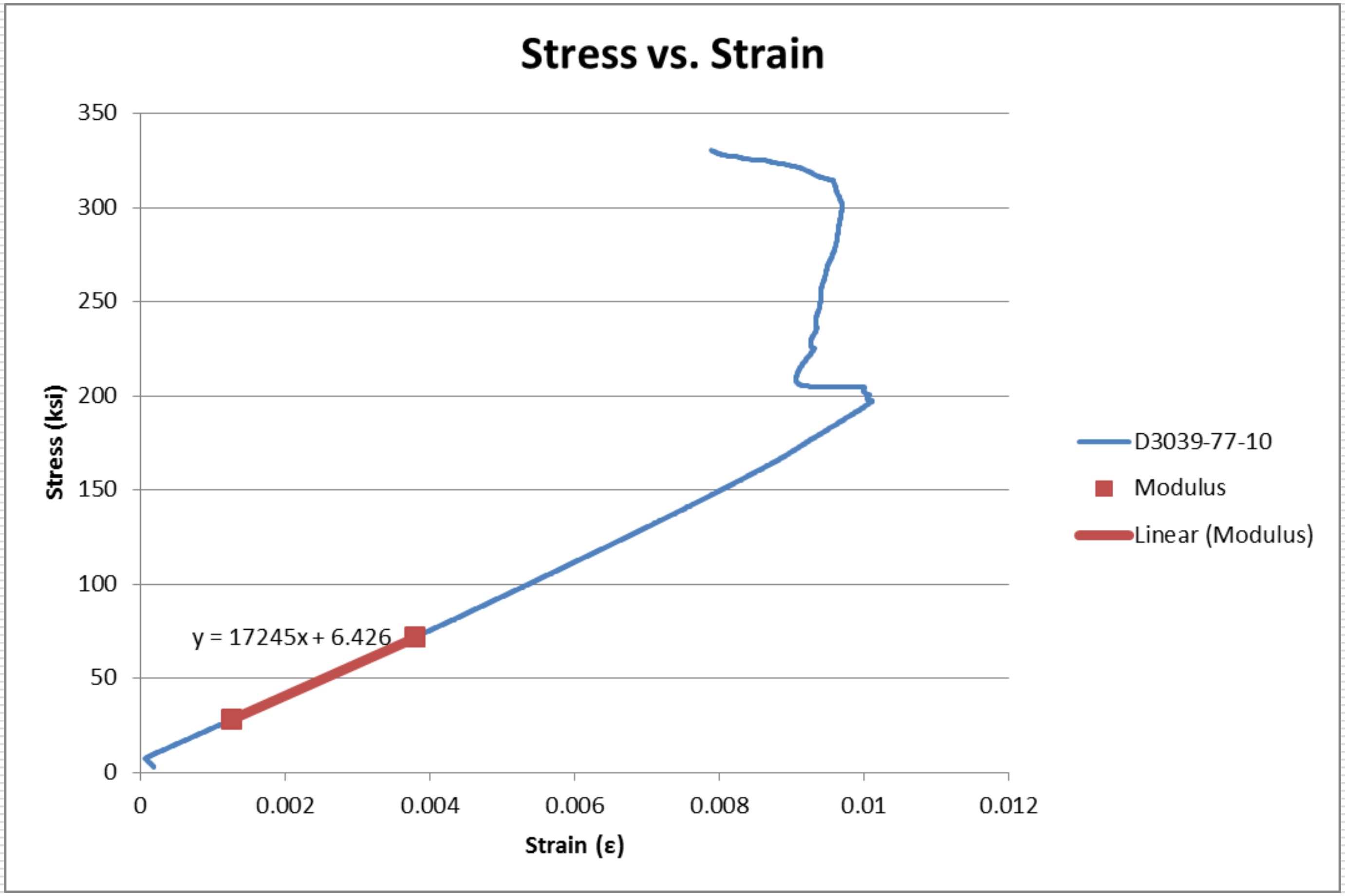
Typical Composite Properties

	Graphite-Polymer	Glass-Polymer	Aluminum
E_1 (Gpa)	155.0	50.0	72.4
E_2 (Gpa)	12.10	15.20	72.4
E_3 (Gpa)	12.10	15.20	72.4
ν_{23}	0.458	0.428	0.3
ν_{13}	0.248	0.254	0.3
ν_{12}	0.248	0.254	0.3
G_{23} (Gpa)	3.20	3.28	27.8
G_{13} (Gpa)	4.40	4.70	27.8
G_{12} (Gpa)	4.40	4.70	27.8
α_1 (10^{-6}/^oC)	-0.018	6.34	22.5
α_2 (10^{-6}/^oC)	24.3	23.3	22.5
α_3 (10^{-6}/^oC)	24.3	23.3	22.5
β_1 (10^{-6}/%M)	146.0	434.	0
β_2 (10^{-6}/%M)	4770	6320	0
β_3 (10^{-6}/%M)	4770	6320	0

