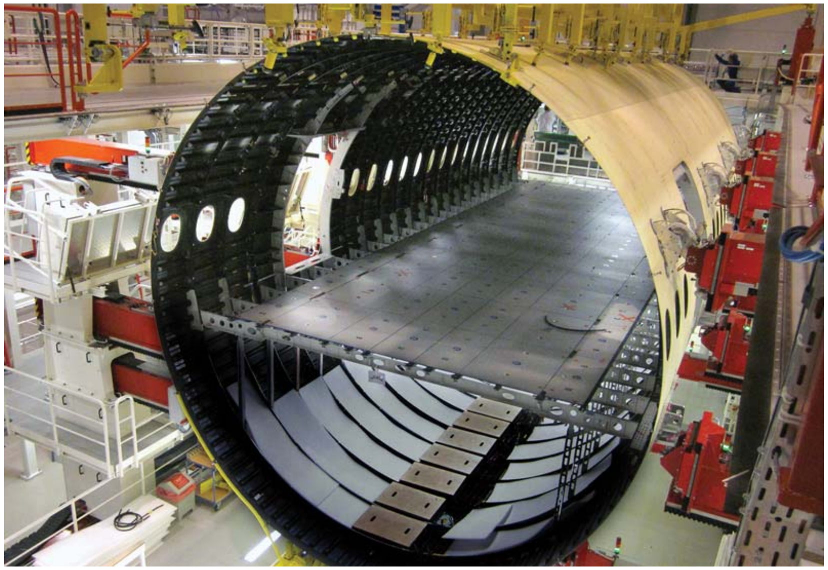
Mechanically Fastened Joint Outline

- Joint Overview
- Overview of Metal Joint for Comparison
- Single Fastener Joint
- Joint with Single Strip of Fasteners
- Multiple Row of Fasteners
- Fastener Loading
- Questions and Discussion

Types of Joining Methods

- **Mechanically Fastened Joints**
  - **Cylindrical Rods: bolt, rivet, pin, or stable**
  - **Primary load bearing**
  - **The load transfer through the fastener is localized**
- **Adhesively Bonded Joints**
  - **Structural Adhesives**
  - **Primary load bearing**
- **Welded Joints**
- **Friction Joints**



Joining Methods

Joint Type	Advantages	Disadvantages
Mechanically Fastened	Straightforward Design Inspectable Repairable No Thickness Restrictions Can be disassembled	Many Parts Stress Concentrations/Weak Fatigue, fretting Prone Prone to corrosion Must be sealed
Adhesive Bonding	Few Parts, Full Load Transfer Repairable, Fatigue Resistant Sealing, Stiff Connection Light-Weight, Smooth Contour, corrosion resistant No Stress Concentrations	Difficult to Inspect, Surface Prep Environmental effects, design Required Skill, Thickness limited Residual Stresses No Disassembly Shear Loading Only
Welding	Permanent, Inspectable Load Transfer, Continuous	Permanent, Microstructural change Residual Stresses, Trade Skill
Friction	Simple	Friction Dependent

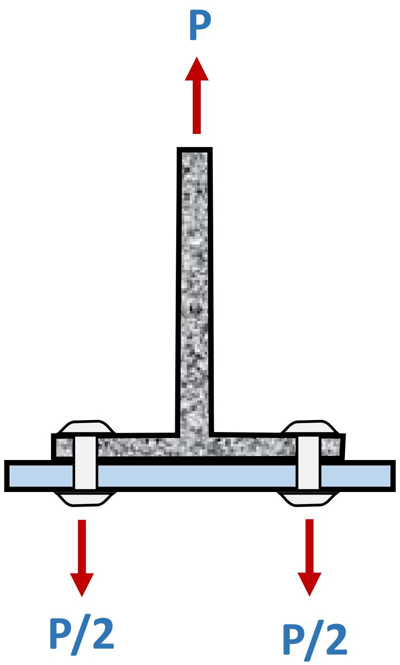
Joint Design Considerations

- **Joint Geometry**
  - Hole diameter, plate width/fastener spacing, plate thickness
- **Clamping Area**
  - CAUTION: Through the Thickness compliance/weakness
- **Ply Fiber Orientation and Ply Stacking Sequence**
- **Hole Stress Concentration**
- **Moisture and Temperature Conditions**
- **Applied Stress**

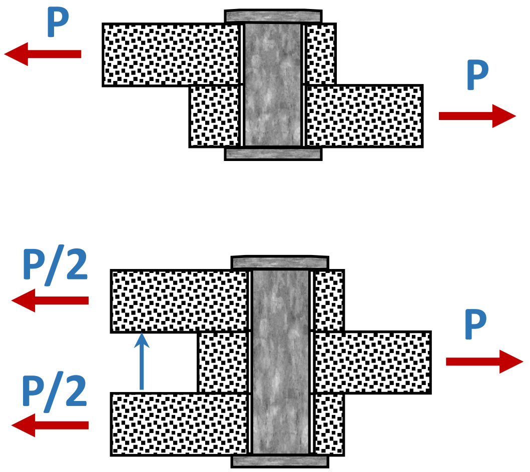
Simple Joint Analysis: Homogeneous and Isotropic

- No Stress Concentrations
  - Static Loading: Factor of Safety of 3
  - Fatigue Loading: Factor of Safety of 4.5
- Multiple Rows of Fasteners
  - Each row takes equal proportion of the applied load
- Failure Stress Analysis
  - Individual Failure Modes
  - Good design promotes preferred failure condition
- Assume Each Fastener in a Row Shares Equal Load

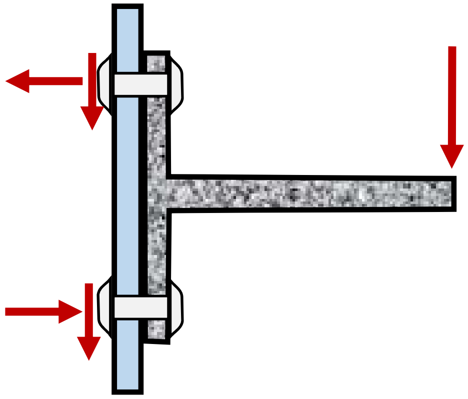
Typical/Idealized Bolted Joint Configurations



Tension

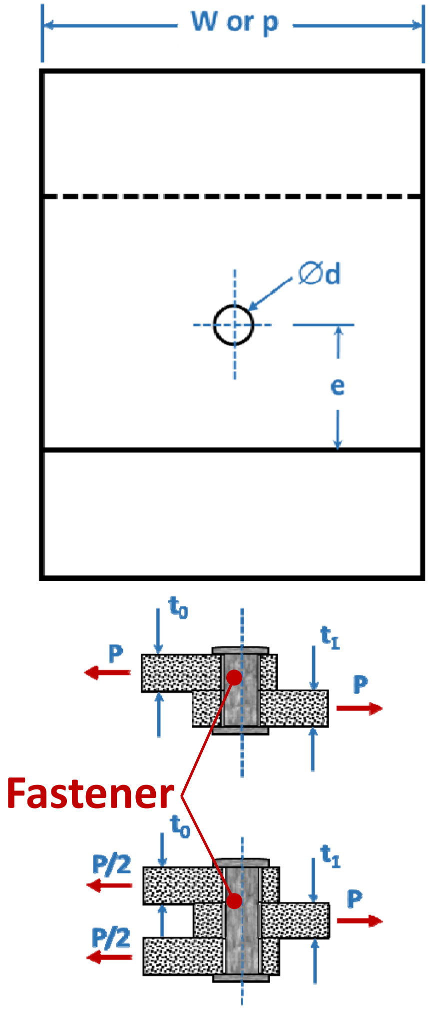


Shear



Complex

Nomenclature for Basic Shear Joint Geometry



d – Hole Diameter

w- Plate Width

p- Fastener Spacing

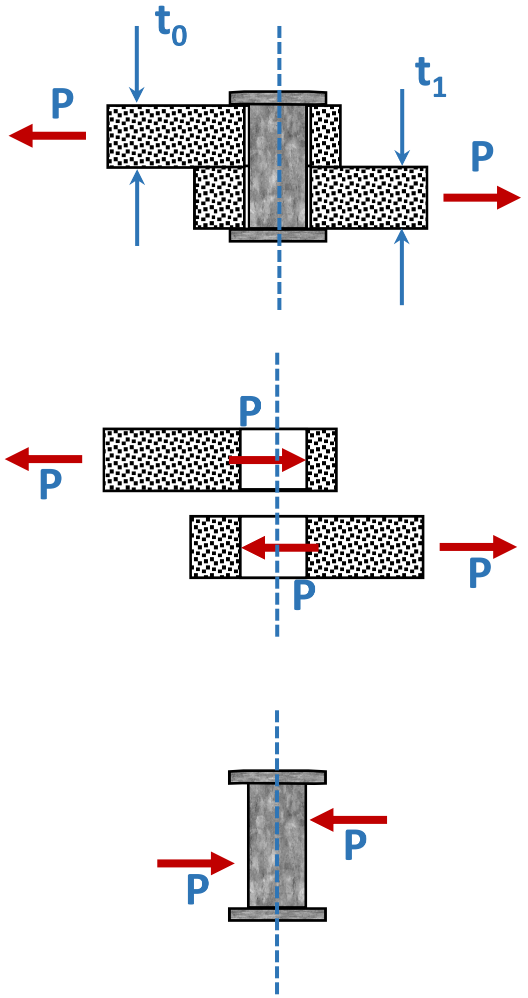
e- Edge Distance

t_0 and t_1 – Plate Thickness

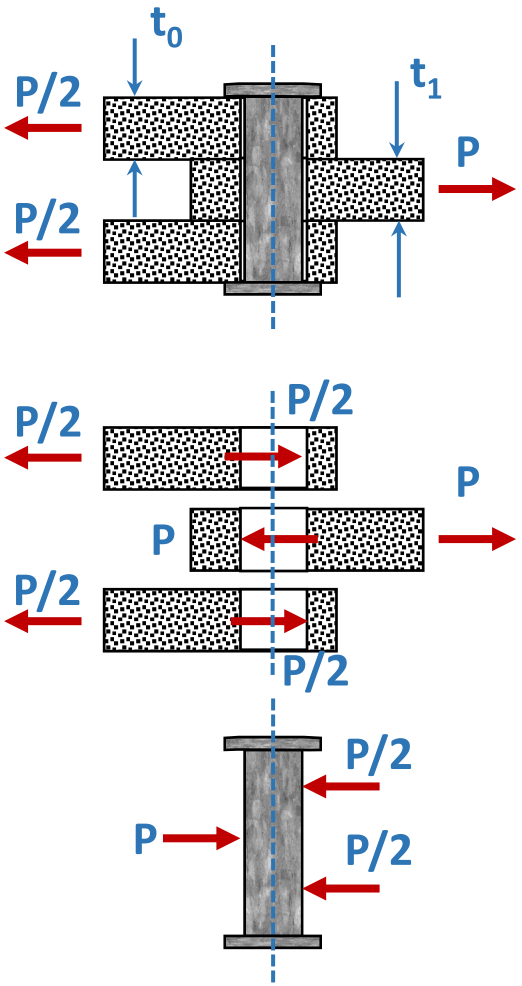
P- Load

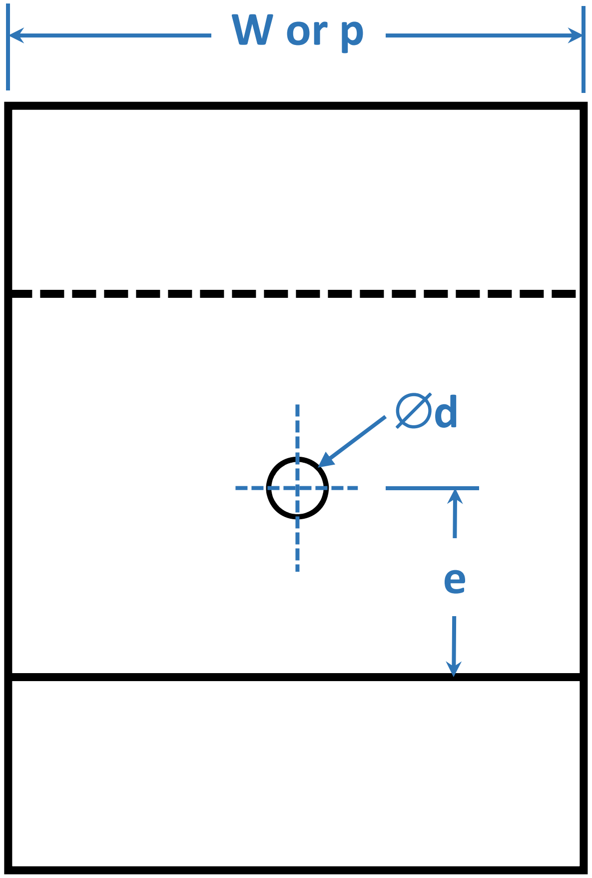
Basic Shear Joint Geometry

Single Lap Configuration

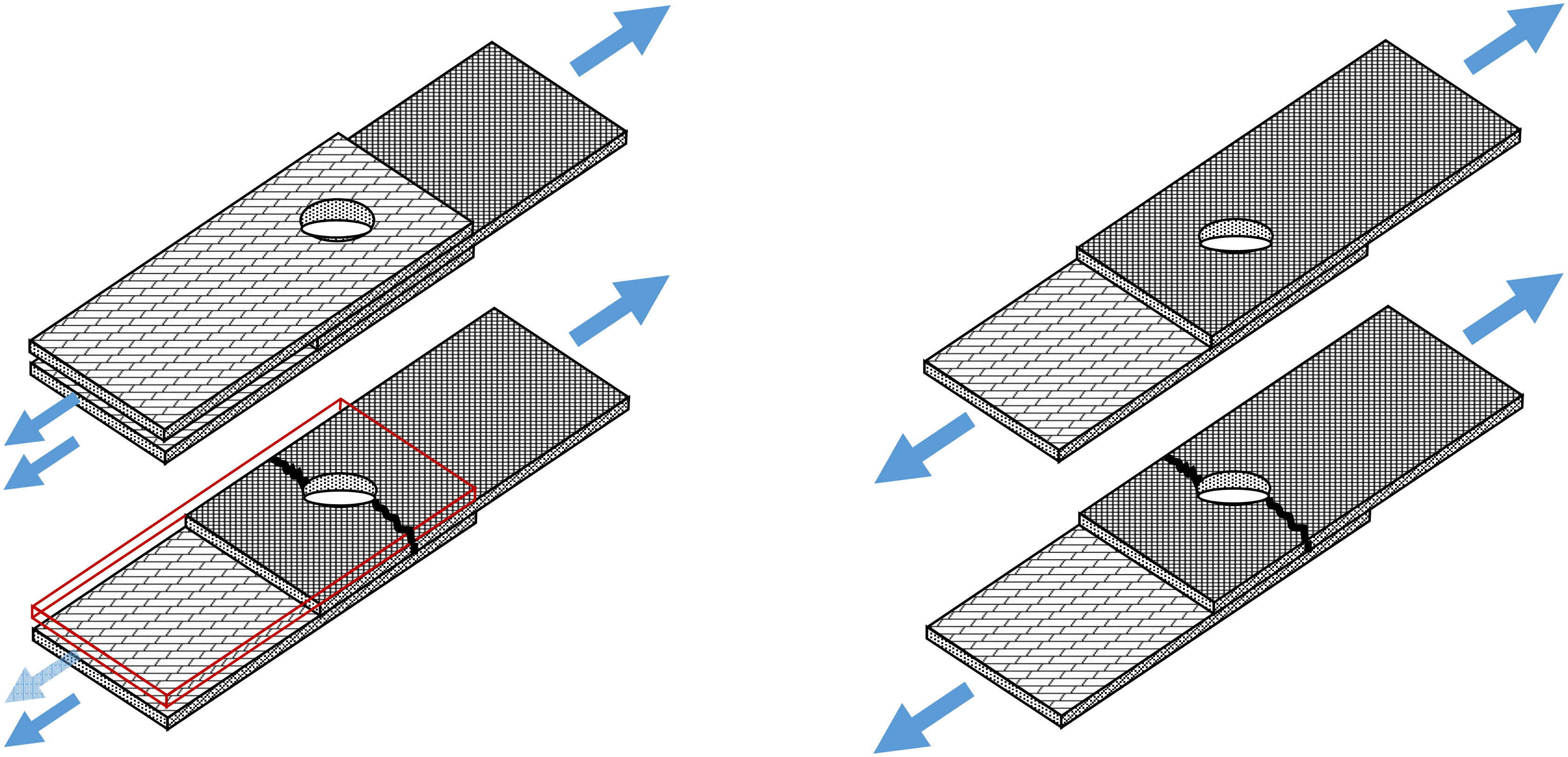


Double Lap Configuration



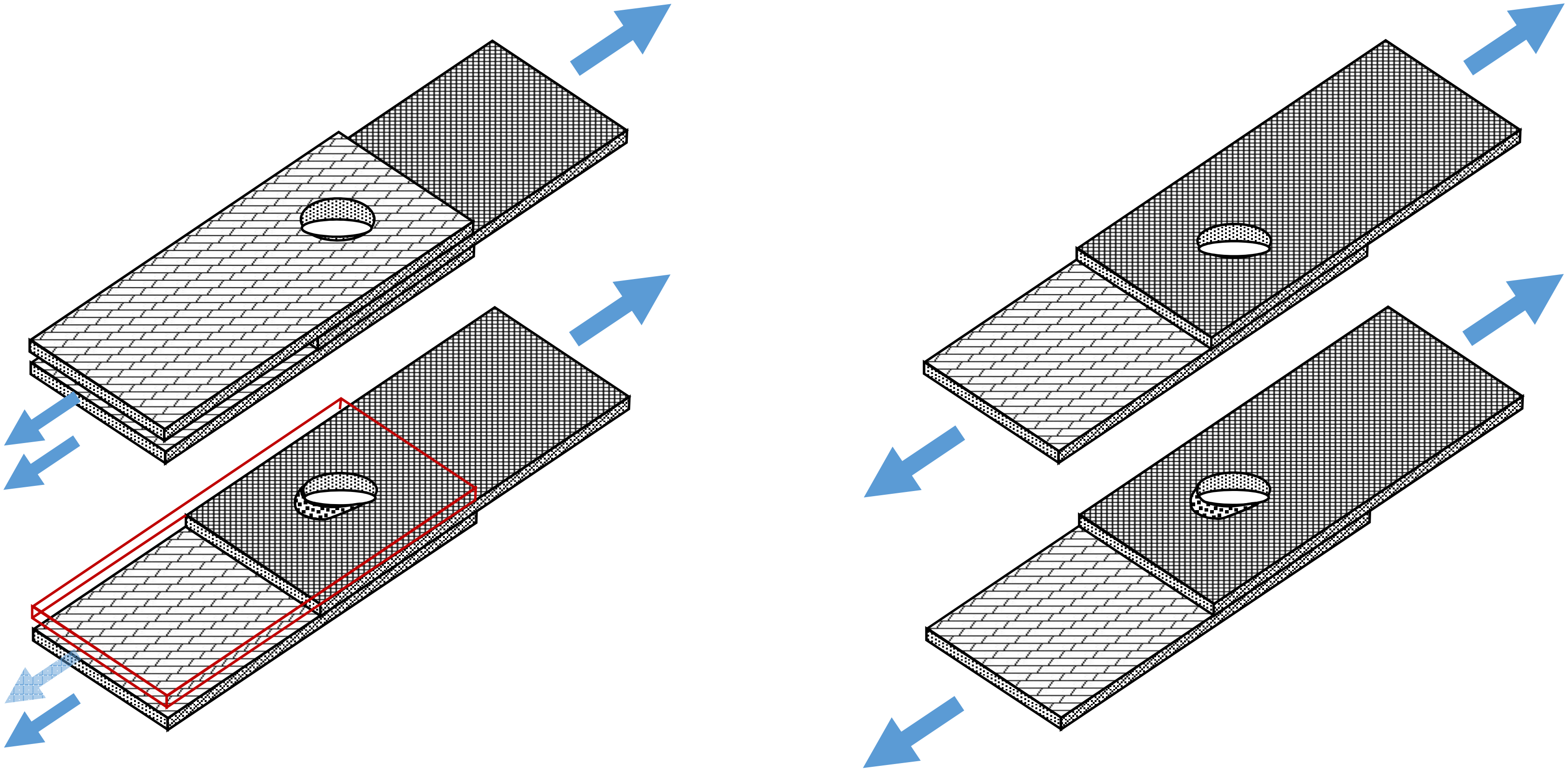


Net Tension Failure



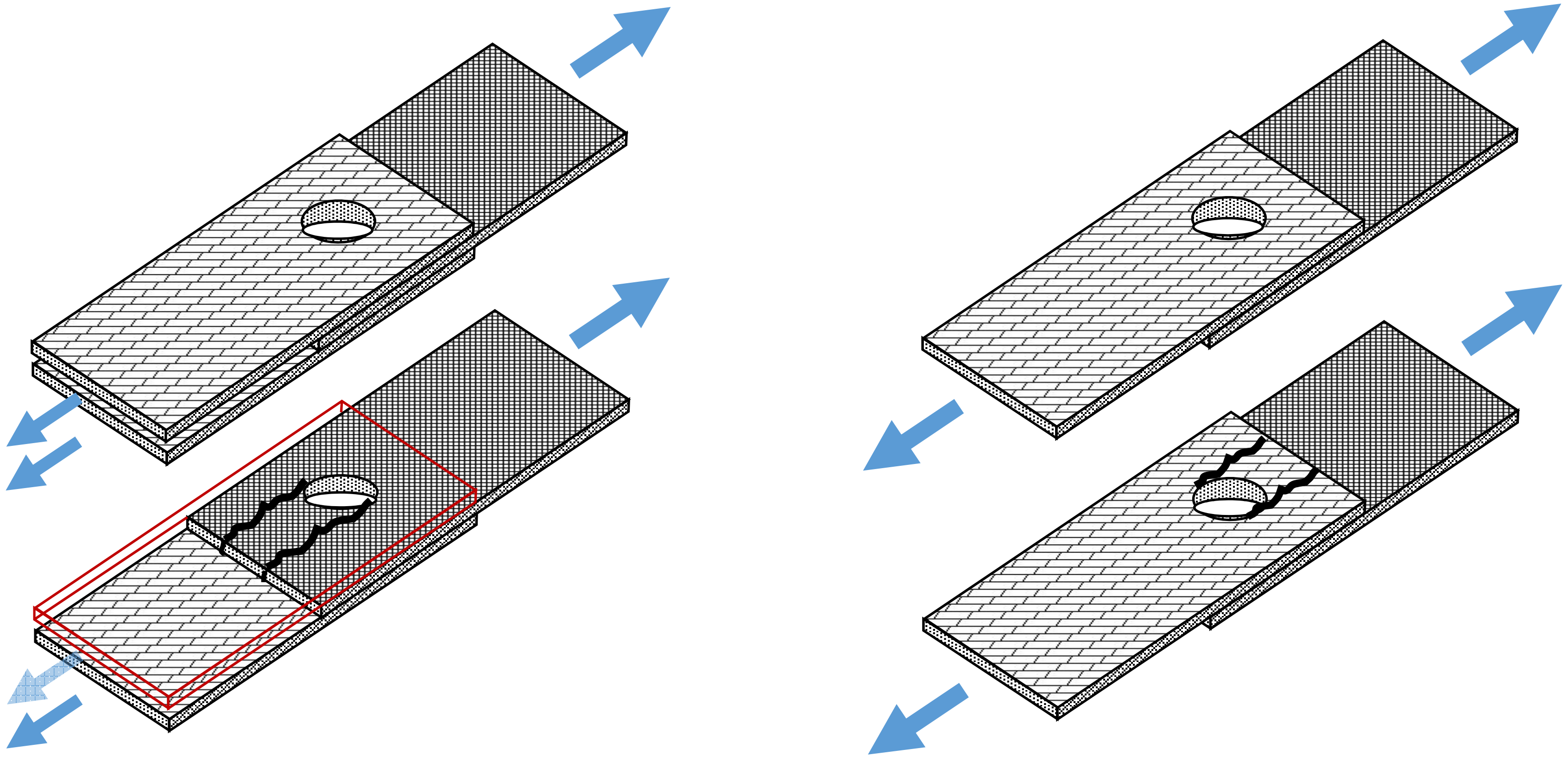
$$P_{all}^{nt} = k \cdot \sigma_{all,tension} \cdot t \cdot \left(w - d \right)$$

Bearing Failure



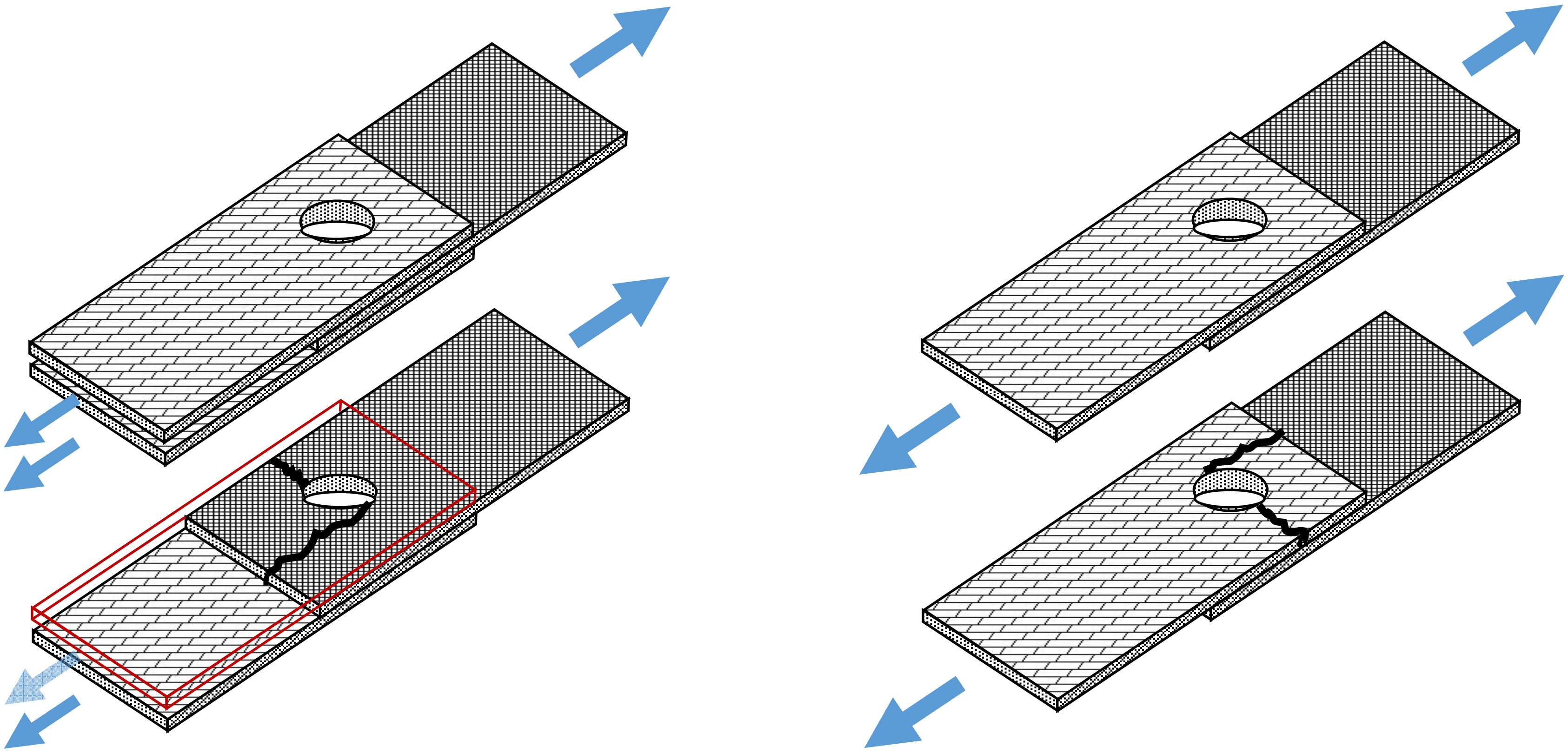
$$P_{all}^{br} = k \cdot \sigma_{all,bearing} \cdot t \cdot d$$

Shear Out Failure



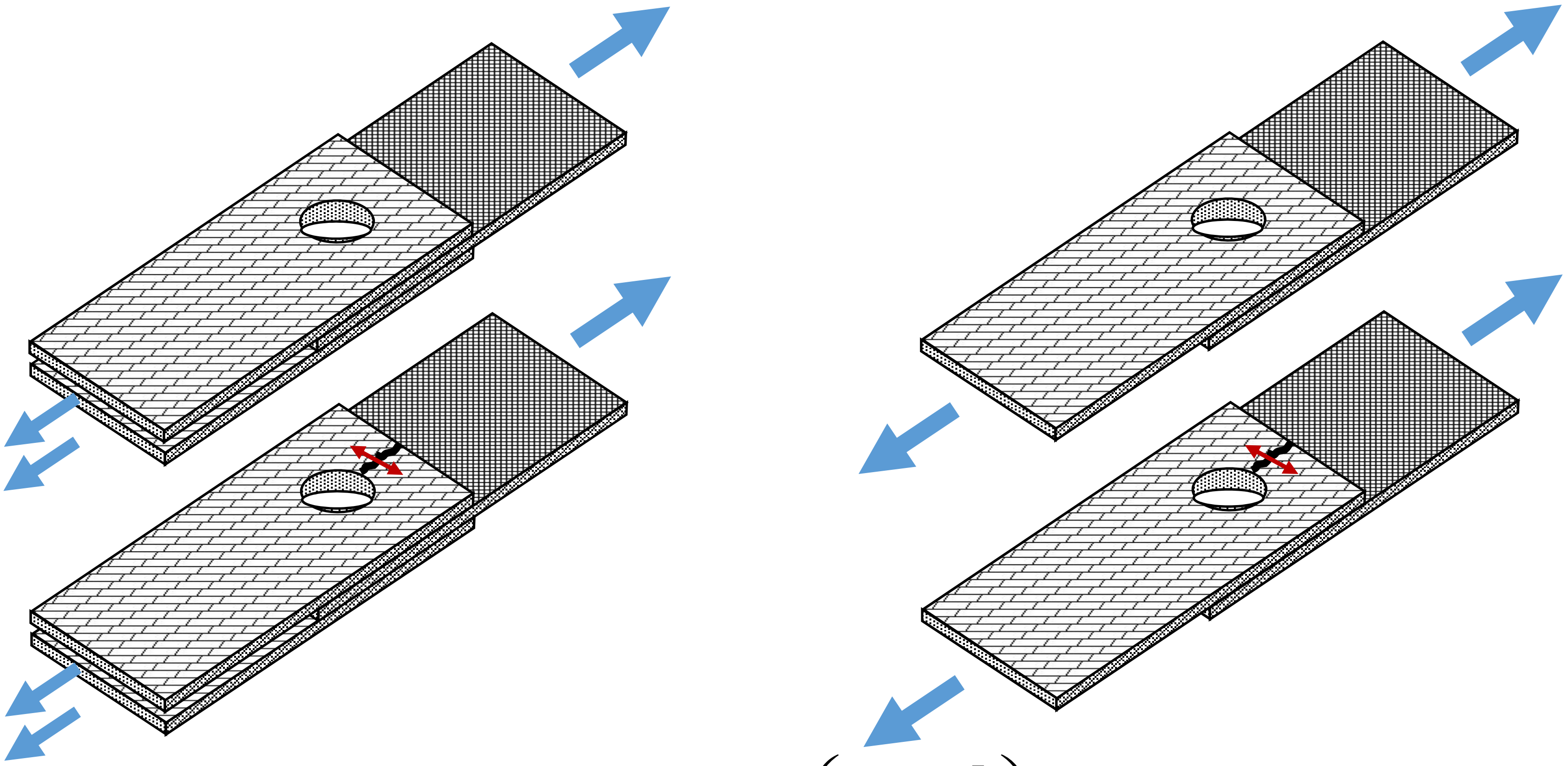
$$P_{all}^{so} = 2 \cdot \tau_{all} \cdot t \cdot e$$

Combined Net Tension and Shear Out Failure



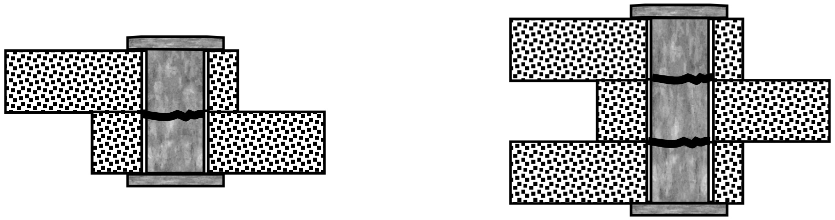
$$P_{all}^{nt/so} = k \cdot \sigma_{all,tension} \cdot \tfrac{1}{2} \cdot t \cdot \left(w - d\right) + \tau_{all} \cdot t \cdot e$$

Cleavage Failure



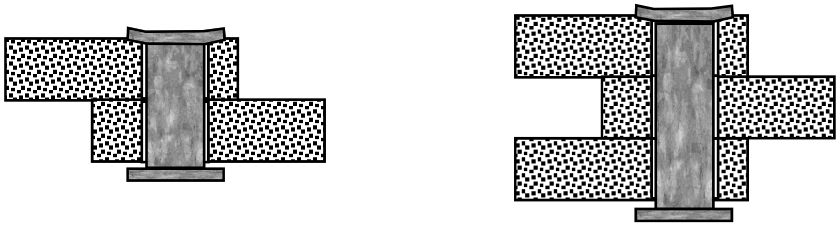
$$P_{all}^{cl} = \cdot \sigma_{all,tension} \cdot t \cdot \left(e - \frac{d}{2} \right)$$

Fastener Failures

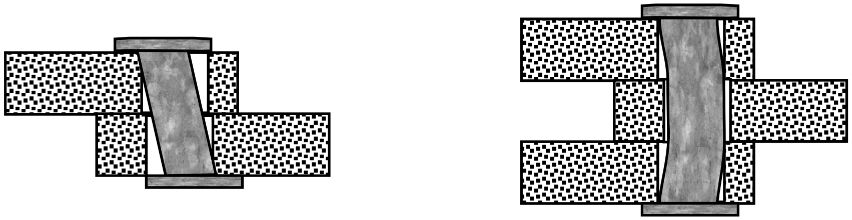


Fastener Shear Failure

$$P_{all}^{fs} = \tau_{all} \cdot A \cdot n$$



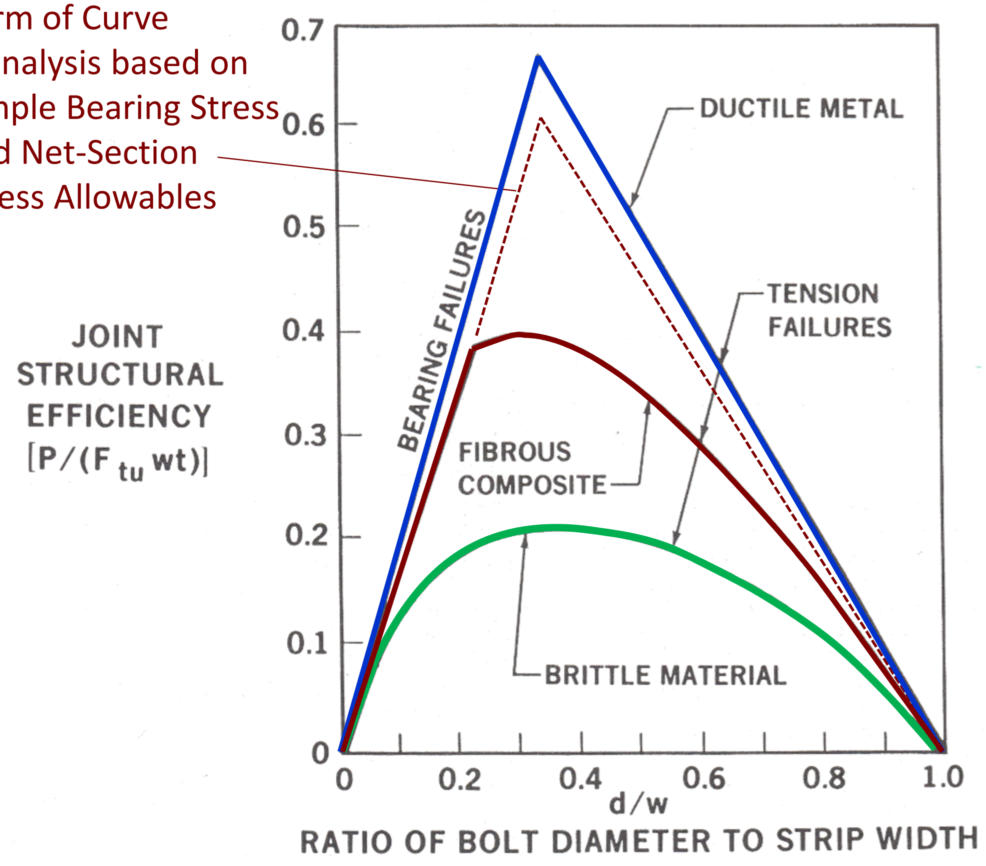
Fastener Pull Through Failure



Fastener Bending Failure

Composite Joint Analysis Neither Perfectly Elastic or Plastic

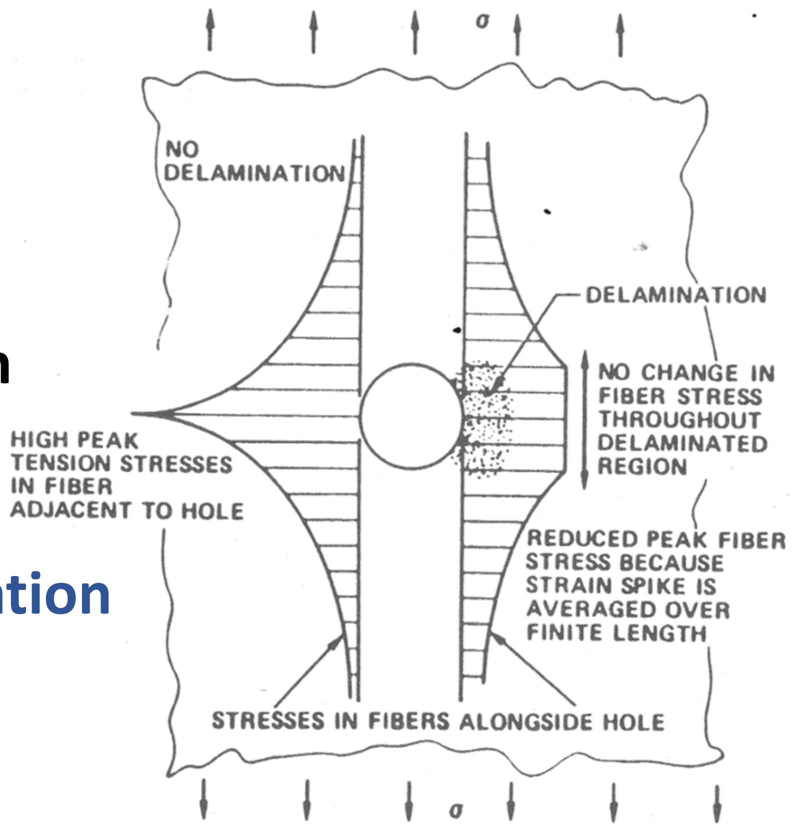
- Single Bolt Joint
- Composite Response Between
  - Window Glass
  - Ductile Metal
- ONLY Fiber Glass comes close to ideal
  - massive delamination prior to failure
- Failure in composite
  - Gross section Failure
  - Not net section failure


Form of Curve if analysis based on Simple Bearing Stress and Net-Section Stress Allowables

Inhomogeneity of Composite Failure Dominates Behavior

Composite Failure Modes

- Tension
    - Fiber/matrix bond broken locally
    - Delamination
    - Splitting of resin between fibers
    - No Fiber Damage
    - Relieves Stress Concentration
- Compression
    - All tensile modes
    - Micro-buckling



Comparison of Metal and Composite Bolted Joints

Simple Comparison between the weights of equivalent metal and composite structures

- Superiority of composites over metals typically based on unnotched specimens
- Sometimes deduced from monolayer properties

For a Common load per unit width, P

$$P = \sigma_a \cdot t_a = \sigma_c \cdot t_c \quad \Rightarrow \quad t_c = \frac{P}{\sigma_c} \text{ and } t_a = \frac{P}{\sigma_a}$$

The weight ratio is given by

$$\frac{w_c}{w_a} = \frac{\rho_c \cdot t_c}{\rho_a \cdot t_a} = \frac{\rho_c \cdot \sigma_a}{\rho_a \cdot \sigma_c} = \frac{\rho_c \cdot \sigma_a}{\rho_a \cdot E_c \cdot \varepsilon_c}$$

Implications to Structural Design

$$\frac{w_c}{w_a} = \frac{\rho_c \cdot t_c}{\rho_a \cdot t_a} = \frac{\rho_c \cdot \sigma_a}{\rho_a \cdot \sigma_c} = \frac{\rho_c \cdot \sigma_a}{\rho_a \cdot E_c \cdot \varepsilon_c}$$