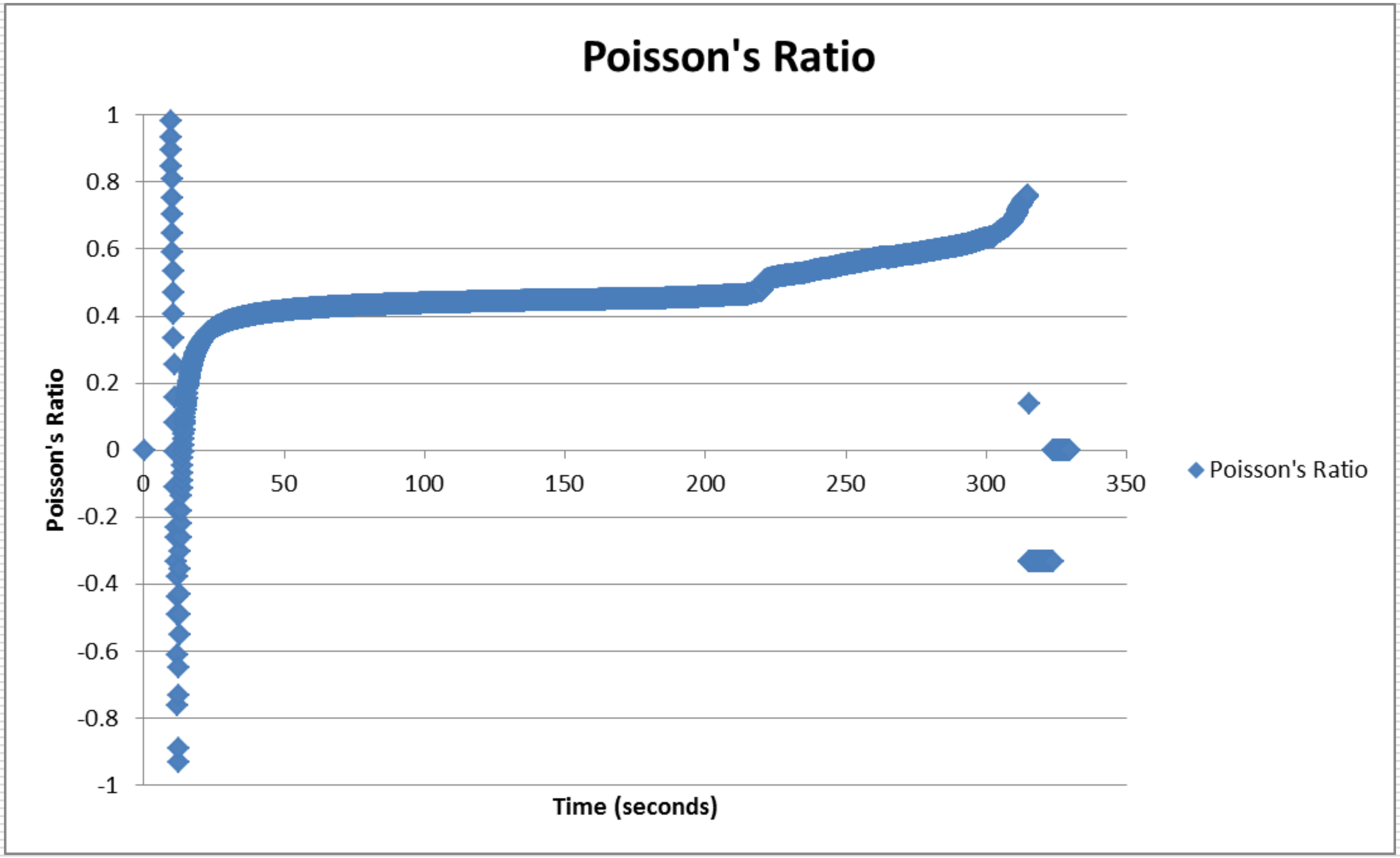
Typical Composite Tensile Data



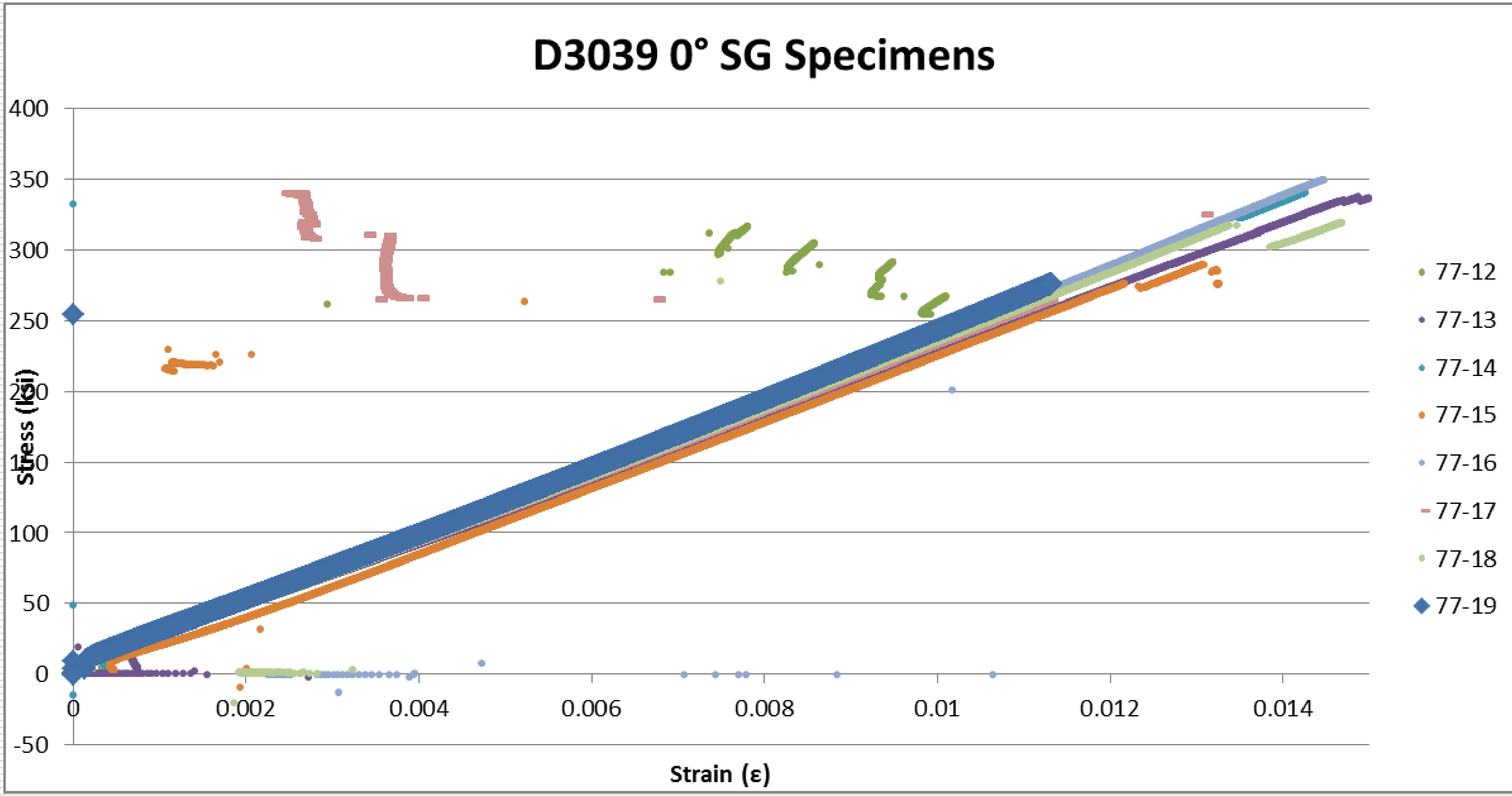
Typical Composite Tensile Data



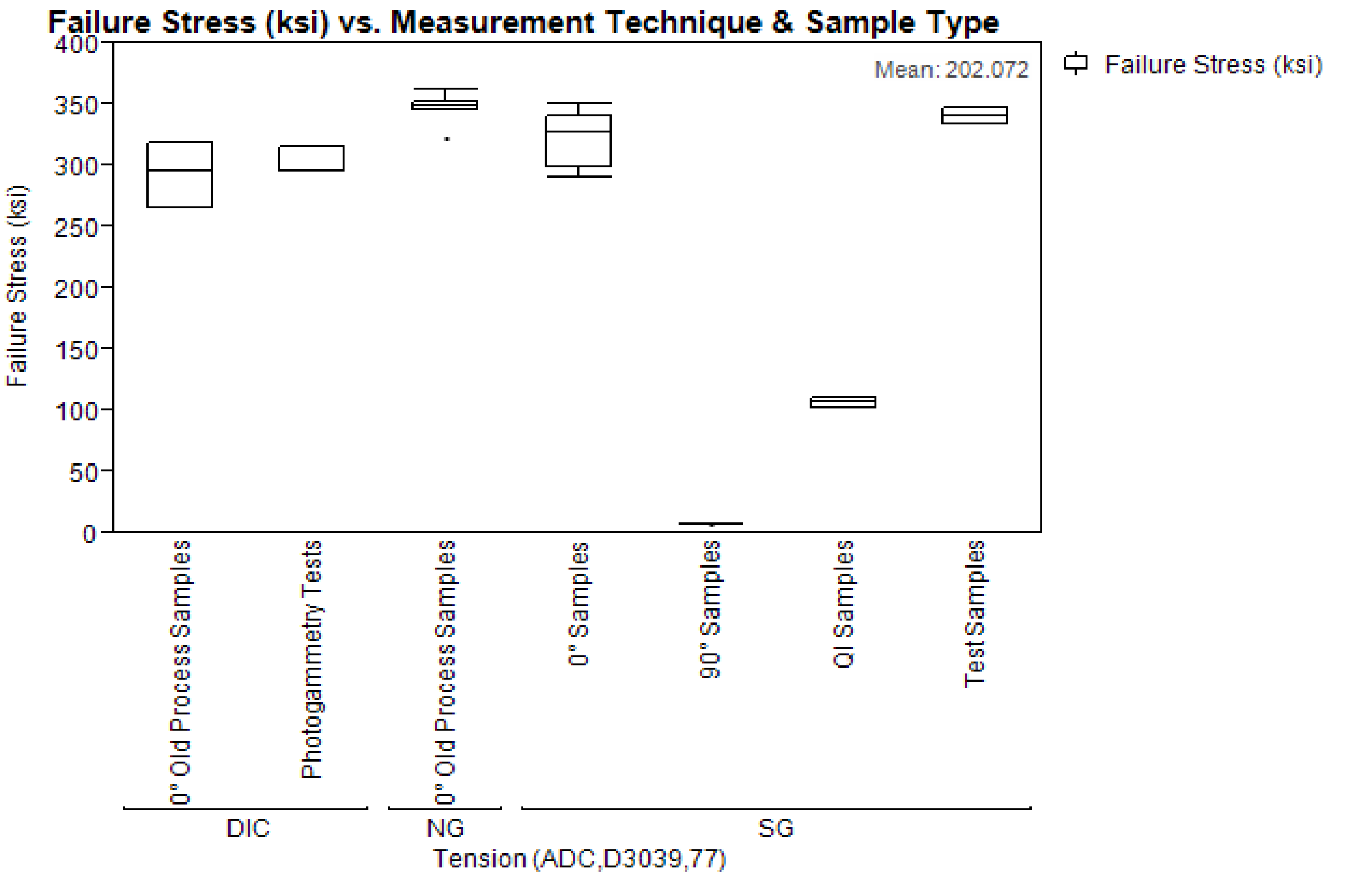
Typical Composite Tensile Data