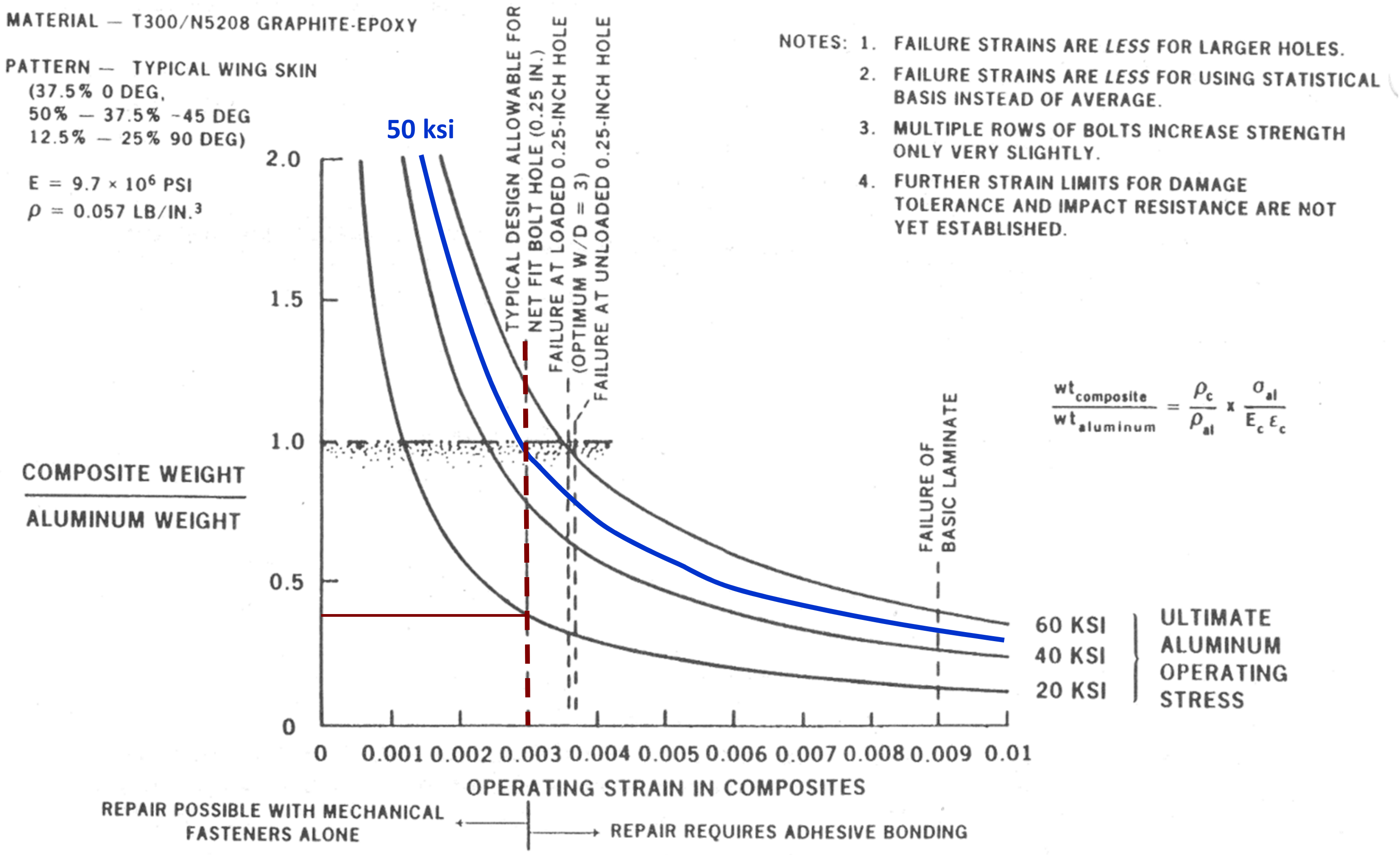
Composite Structures are preferred

- Aluminum stress σ_a is Low
- Strain in composite ε_c is kept High

Aluminum Structures are preferred

- Aluminum stress σ_a is High

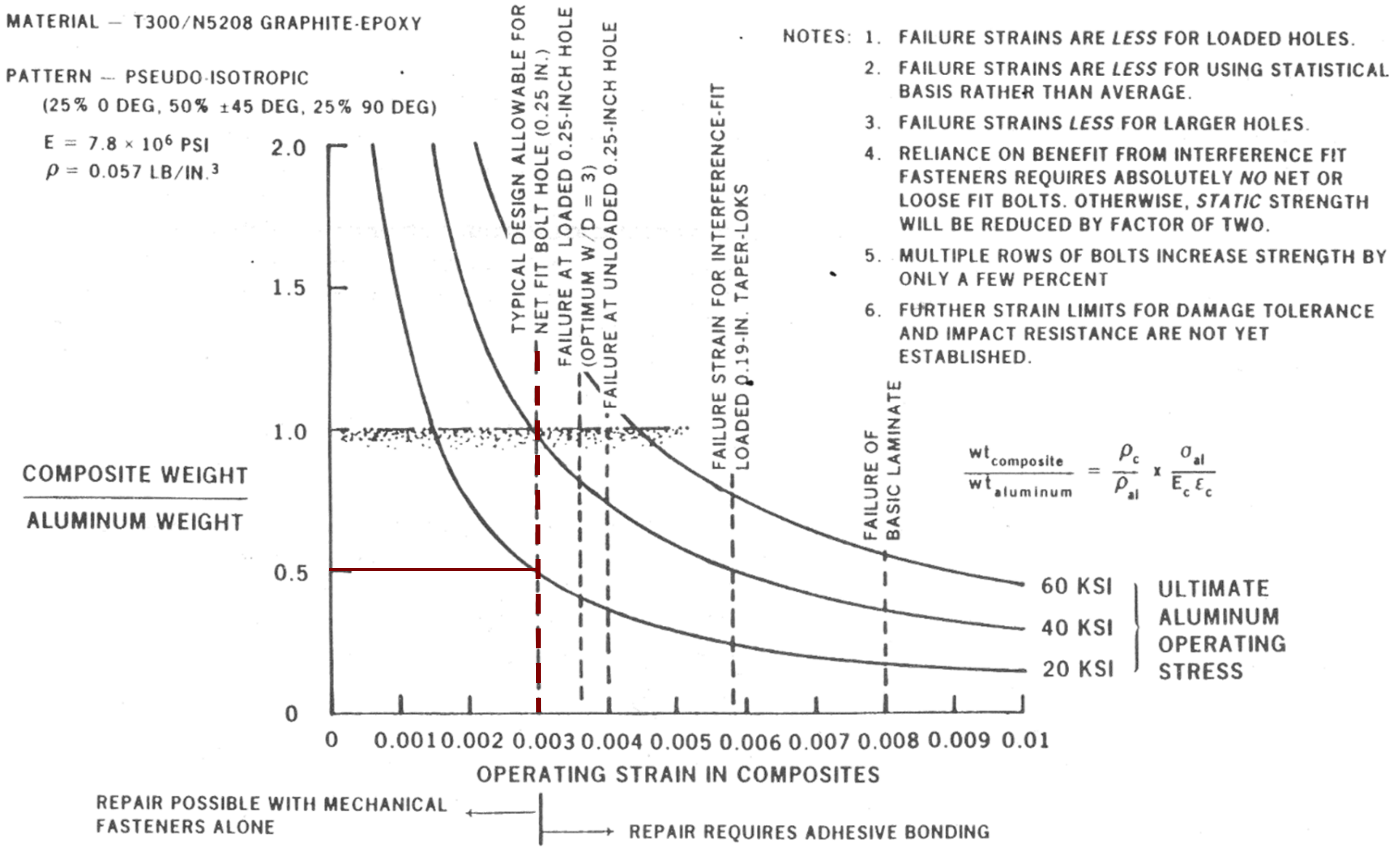
Illustrations of Trade-Offs on a Lay-Up Suitable for Wing Skins



NOTES:
1. FAILURE STRAINS ARE *LESS* FOR LARGER HOLES.
2. FAILURE STRAINS ARE *LESS* FOR USING STATISTICAL BASIS INSTEAD OF AVERAGE.
3. MULTIPLE ROWS OF BOLTS INCREASE STRENGTH ONLY VERY SLIGHTLY.
4. FURTHER STRAIN LIMITS FOR DAMAGE TOLERANCE AND IMPACT RESISTANCE ARE NOT YET ESTABLISHED.

$$\frac{wt_{composite}}{wt_{aluminum}} = \frac{\rho_c}{\rho_{al}} \times \frac{\sigma_{al}}{E_c\,\varepsilon_c}$$

Illustrations of Trade-Offs on a Quasi-Isotropic Lay-Up



MATERIAL — T300/N5208 GRAPHITE-EPOXY

PATTERN — PSEUDO-ISOTROPIC
(25% 0 DEG, 50% ±45 DEG, 25% 90 DEG)
$E = 7.8 \times 10^6$ PSI
$\rho = 0.057$ LB/IN.^{3}

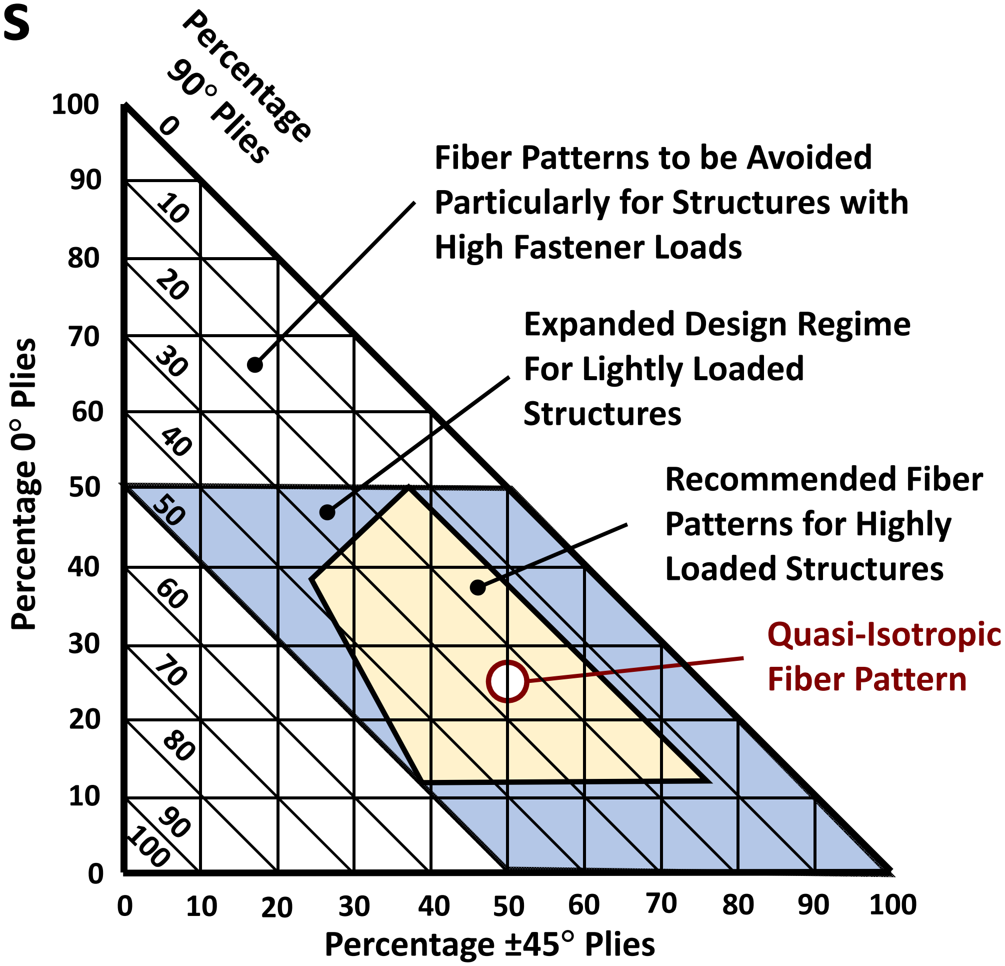
NOTES:
1. FAILURE STRAINS ARE *LESS* FOR LOADED HOLES.
2. FAILURE STRAINS ARE *LESS* FOR USING STATISTICAL BASIS RATHER THAN AVERAGE.
3. FAILURE STRAINS *LESS* FOR LARGER HOLES.
4. RELIANCE ON BENEFIT FROM INTERFERENCE FIT FASTENERS REQUIRES ABSOLUTELY *NO* NET OR LOOSE FIT BOLTS. OTHERWISE, *STATIC* STRENGTH WILL BE REDUCED BY FACTOR OF TWO.
5. MULTIPLE ROWS OF BOLTS INCREASE STRENGTH BY ONLY A FEW PERCENT
6. FURTHER STRAIN LIMITS FOR DAMAGE TOLERANCE AND IMPACT RESISTANCE ARE NOT YET ESTABLISHED.

$$\frac{wt_{composite}}{wt_{aluminum}} = \frac{\rho_c}{\rho_{al}} \times \frac{\sigma_{al}}{E_c \varepsilon_c}$$

Recommended Fiber Patterns for Bolted or Riveted Joints

Pattern Recommendations

- Quasi-Isotropic Pattern
  - 0° → 25%
  - 90° → 25%
  - ±45° → 50%
- Dispersed Plies
- Range
  - No more than 3/8 of the fibers in any one of the identified directions
  - No less than 1/8 of the fibers in any one of the identified directions



Fiber Patterns to be Avoided Particularly for Structures with High Fastener Loads

Expanded Design Regime For Lightly Loaded Structures

Recommended Fiber Patterns for Highly Loaded Structures

Quasi-Isotropic Fiber Pattern

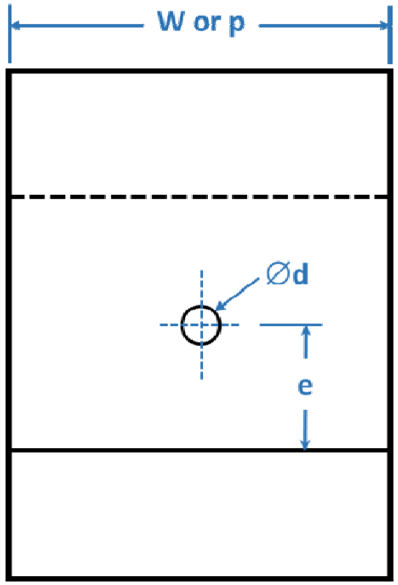
Elastic-Isotropic Stress Concentrations for Single Holes

A Wealth of Data Exists

- Single Fastener
- Single Lap/Shear
- Double Lap/Shear

Theories Developed to Explain Data

- Performance/Design of more complex structural joints checked against data
- THIS IS A LOGICAL PLACE TO START THE DEVELOPMENT



An additional variable is clamping torque

Elastic-Isotropic, Single Hole Net-Section Tension Stress Concentration

Stress Concentration Based on

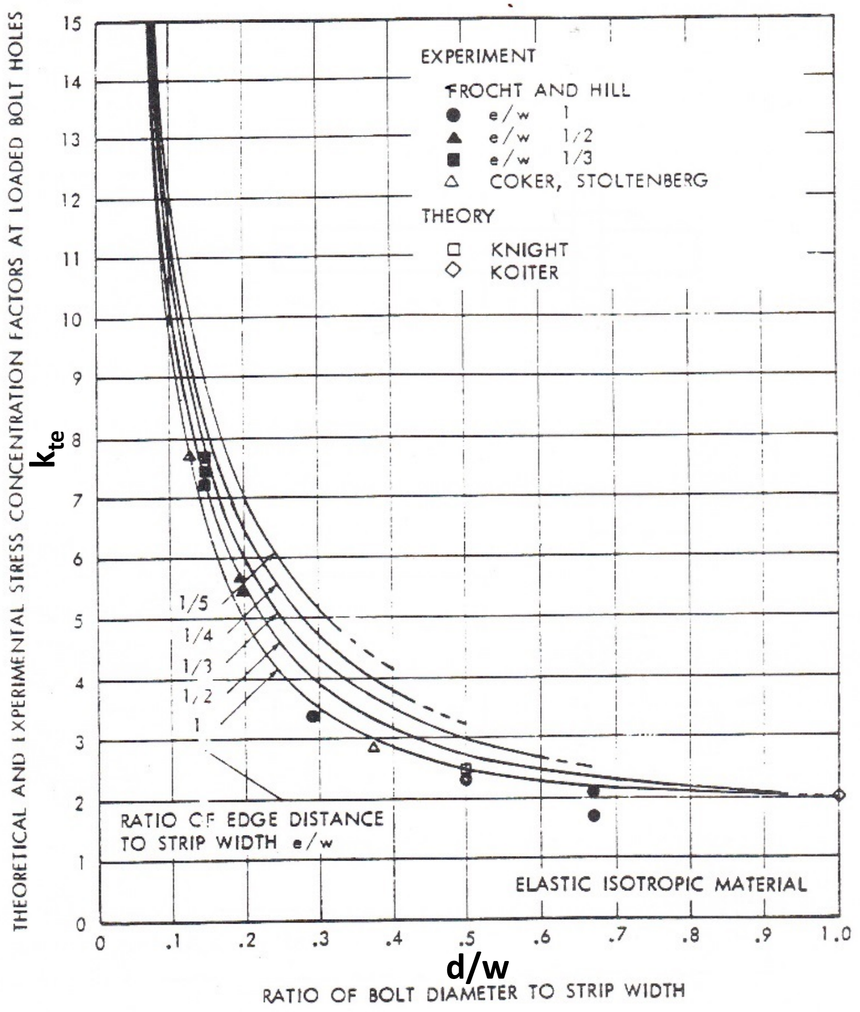
- Experimental data
- Analytical results

$$k_{te} = \left(\frac{w}{d}+1\right) - \frac{3}{2}\cdot\frac{\left(\frac{w}{d}-1\right)}{\left(\frac{w}{d}+1\right)}\cdot\Theta$$

$$\Theta = \frac{3}{2} - \frac{1}{2}\cdot\left(\frac{w}{e}\right) \quad \text{for} \quad \frac{e}{w} \le 1$$

$$\Theta = 1 \qquad \text{for} \quad \frac{e}{w} \le 1$$

$$\sigma_{\max} = k_{te}\cdot\frac{P}{t\cdot(w-d)}$$



Elastic-Isotropic, Single Hole Average Bearing Stress Concentration

Stress Concentration Based on

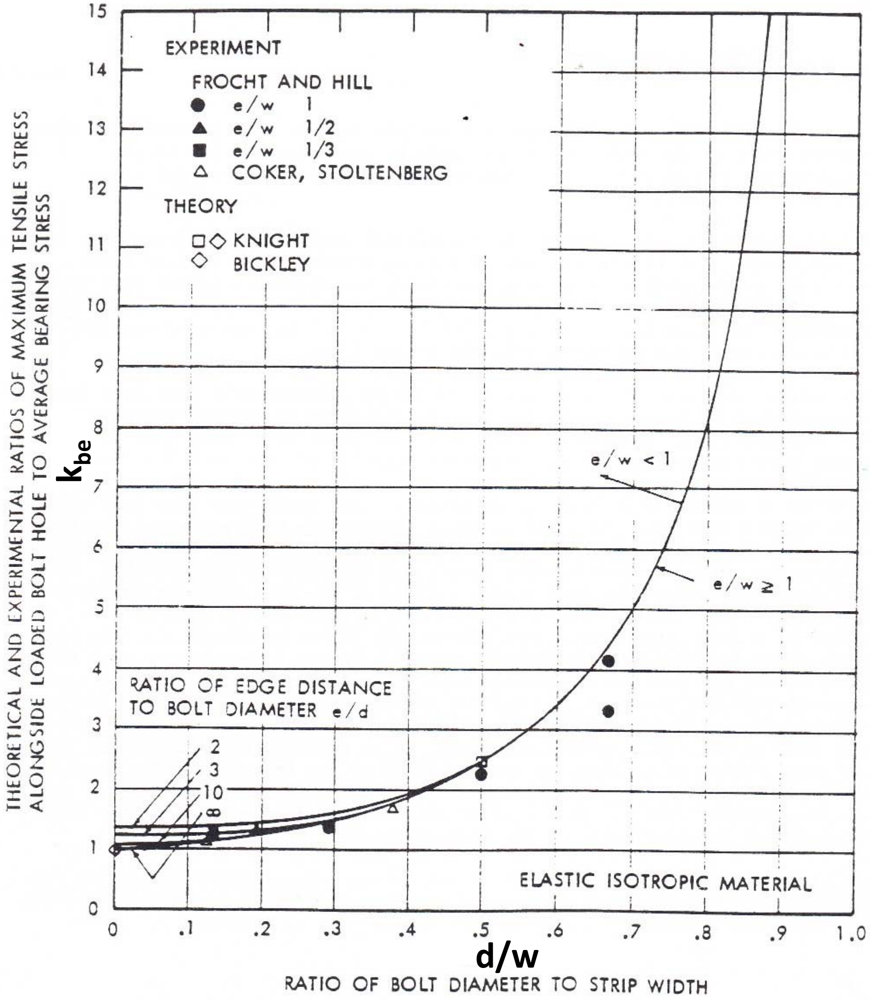
- **Experimental data**
- **Analytical results**

$$k_{be} = \frac{\sigma_{\max}}{P/_{t\cdot d}} = \frac{k_{te}}{w/_{d} - 1}$$

$$= 1 + \frac{2}{w/_{d} - 1} - \frac{3}{2}\cdot\frac{1}{w/_{d} + 1}\cdot\Theta$$

$$\Theta = \frac{3}{2} - \frac{1}{2}\cdot\left(\frac{w}{e}\right) \quad \text{for} \quad \frac{e}{w} \le 1$$

$$\Theta = 1 \quad \text{for} \quad \frac{e}{w} \le 1$$



Elastic-Isotropic, Single Hole Average Bearing Stress Concentration

Stress Concentration

- **Well behaved for small d/w**

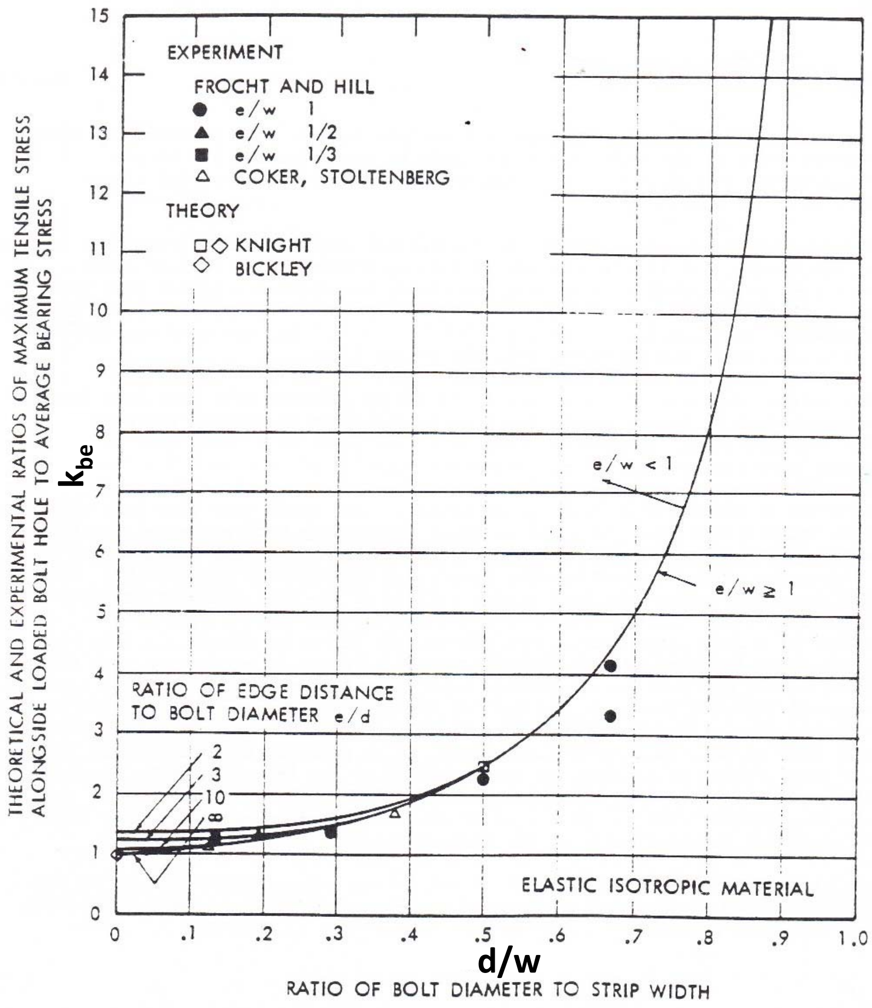
$$k_{be} = 1 + \frac{2}{{}^{w}\!/_{d} - 1} - \frac{3}{2}\cdot\frac{1}{{}^{w}\!/_{d} + 1}\cdot\Theta$$