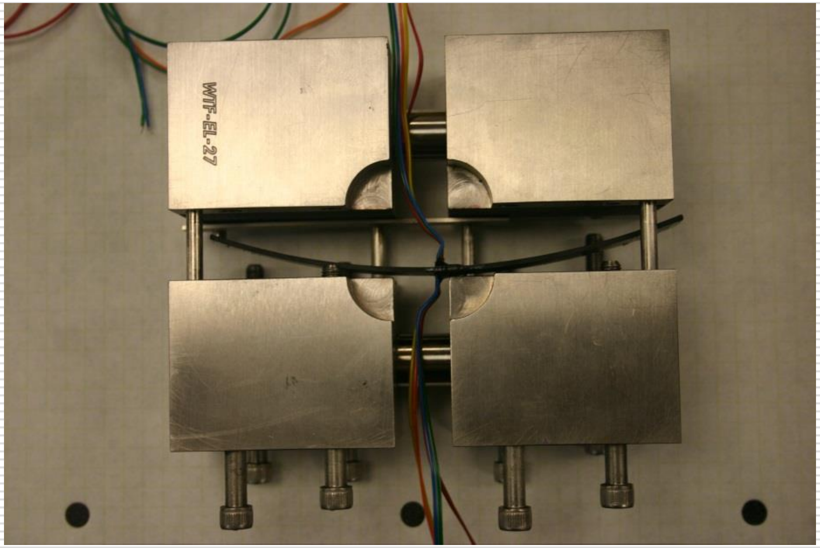
Typical Composite Tensile Data

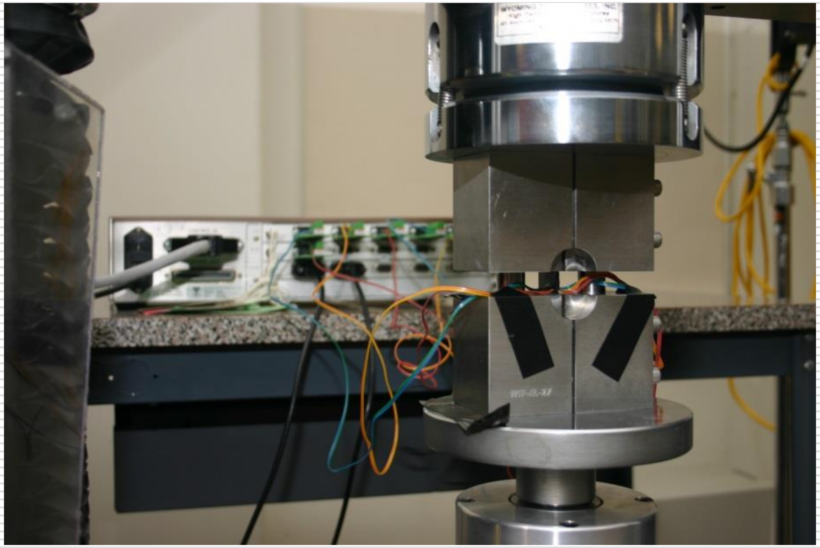


Typical Composite Data Summary

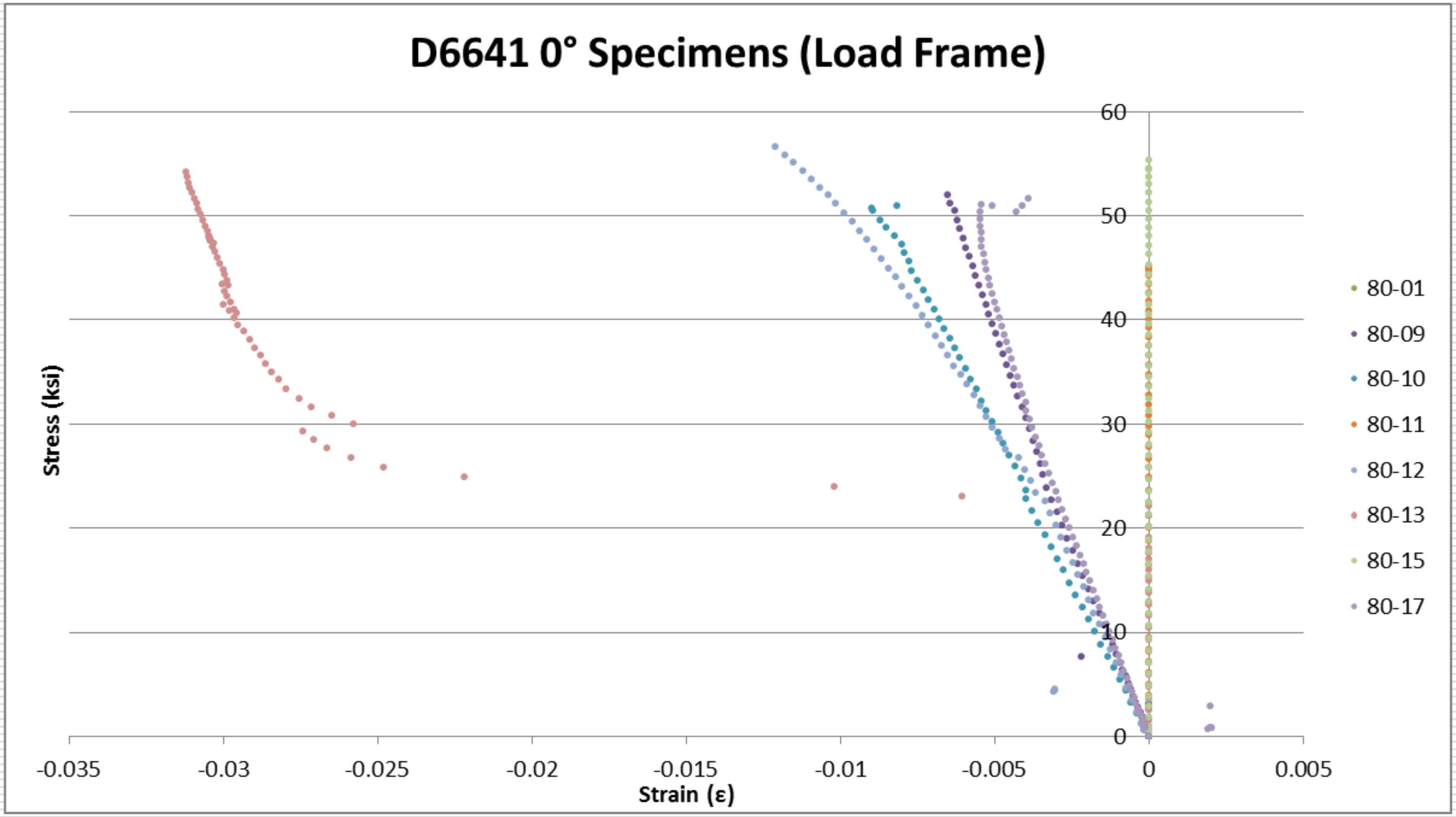