- **d/w→0**

$$k_{be} = 1$$

- **d/w→0, accounting for e**

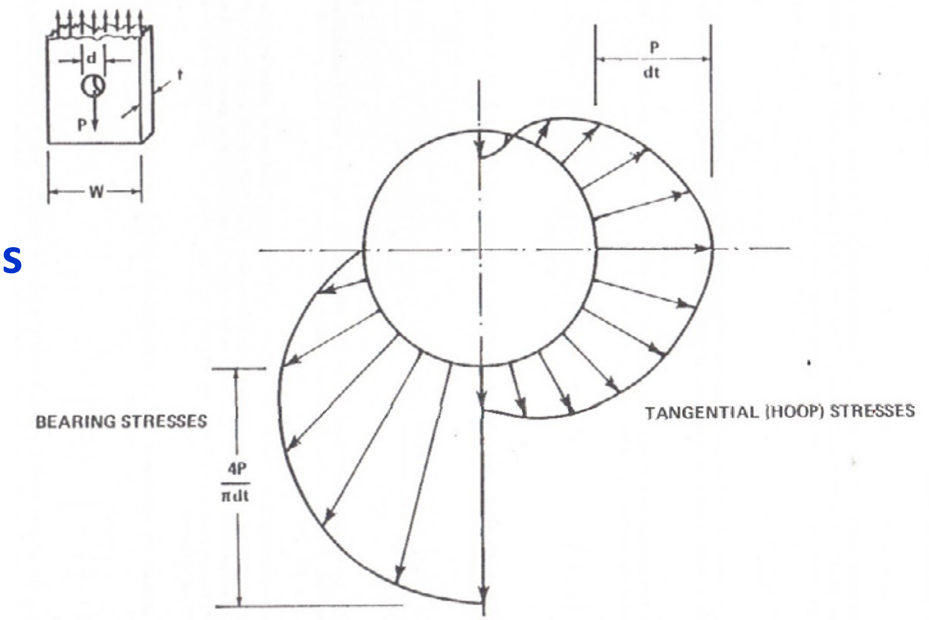
$$k_{be} = 1 + \frac{{}^{3}\!/_{4}}{{}^{e}\!/_{d}}$$



Elastic-Isotropic Stress Field Around a Bolt Hole

For Nominal Net Fit Fastener

- **Bearing Stress Peaks 27% higher than average**
- **Peak Hoop Tensile Stress**
  - **Equal to average Bearing Stress**
- **Distance to Edge Not Accounted for**
- **Potential Failure Sites**
  - **Bearing**
  - **Back Side of Fastener**
  - **Tension on Side of Hole**
- **With Shorter Edge Distance**
  - **Cleavage Failure Possible**



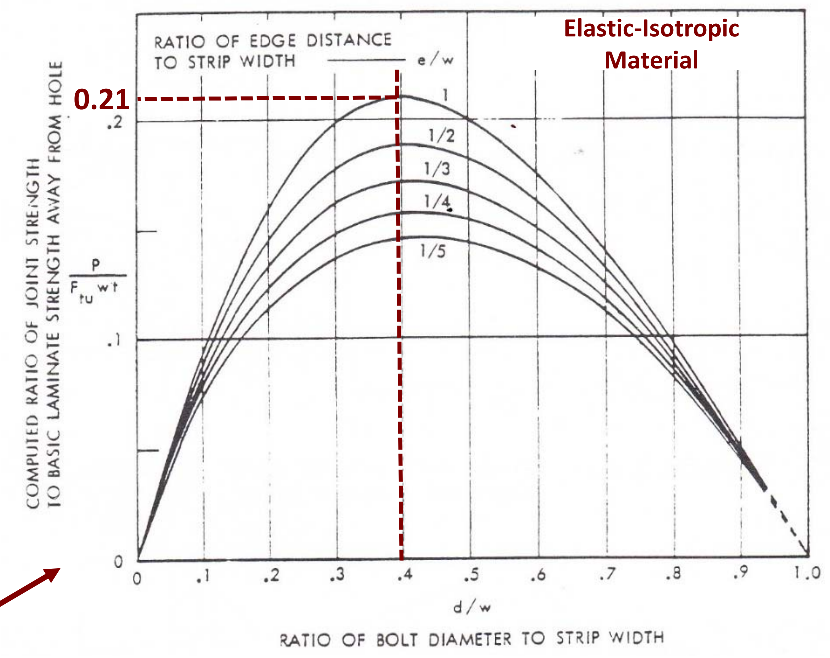
Impact of Stress Concentration on Strip Strength

Solving k_{te} and $k_{be}(k_{te})$ for Joint Strength P

$$P = \left(\sigma_{net}\right)_{avg} \cdot \left(w-d\right)\cdot t = \frac{\sigma_{\max}}{k_{te}} \cdot \left(w-d\right)\cdot t$$

$$= \frac{\sigma_{\max} \cdot t \cdot w}{\frac{2}{\left(1-\frac{d}{w}\right)} + \frac{1}{\left(\frac{d}{w}\right)} - \frac{3}{2}\cdot\frac{\Theta}{\left(1+\frac{d}{w}\right)}}$$

$$\boxed{\frac{P}{\sigma_{\max} \cdot t \cdot w} = \frac{1}{\frac{2}{\left(1-\frac{d}{w}\right)} + \frac{1}{\left(\frac{d}{w}\right)} - \frac{3}{2}\cdot\frac{\Theta}{\left(1+\frac{d}{w}\right)}}}$$



Max Load at d/w=0.4, bolt pitch p=2.5in

$$P_{\max} \le 0.21 \cdot \sigma_{\max} \cdot w \cdot t$$

(Advanced Composites are NOT this brittle)

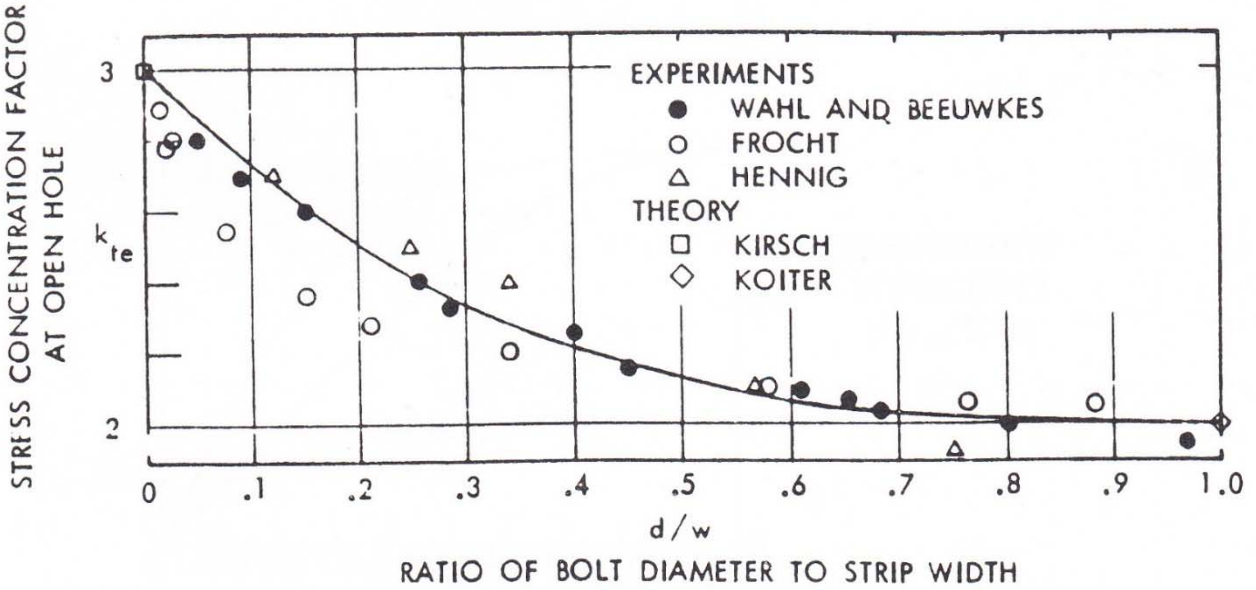
Net-Section Strength of a Strip Containing UNLOADED Hole

k_{te} for UNLOADED Hole

$$k_{te} = 2 + \left(1 - \frac{d}{w}\right)^3$$

Net Section Strength of Hole

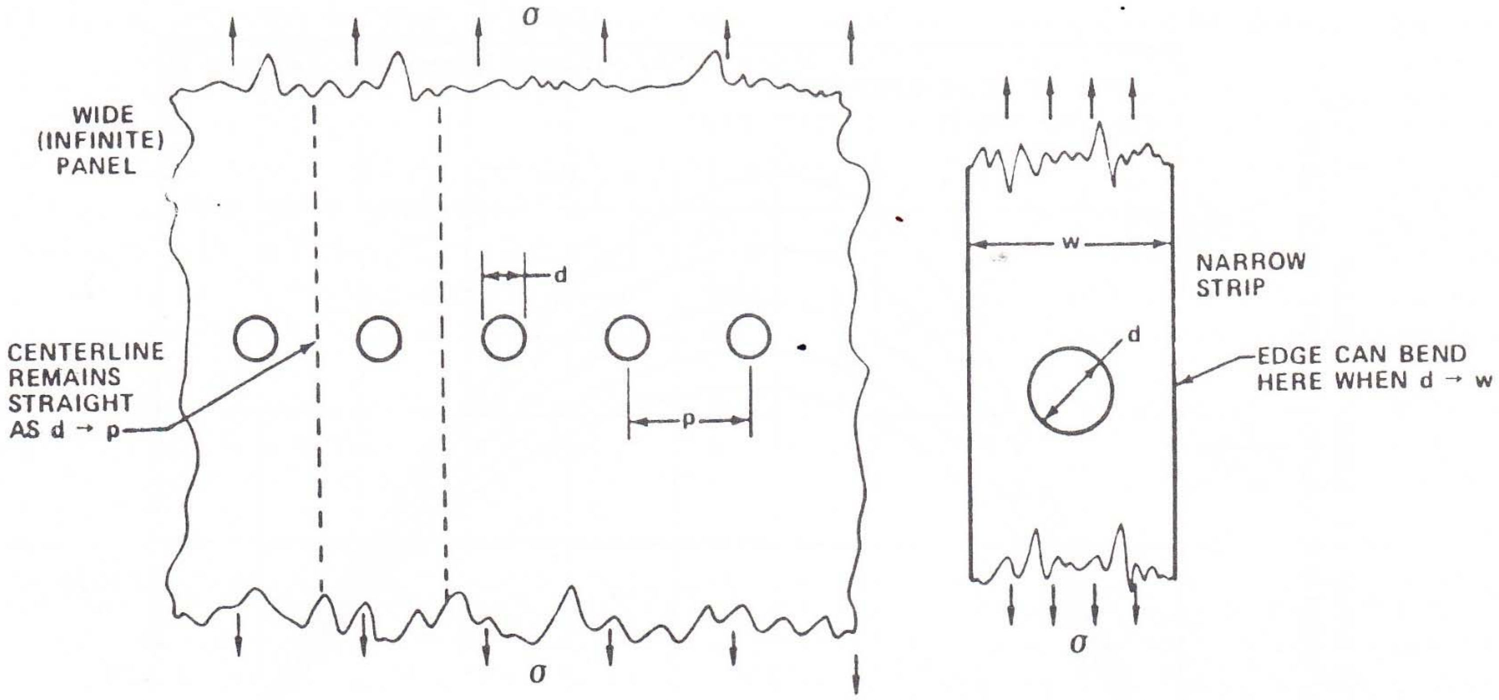
$$\frac{P}{\sigma_{\max} \cdot t \cdot w} = \frac{\left(1 - \frac{d}{w}\right)}{k_{te}} = \frac{\left(1 - \frac{d}{w}\right)}{2 + \left(1 - \frac{d}{w}\right)^3}$$



Differences Between Isolated Hole and Wide Seam of Holes

Differences Small for d/w→0

Differences Substantial as d/w →1



Isolated versus Seam of Hole Comparison, Unloaded Hole

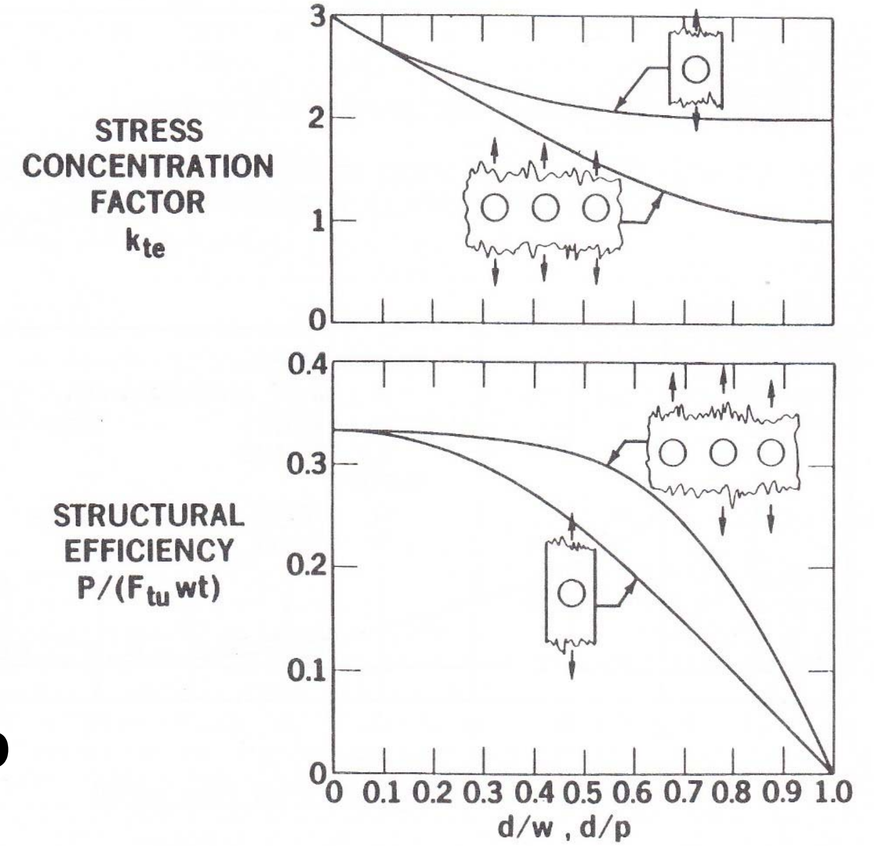
Suggested Form of Seam k_{te}

$$k_{te} = 1 + \left(1 - \frac{d}{p}\right)^3 + \left[1 - \left(\frac{d}{p}\right)^2\right]^{2.41}$$

k_{te} for UNLOADED Hole

$$k_{te} = 2 + \left(1 - \frac{d}{w}\right)^3$$

Difference Between the Two Significant, (d/w≤0.5)



Isolated versus Seam of Hole Comparison, Loaded Hole

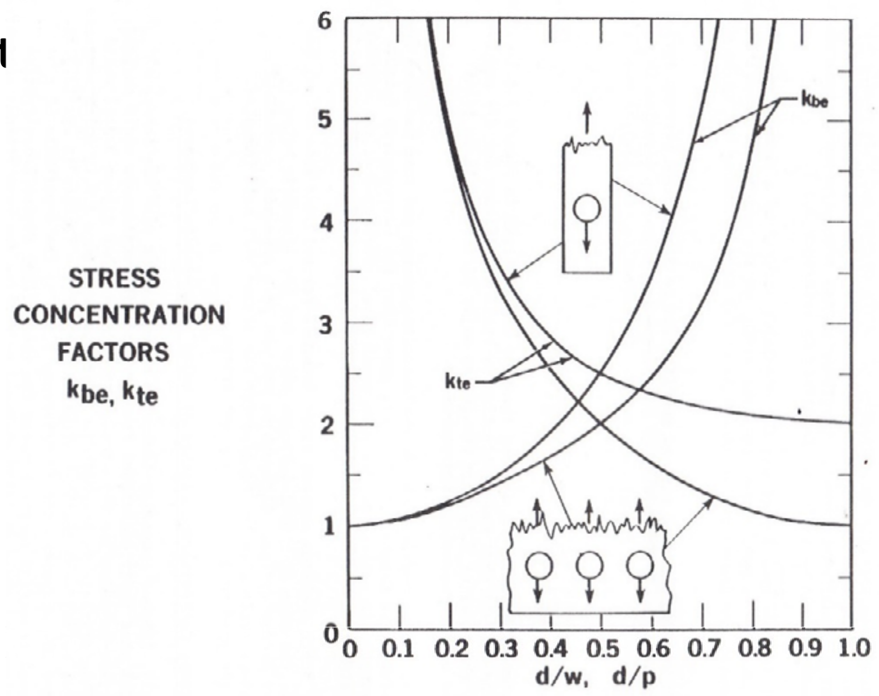
Suggested Form of Seam k

$$k_{te} = \frac{p}{d} + \left[1 - \left(\frac{d}{p}\right)^2\right]^{2.41} - \frac{3}{2}\cdot\frac{\left(p/d - 1\right)}{\left(p/d + 1\right)}\cdot\Theta$$

k_{te} for LOADED Single Hole

$$k_{te} = \left(\frac{w}{d} + 1\right) - \frac{3}{2}\cdot\frac{\left(\frac{w}{d} - 1\right)}{\left(\frac{w}{d} + 1\right)}\cdot\Theta$$

$$k_{be} = \frac{\sigma_{\max}}{P/t\cdot d} = \frac{k_{te}}{w/d - 1}$$

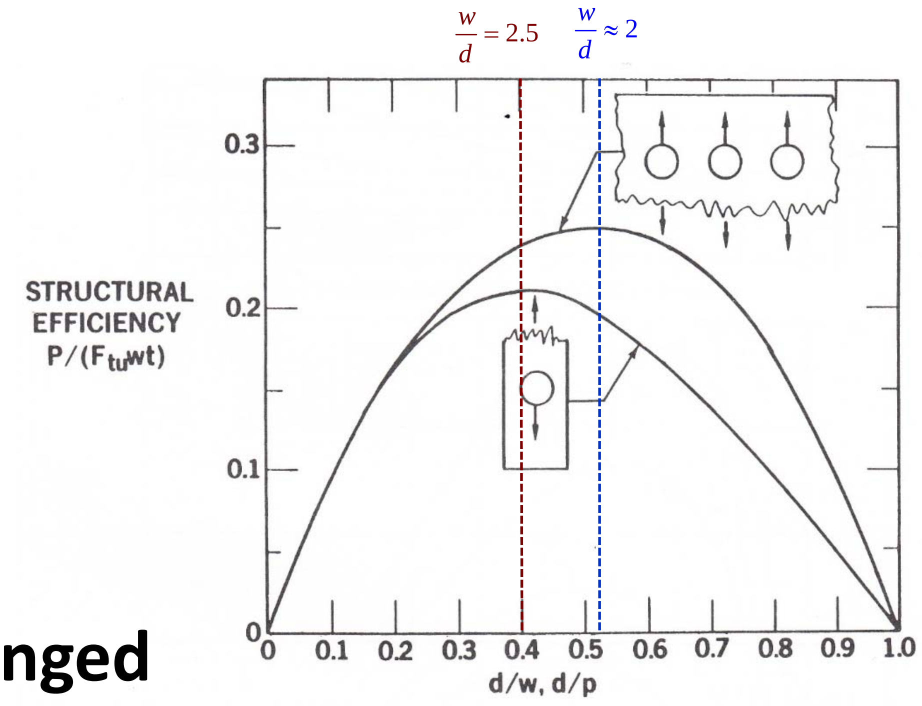


Impact of Stress Concentrations on Strip Strength, Updated

$$P = \left(\sigma_{net}\right)_{avg}\cdot\left(w-d\right)\cdot t = \frac{\sigma_{\max}}{k_{te}}\cdot\left(w-d\right)\cdot t$$

$$= \frac{\sigma_{\max}\cdot t\cdot w}{\frac{2}{\left(1-\frac{d}{w}\right)}+\frac{1}{\left(\frac{d}{w}\right)}-\frac{3}{2}\cdot\frac{\Theta}{\left(1+\frac{d}{w}\right)}}$$

$$\frac{P}{\sigma_{\max}\cdot t\cdot w} = \frac{1}{\frac{2}{\left(1-\frac{d}{w}\right)}+\frac{1}{\left(\frac{d}{w}\right)}-\frac{3}{2}\cdot\frac{\Theta}{\left(1+\frac{d}{w}\right)}}$$



Optimum Bolt Spacing Changed

Open Hole Composite Laminate Stress Concentration Factor

Characteristics of Composite Laminates

- No local yielding
- High Interlaminar stresses induced at hole boundaries
- Above conditions result in greater and varying stress concentration effect

Ultimate Stress in a Composite Structure

$$\sigma_{ult} = \frac{k_{tc} \cdot P_{app}}{\left(w - d\right) \cdot t}$$

$$k_{tc} \equiv \text{ Composite Stress Concentration}$$

Open Hole Stress Concentration for Orthotropic Laminate

Net Tension Stress Concentration Factor Dependent On Ply Angles

$$k_{tc} = 1 + \sqrt{2 \cdot \sqrt{\frac{E_1}{E_2}} - \nu_{21} + \frac{E_1}{G_{12}}}$$

$E_1 \equiv$ longitudinal Young's Modulus

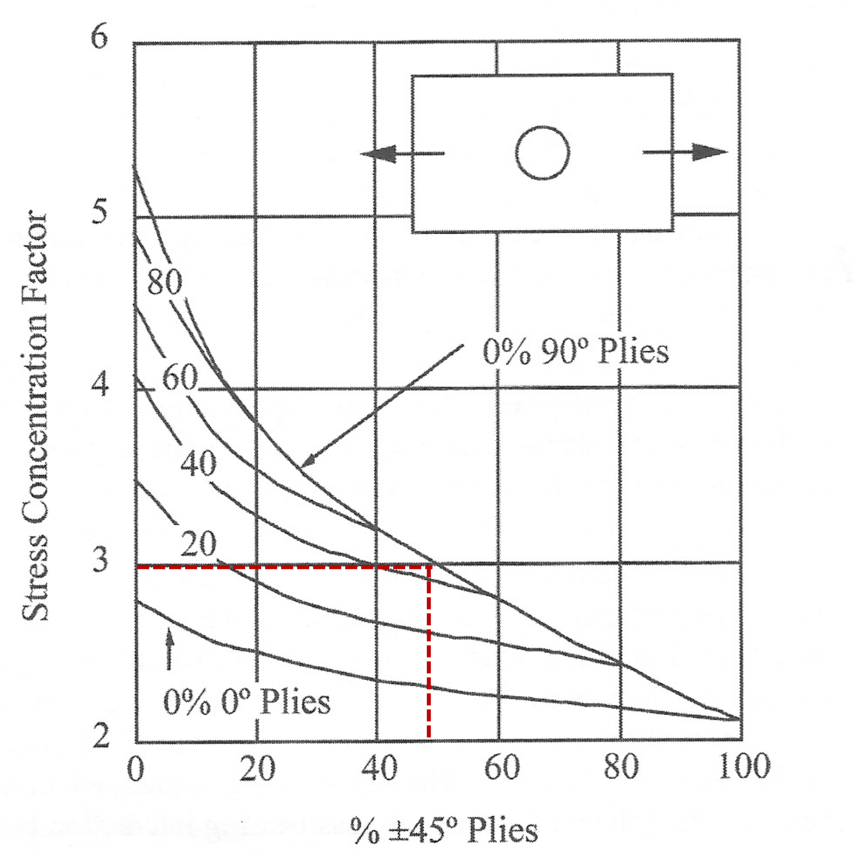
$E_2 \equiv$ transverse Young's Modulus

$G_{12} \equiv$ lin-plane shear modulus

$\nu_{21} \equiv$ major in-plane Poisson's Ratio

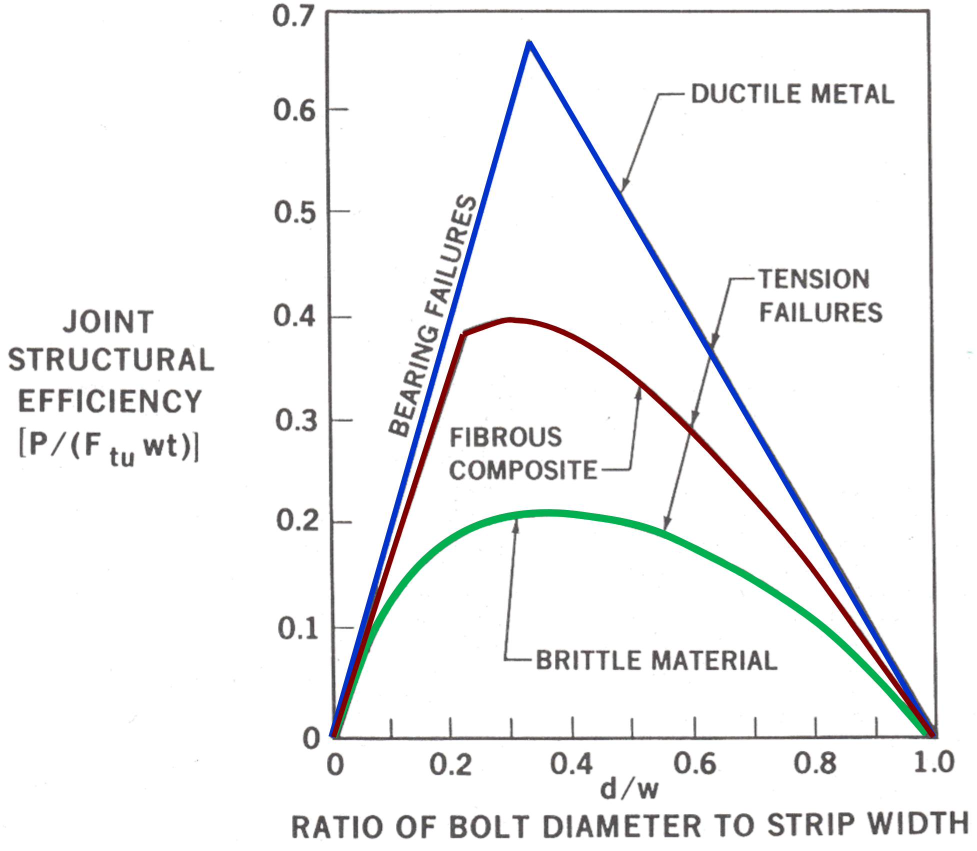
Composite Laminate k_{tc} Vary with Ply Geometry

- k_{tc}=7.5 for 100% 0° plies
- k_{tc}=1.8 for 100% ±45° plies
- k_{tc}=3 for 50% ±45 ° plies
  Isotropic Value



Stress Concentrations in Composite Materials Bolted Joint

- **Composite Laminates**
  - **Do not behave as brittle as Fiber and Matrix would suggest**
  - **Bolted joints are stronger**
  - **Joint structural efficiency a result of stress concentration relief**



Proposed Relationship Between k_{te} and k_{tc}, Loaded Holes

Experimentally Determined Relationship

$$k_{tc} - 1 = C \cdot \left(k_{te} - 1 \right)$$

Assumptions

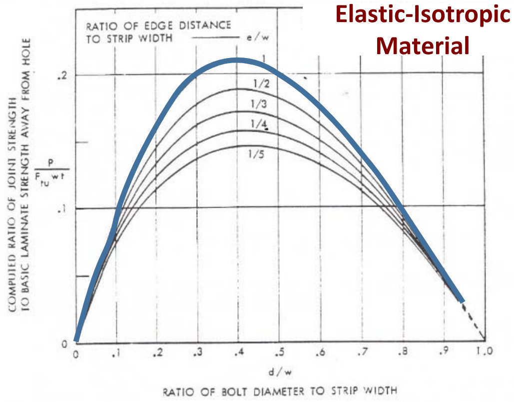
- Joint geometry will cause NET-TENSION failure
- Fiber pattern will cause NET-TENSION failure

Experimental Measurement of k_{tc}

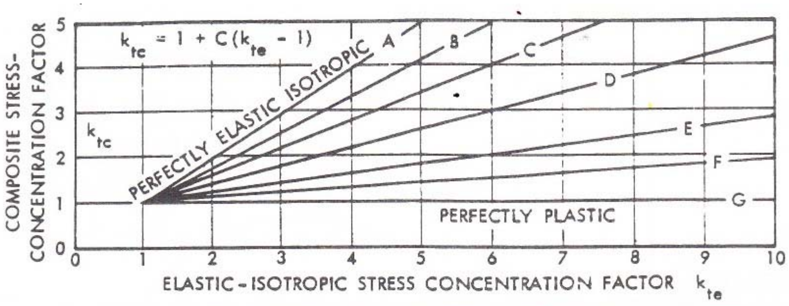
$$k_{tc} = F_{tu} \cdot t \cdot \left(w - d \right) / P$$

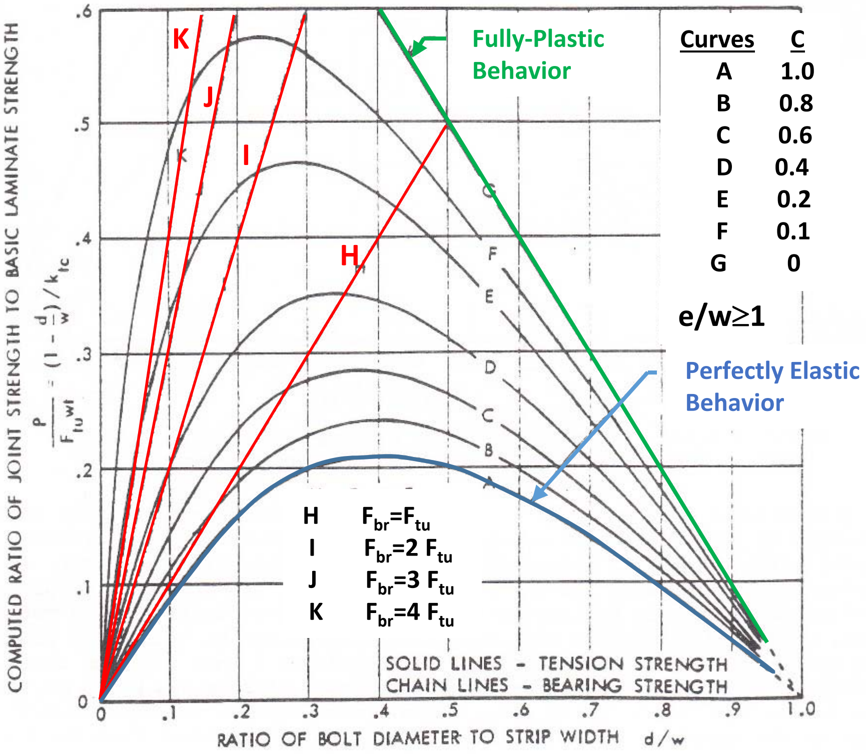
Composite Joint Efficiency versus d/w, Single Hole in Narrow Strip

$$\frac{P}{F_{tu}\cdot t\cdot w}=\frac{\left(1-\frac{d}{w}\right)}{k_{tc}}$$



Elastic-Isotropic Material





Fully-Plastic Behavior

Curves	C
A	1.0
B	0.8
C	0.6
D	0.4
E	0.2
F	0.1
G	0

$e/w \geq 1$

Perfectly Elastic Behavior

H	$F_{br}=F_{tu}$
I	$F_{br}=2\ F_{tu}$
J	$F_{br}=3\ F_{tu}$
K	$F_{br}=4\ F_{tu}$

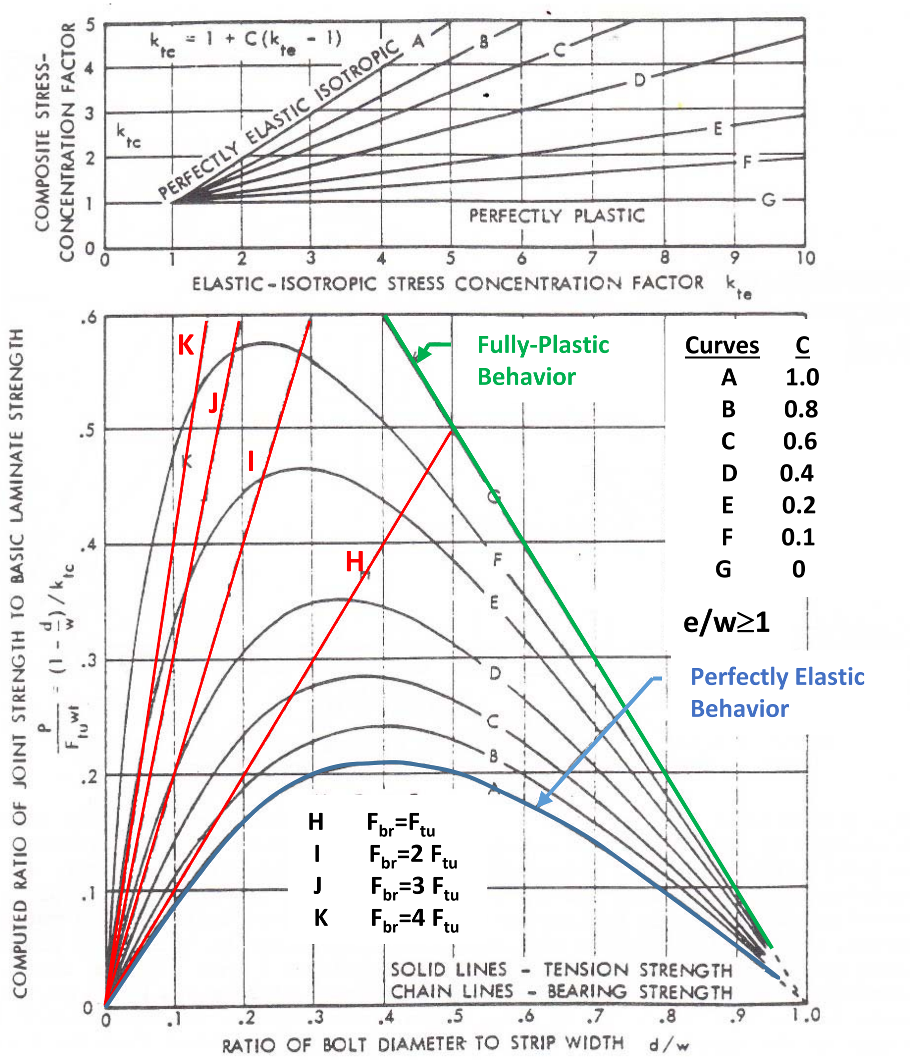
Composite Joint Efficiency versus d/w, Single Hole in Narrow Strip

Eventually Bolt Spacing Becomes Large,

- No-longer Net Tension
- Bearing Failure Dominate

To use this figure all that is needed is the Experimental Determination of

- Laminate Strength, F_{tu}
- Bearing Strength, F_{br}
- Correlation Factor, C



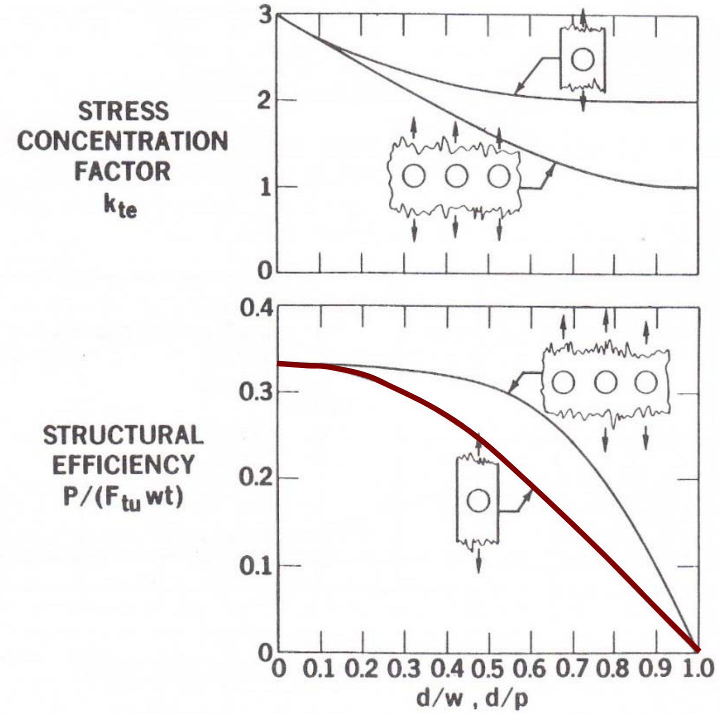
Curves	C
A	1.0
B	0.8
C	0.6
D	0.4
E	0.2
F	0.1
G	0

H	$F_{br}=F_{tu}$
I	$F_{br}=2\ F_{tu}$
J	$F_{br}=3\ F_{tu}$
K	$F_{br}=4\ F_{tu}$

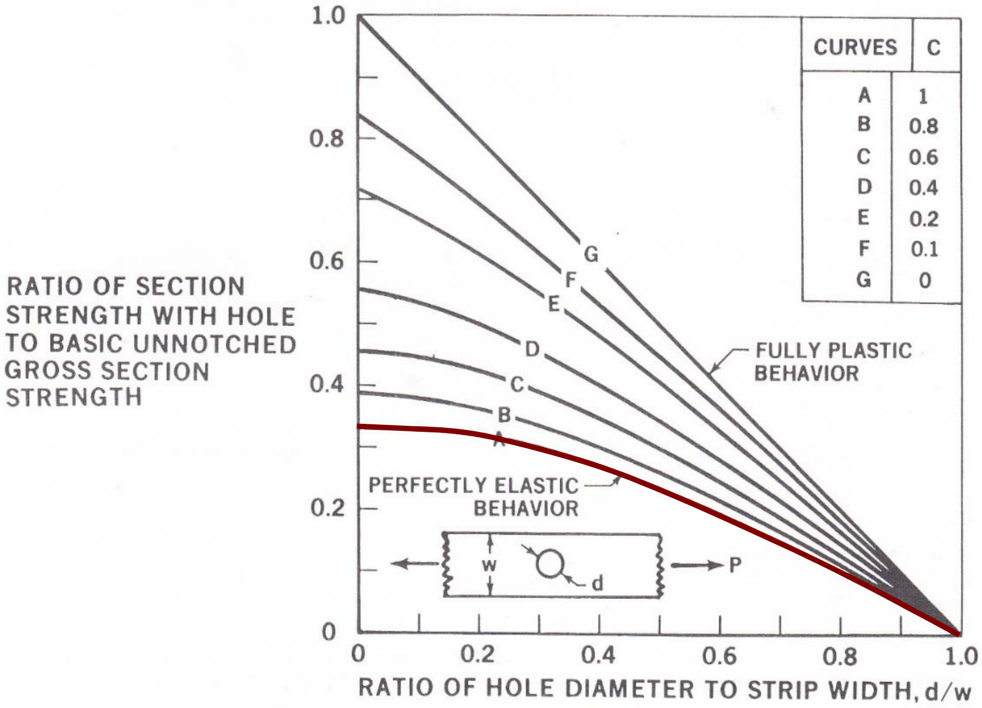
Proposed Relationship Between k_{te} and k_{tc}, Un-Loaded Holes