Typical Composite Tensile Data

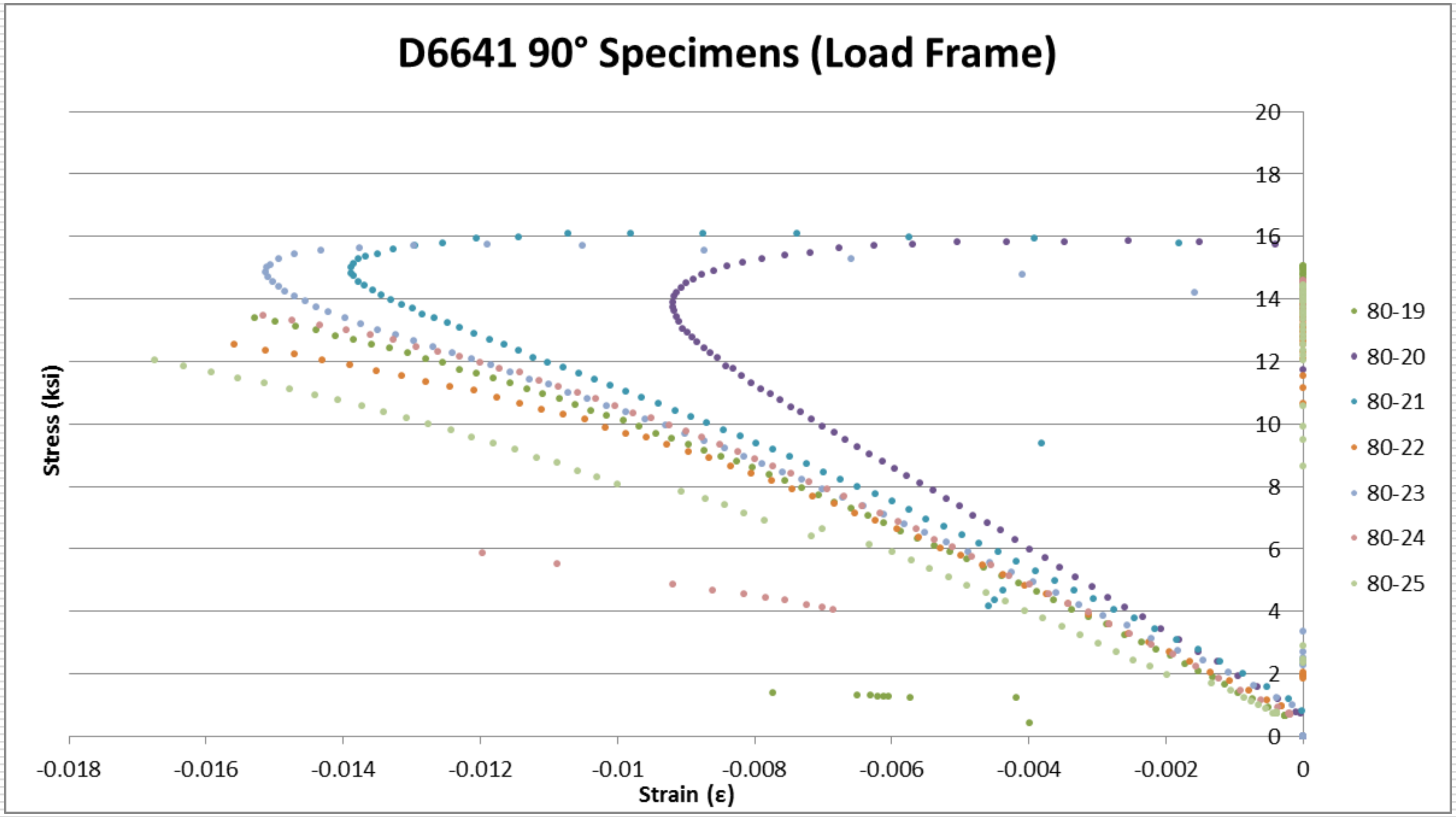




Typical Composite Compression Data



Typical Composite Compression Data



Restrictions on Engineering Constants - Isotropic

- **Shear Modulus**
  - $G=E/2(1+\nu)$
  - $\nu>-1$
- **Hydrostatic Pressure** $\sigma_x=\sigma_y=\sigma_z=-p$
  - The sum of the normal or extensional strains
  - $\theta=\varepsilon_x+\varepsilon_y+\varepsilon_z=p/\ (E/\ 3(1-2\nu)) = p/K$
  - K- Bulk Modulus = $E/\ 3(1-2\nu)$
  - $\nu<1/2$

Restrictions on Engineering Constants - Orthotropic

- $S_{11}, S_{22}, S_{33}, S_{44}, S_{55}, S_{66} > 0$
- $C_{11}, C_{22}, C_{33}, C_{44}, C_{55}, C_{66} > 0$
- $(1-\nu_{23}\nu_{32})>0, (1-\nu_{13}\nu_{31})>0, (1-\nu_{12}\nu_{21})>0$
- $\Delta=1-\nu_{12}\nu_{21}-\nu_{23}\nu_{32}-\nu_{13}\nu_{31}-2\nu_{21}\nu_{32}\nu_{13}>0$
- See Jones pp 68-69