$$k_{tc} - 1 = C \cdot \left(k_{te} - 1\right)$$

Elastic-Isotropic

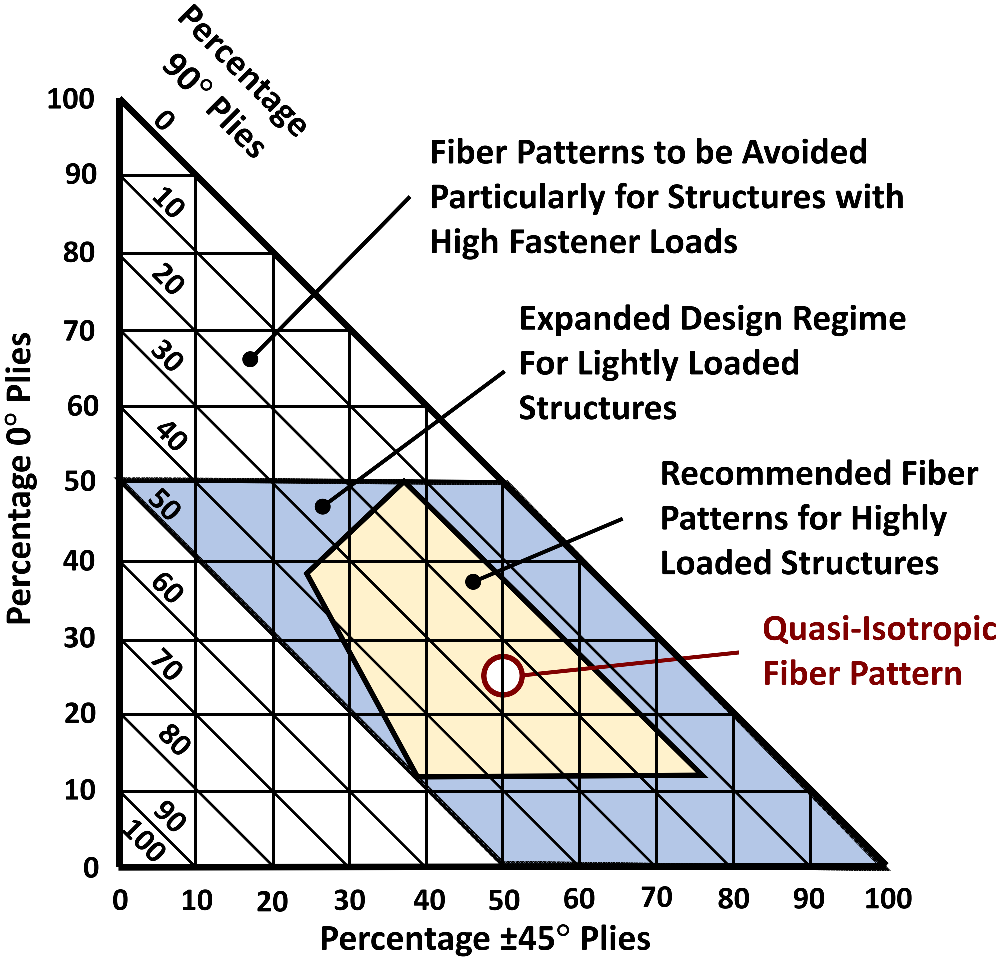


Composite



Experimental Program Conducted to find C

All Tests Conducted within Recommended Region



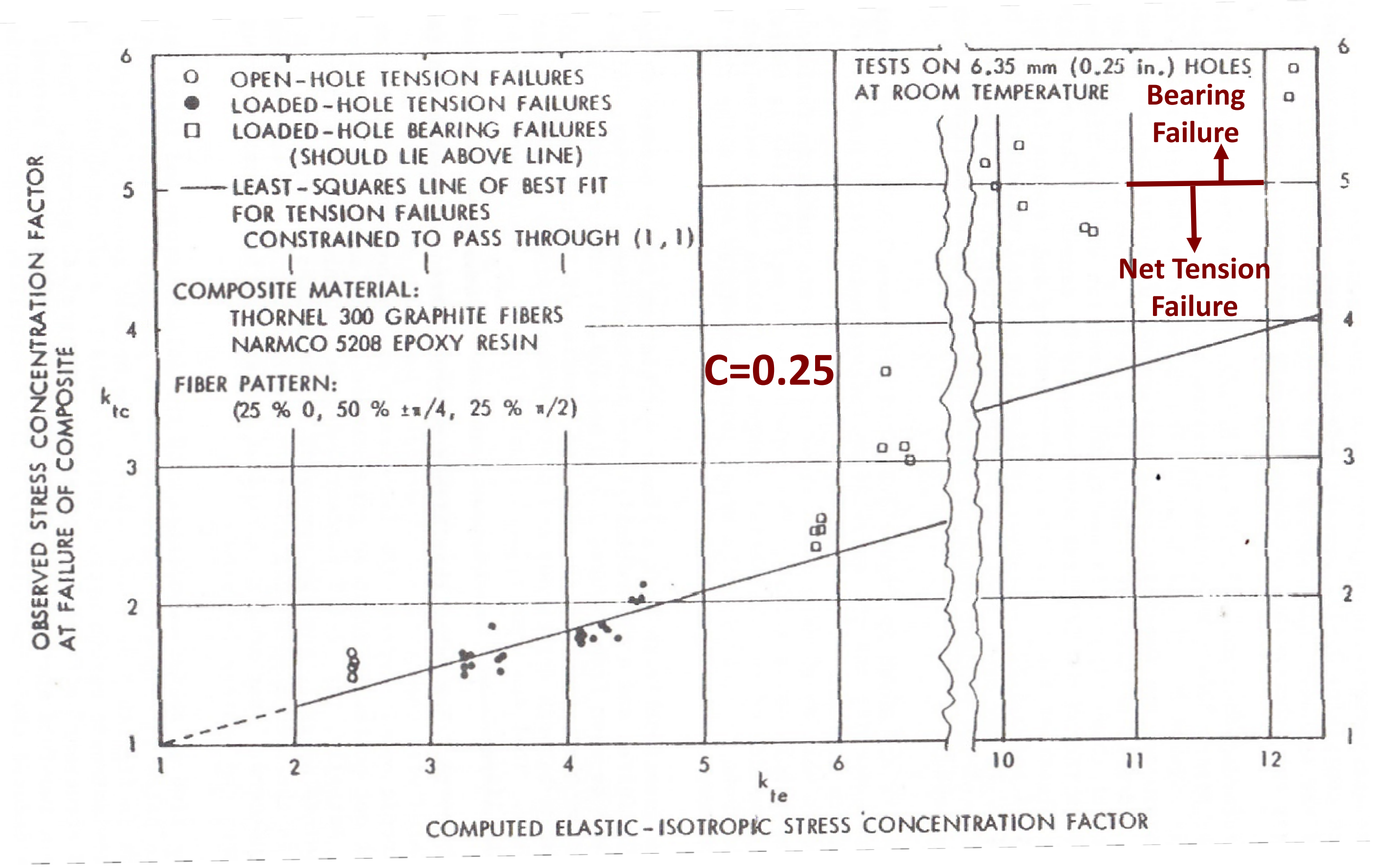
Fiber Patterns to be Avoided Particularly for Structures with High Fastener Loads

Expanded Design Regime For Lightly Loaded Structures

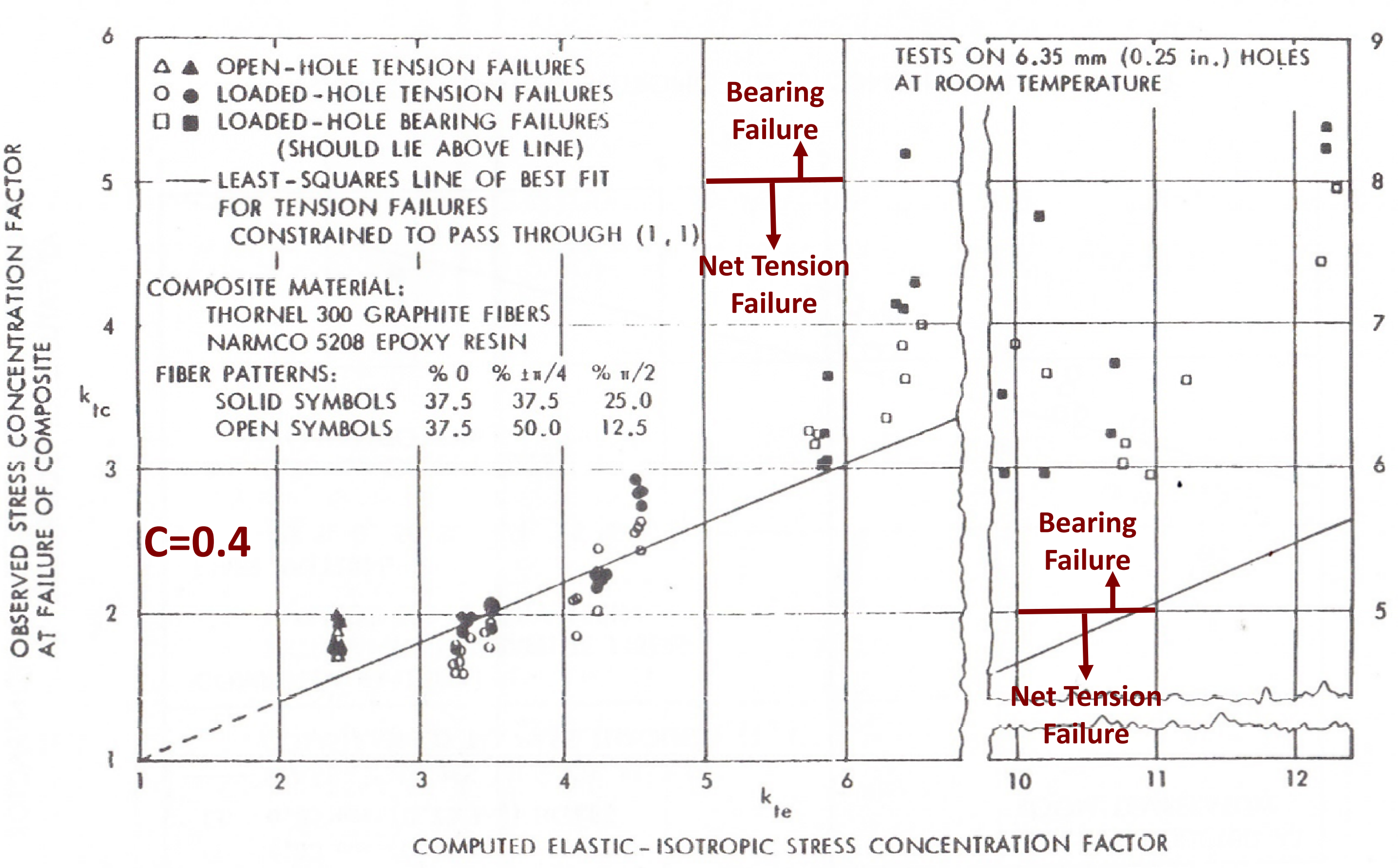
Recommended Fiber Patterns for Highly Loaded Structures

Quasi-Isotropic Fiber Pattern

Quasi-Isotropic Gr/Ep: Thornel 300/Narmco 5280



Orthotropic Gr/Ep: Thornel 300/Narmco 5280



Bearing Failure

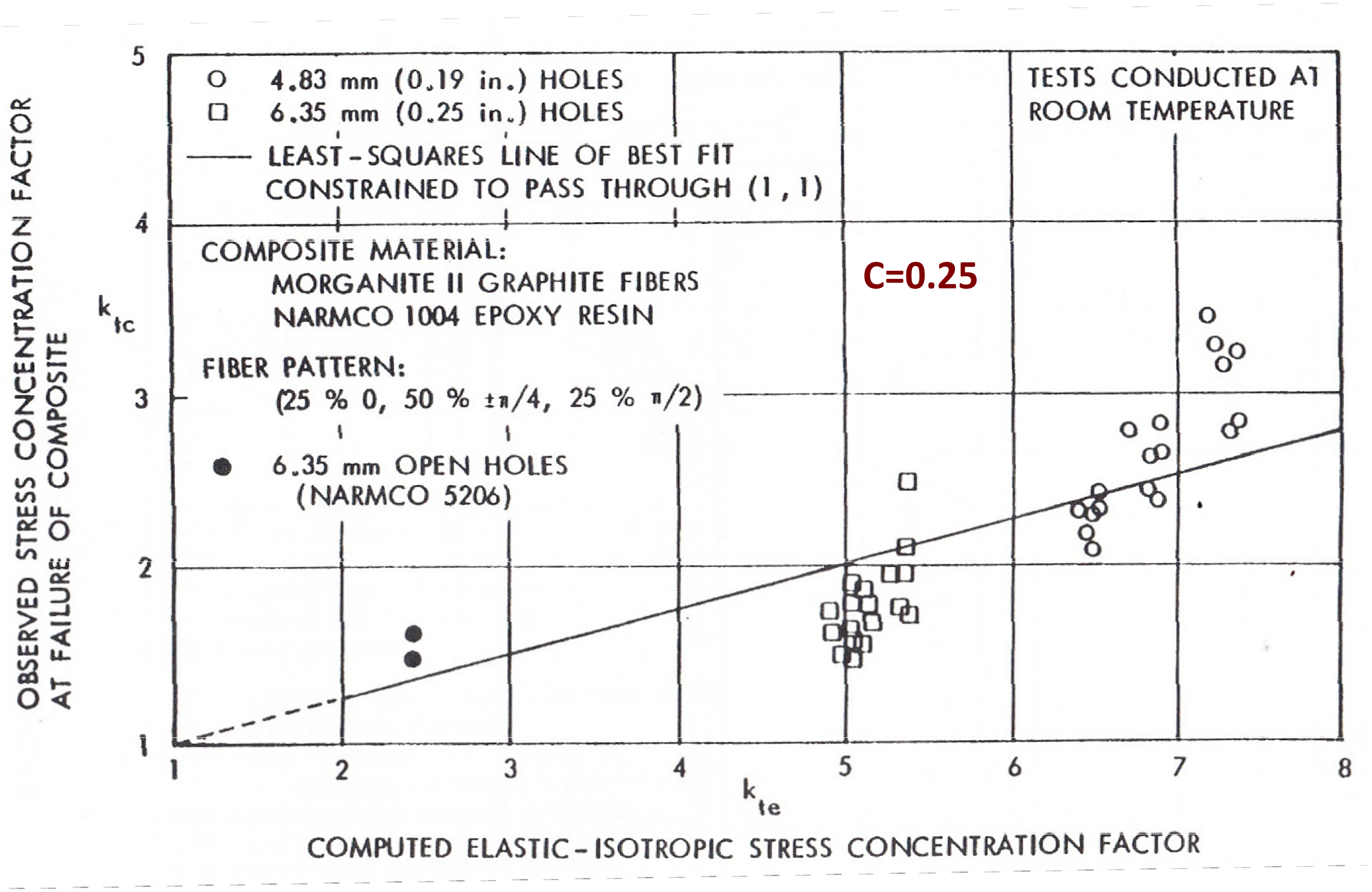
Net Tension Failure

C=0.4

Bearing Failure

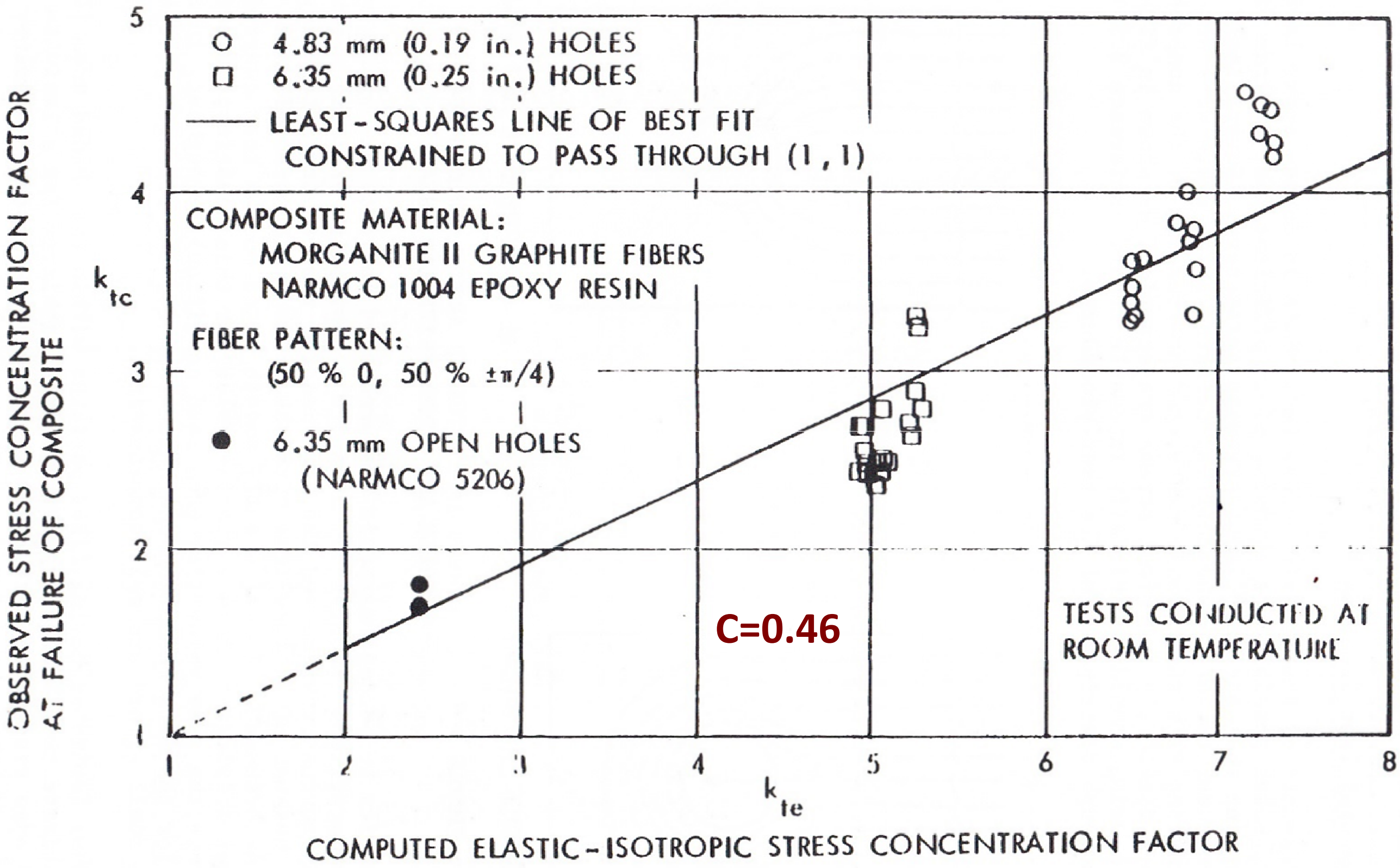
Net Tension Failure

Quasi-Isotropic Gr/Ep: Morganite II/Narmco 1004



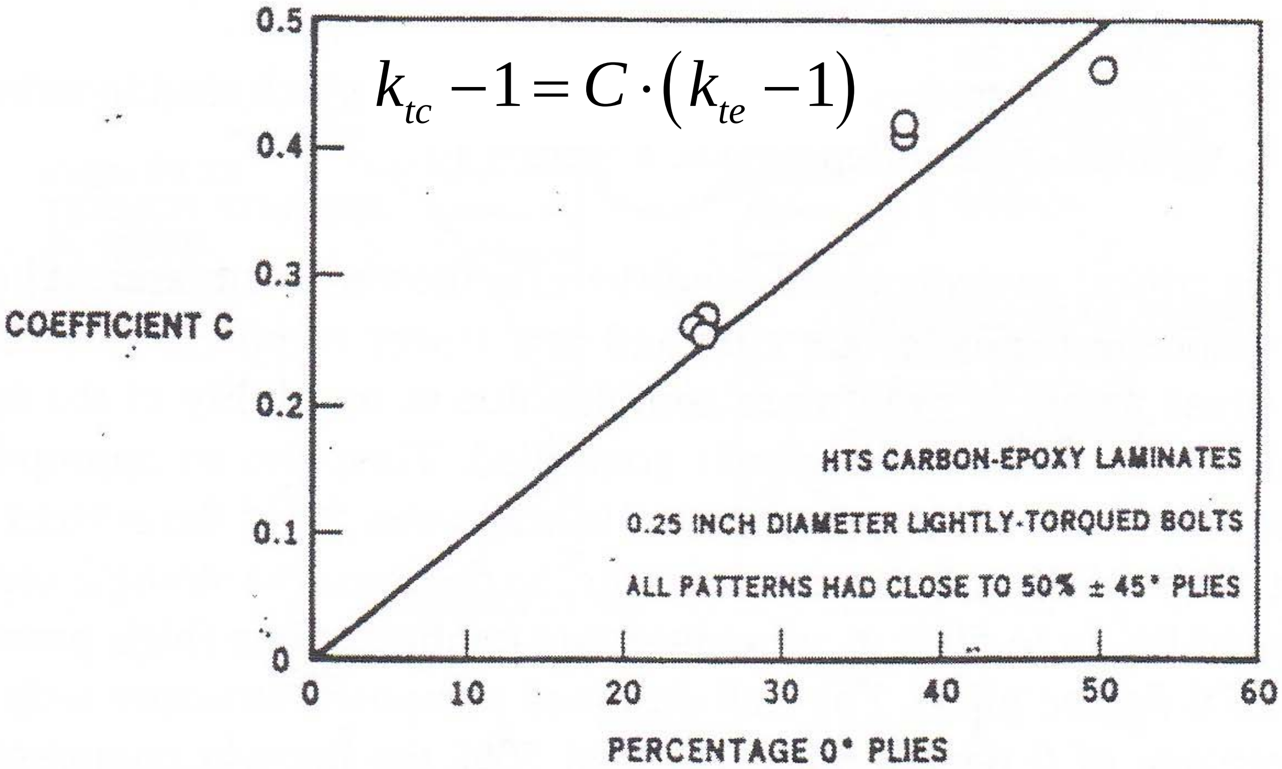
C=0.25

Orthotropic Gr/Ep: Morganite II/Narmco 1004



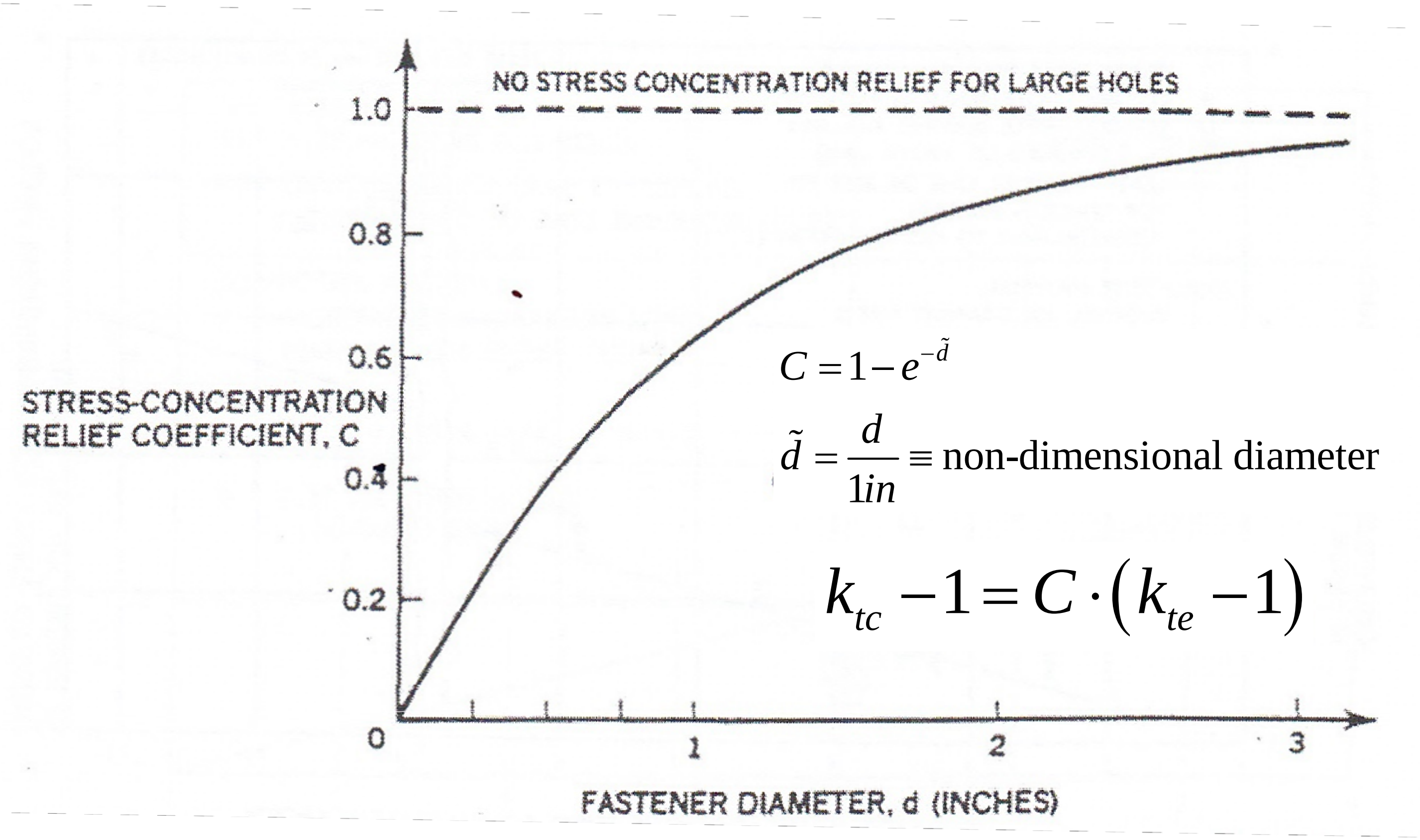
C=0.46

C as a Function of 0° Plies



$$k_{tc} - 1 = C \cdot \left(k_{te} - 1\right)$$

Quasi-Isotropic Values of C versus Fastener Diameter



$$C = 1 - e^{-\tilde{d}}$$

$$\tilde{d} = \frac{d}{1in} \equiv \text{non-dimensional diameter}$$

$$k_{tc} - 1 = C \cdot \left(k_{te} - 1\right)$$

Stress Concentration Around a Hole Complex

- Peak stresses actually occur at 45° to the hole
  - k_{tc}=2 for common laminate configurations

- Individual ply stresses will vary with orientation

- Complex Three Dimensional state of stress

- Interlaminar stresses result in delamination
  - tends to reduce k_{tc} through local softening

- Compressive Loading Even More Complex

Shear-Out Failures

In Metal Structures

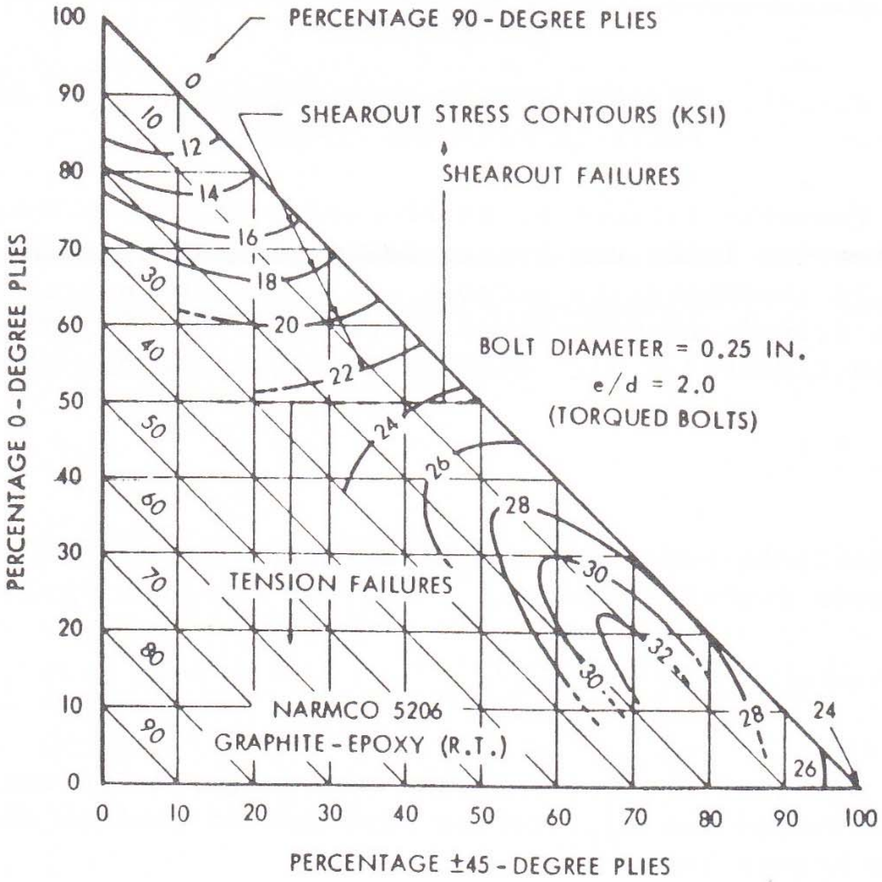
$$\tau_{ult} = \frac{P_{app}}{2 \cdot e \cdot t}$$

Relationship Holds in Composites as long as

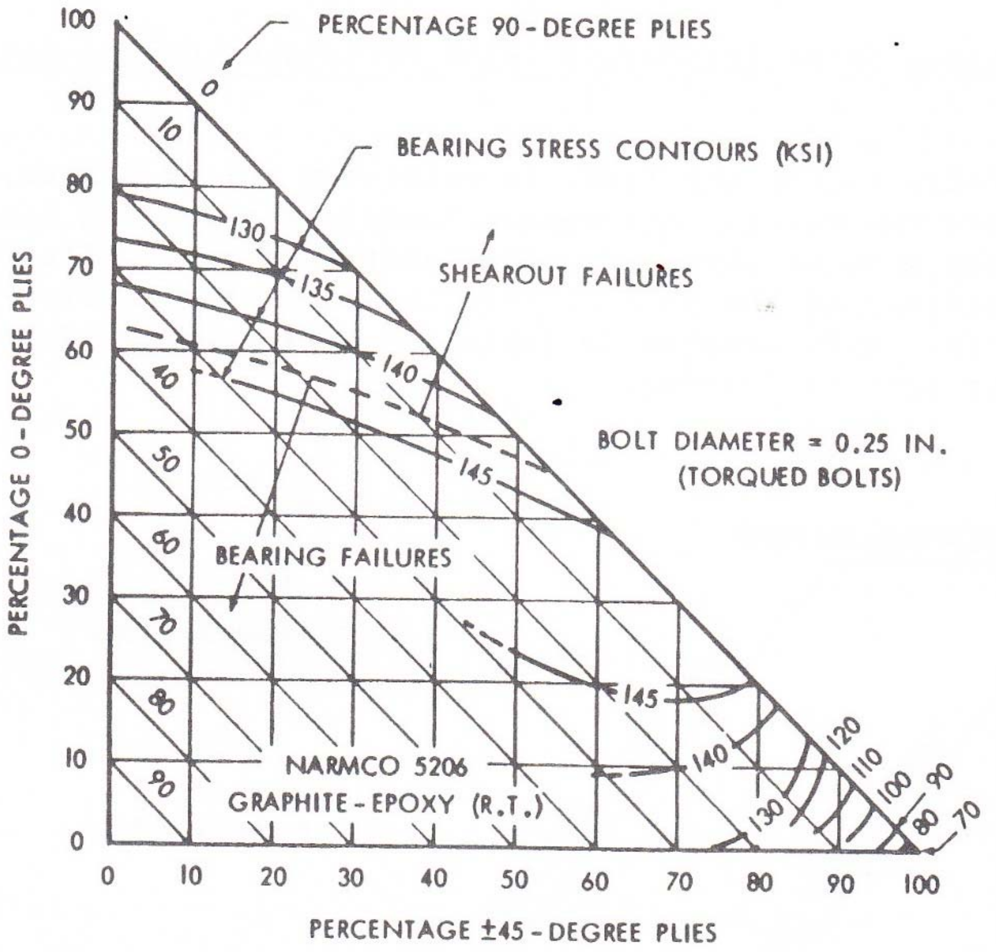
- **% of ±45 plies significantly large**
- **optimal 50% ±45 plies**

Variations in ply percentages will cause changes in Shear Out strength

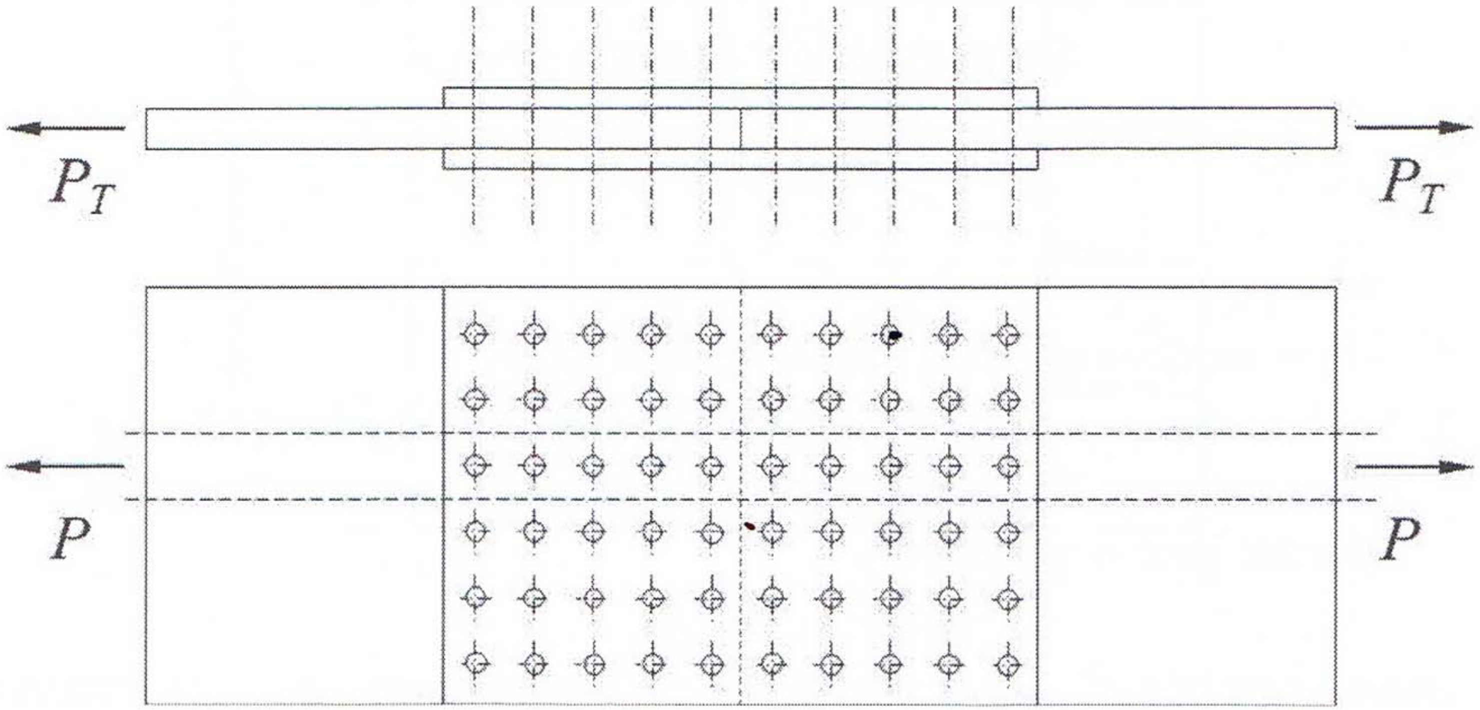
Failure Mode Results for Short Edge Distance, e/d=2



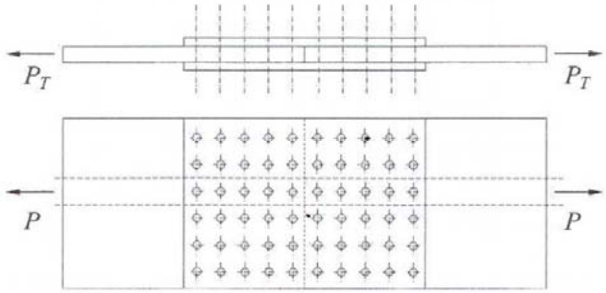
Failure Mode Results for Large Edge Distances, e/d>8



Multiple Rows of Fasteners requires Non-Linear FEA



First Estimate of Joint Strength for Multiple Rows of Fasteners



- **Divide Panel into Strips of Equal Width**
  - **Strip Width = Fastener Pitch p**
  - **Strip Load P**
  - **m =number of Strips/Fasteners**
  - **Total Load P_t**

$$P = \frac{P_T}{m}$$

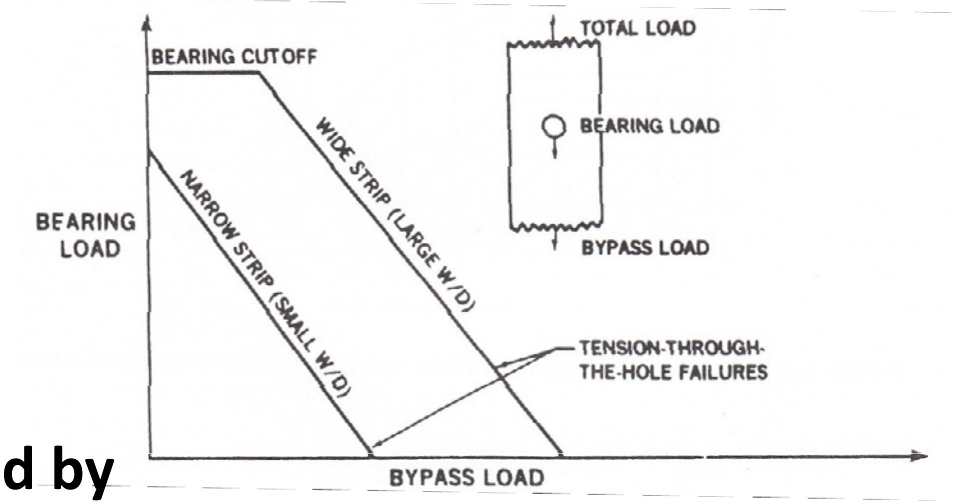
- **Each Fastener in the Strip will take a Portion of the load P**

$$P_{fastener} = \frac{P_T}{\text{Total Number of Fasteners}}$$

Assuming Linear Interaction between Bearing & By-Pass Loads

Linear Interaction Initially Assumed

$$\sigma_{\max} = k_b \cdot \sigma_b + k_t \cdot \sigma_t \leq \sigma_{tu}$$



σ_b = fastener bearing stress

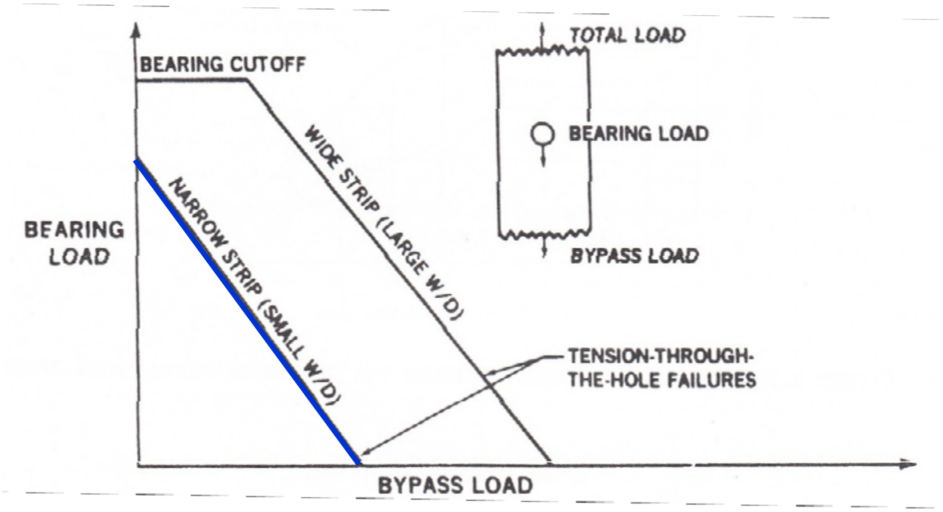
σ_t = net tension stress caused by load not reacted by the fastener

k_t, k_b = constants of proportionality

σ_{tu} = ultimate gross (far field) strength

Interaction between Bearing and Bypass In Multi-Row Joints

$$\sigma_{\max} = k_b \cdot \sigma_b + k_t \cdot \sigma_t \le \sigma_{tu}$$



Alleviation Factors and Stress Concentrations Factors

$$k_b = \frac{1}{\frac{w}{d} - 1} \cdot \left\{ 1 + C \left[k_{be} \cdot \left(\frac{w}{d} - 1 \right) - 1 \right] \right\}$$

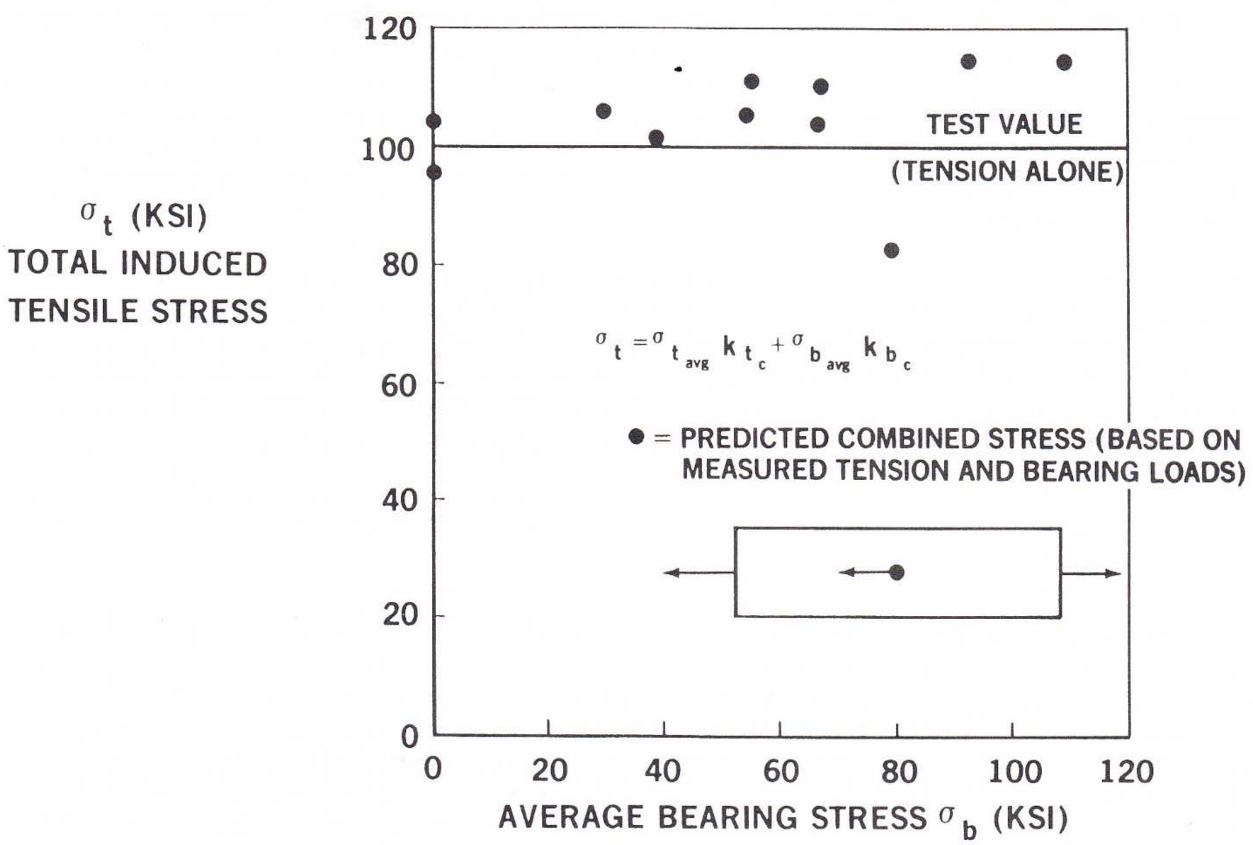
$$k_{be} = \frac{\sigma_{\max}}{\frac{P}{t \cdot d}} = \frac{k_{te}}{\frac{w}{d} - 1} = 1 + \frac{2}{\frac{w}{d} - 1} - \frac{3}{2} \cdot \frac{1}{\frac{w}{d} + 1} \cdot \Theta$$

$$k_t = 1 + C \cdot \left(k_{te} - 1 \right)$$

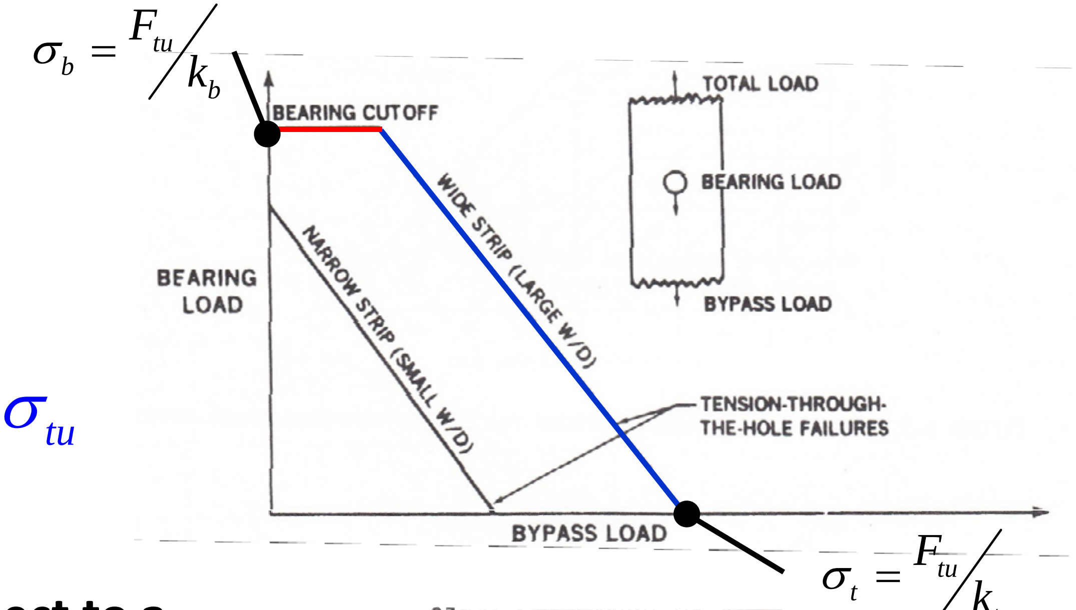
$$k_{te} = 2 + \left(1 - \frac{d}{w} \right)^3$$

Experimental Conformation of Linear Relationship

$$\sigma_{\max} = k_b \cdot \sigma_b + k_t \cdot \sigma_t \leq \sigma_{tu}$$



Interaction between Bearing and Bypass In Multi-Row Joints



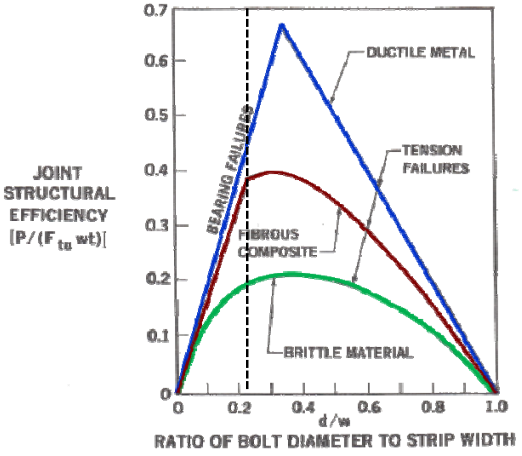
Even if w/d too large to exhibit tensile failure under pure bearing load it will still do so for bypass load

$$\sigma_{\max} = k_b \cdot \sigma_b + k_t \cdot \sigma_t \le \sigma_{tu}$$

$$\sigma_b = F_{tu} - k_t \cdot \sigma_t \le F_{br}$$

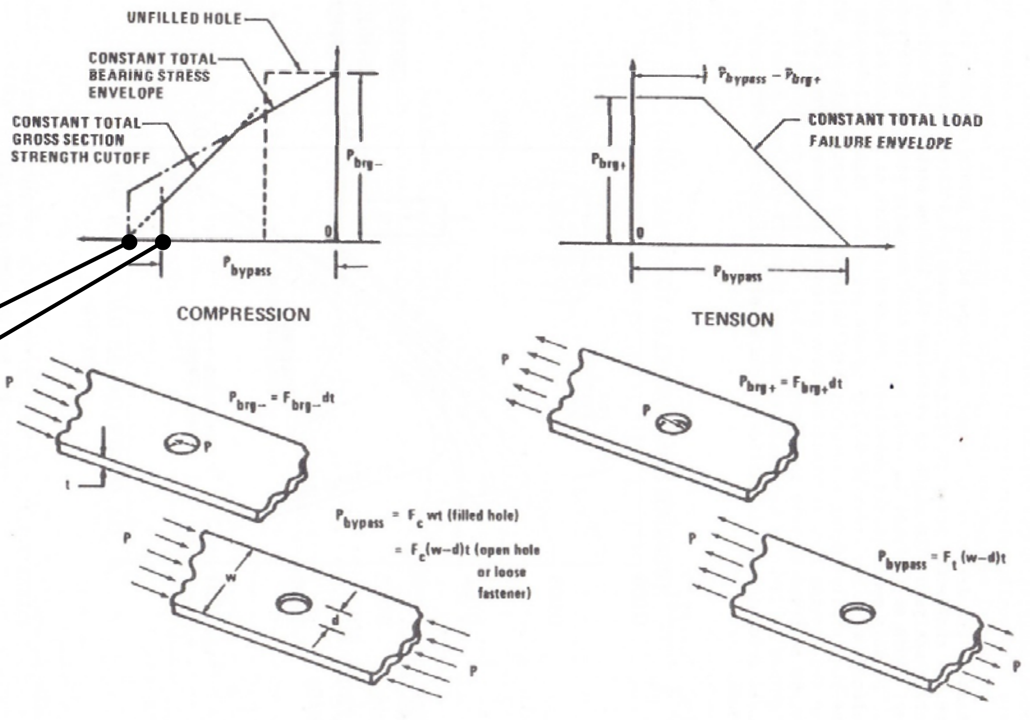
Most Efficient Joint: One bolt subject to a pure Bearing load (no Bypass) with minimum Bearing Load

Maximum Joint Strength: Proportioning the joint to cause tension (rather than bearing) failures at all bolt holes



Interactions for Compressive Loading Has Different Form``

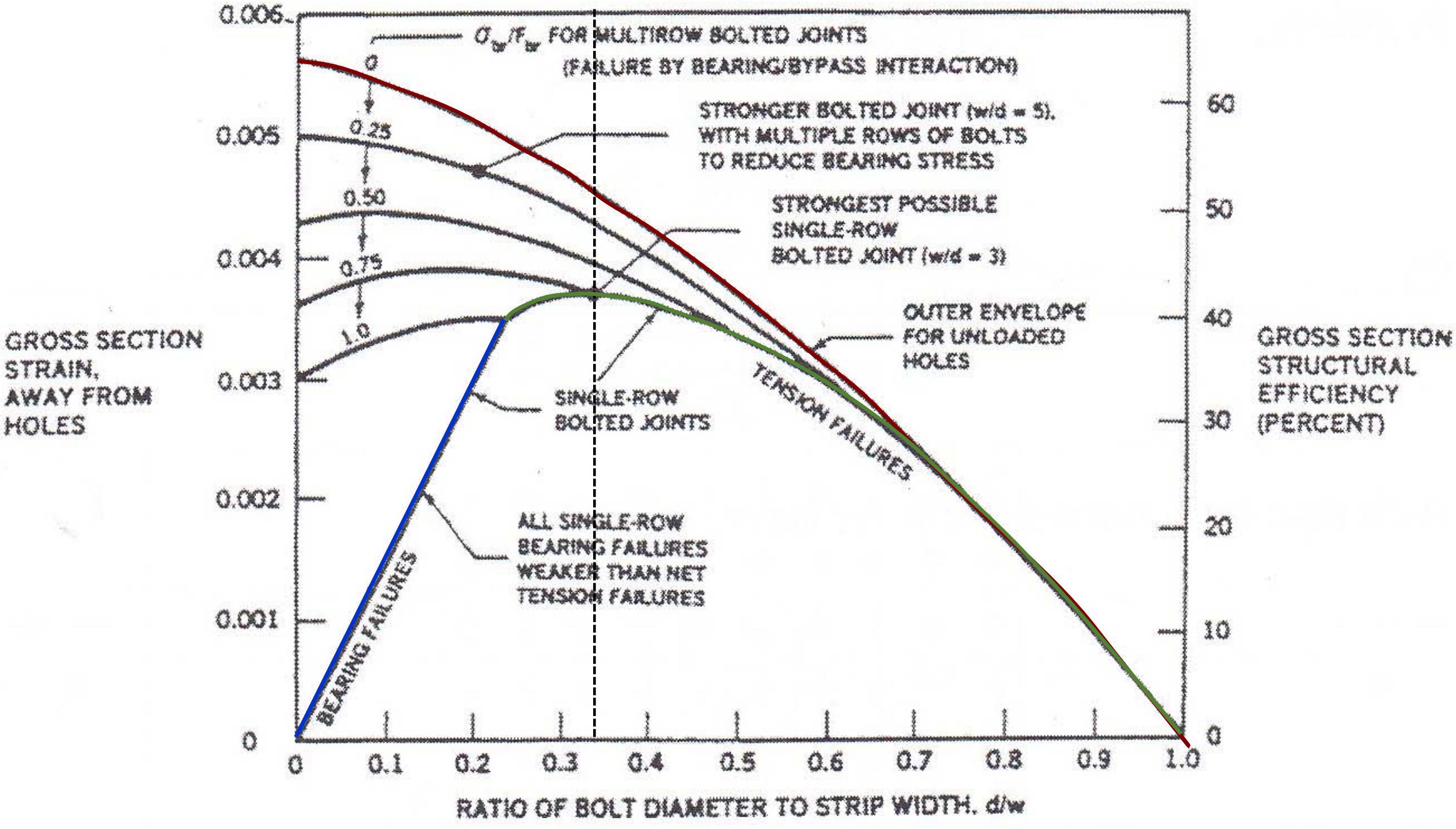
$$F_{br} = \sigma_b + \sigma_c$$



Ultimate Compressive Load, F_{cu}
Tight Fitting Bolt
Loose Fitting Bolt

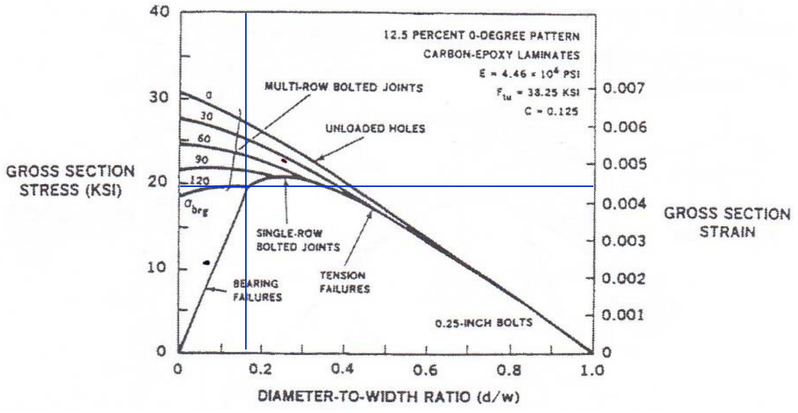
σ_C = Gross-Section Compressive Stress
F_{br} = Bearing Strength
σ_b = Bearing Load

Multi-Rows of Fasteners Show Small Improvement Over Single Row

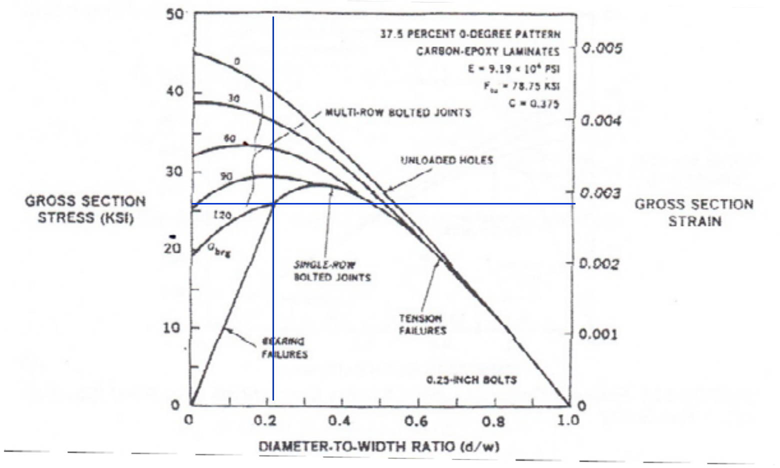


Impact of 0° Plies on Stress

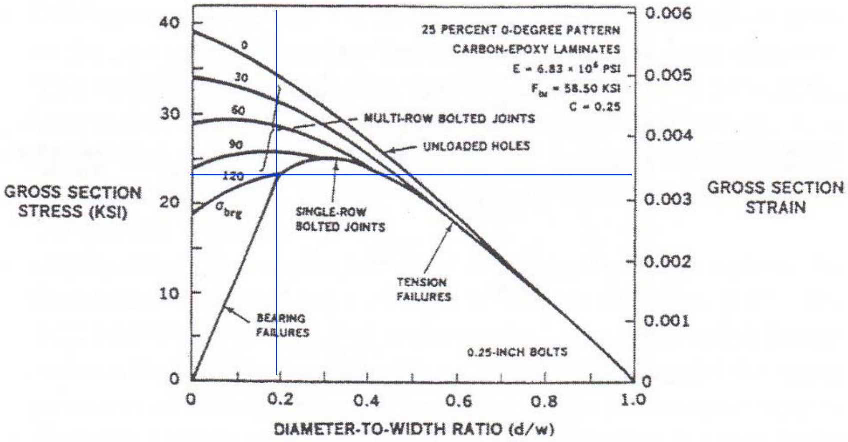
12.5% 0° Plies



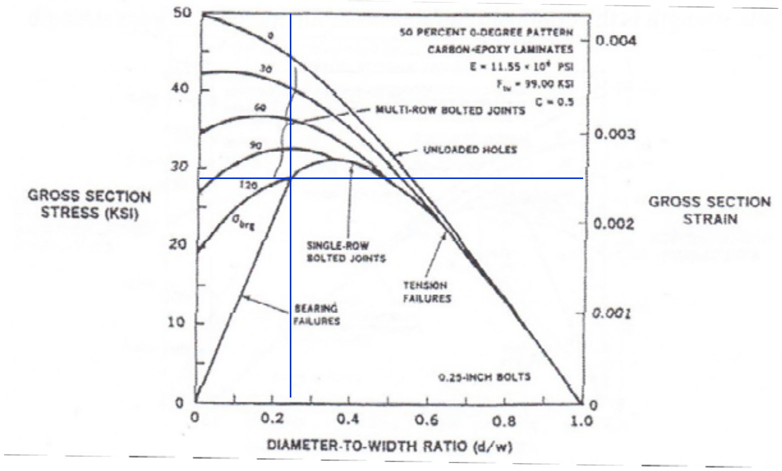
37.5% 0° Plies



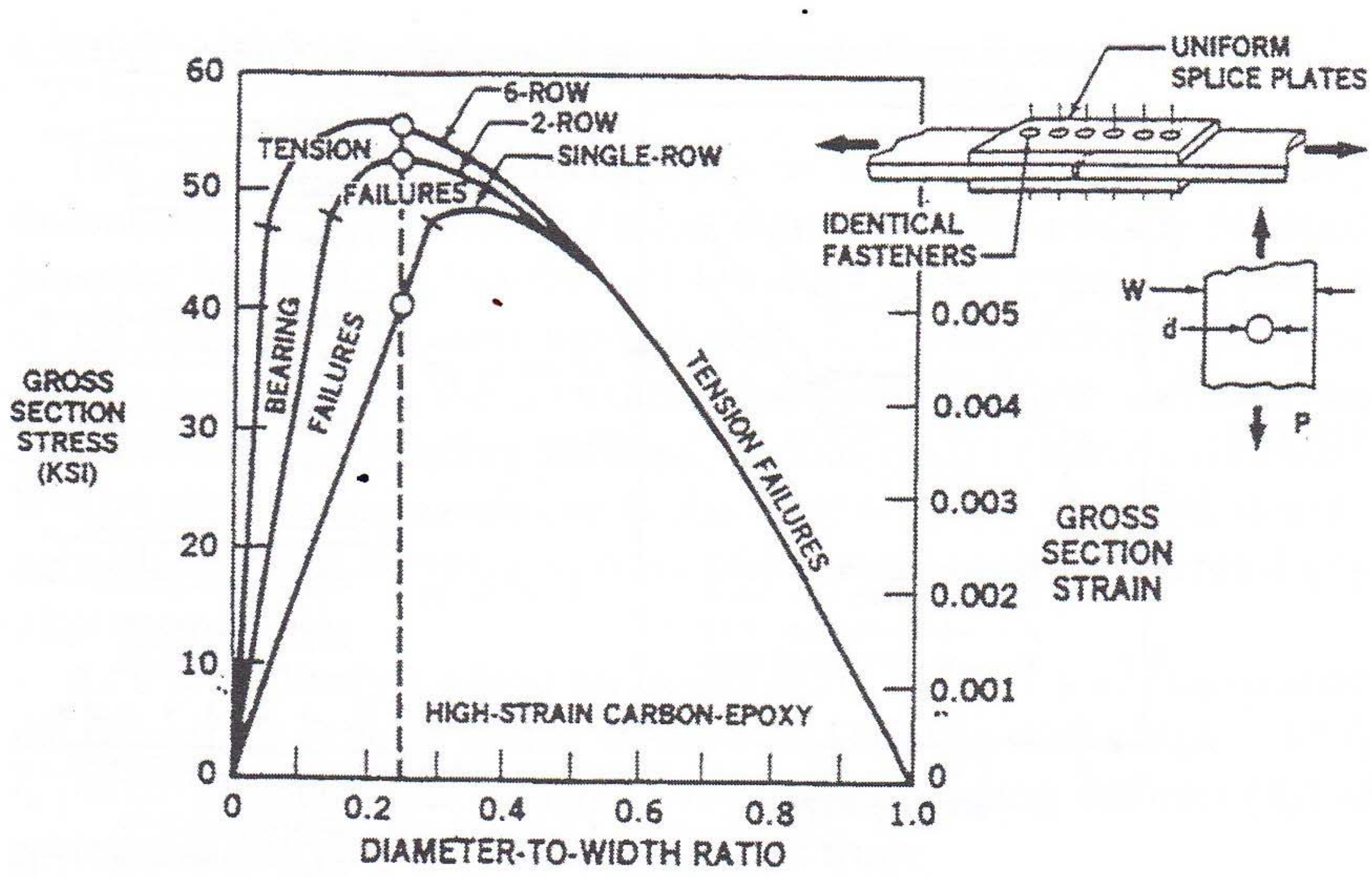
25% 0° Plies



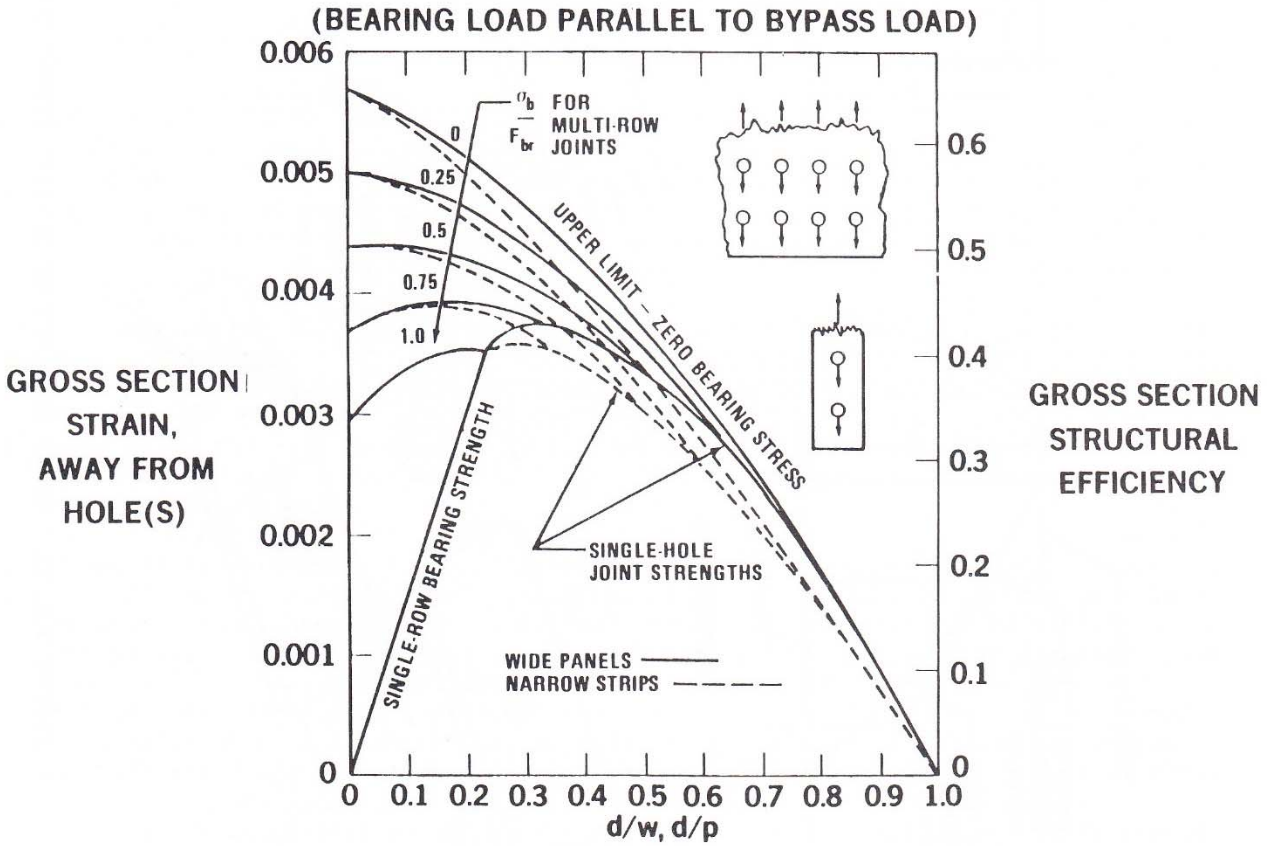
50% 0° Plies



Multi-Row Improvements Only Until Bearing/Tension Transition



Comparison Between Narrow Strip and Wide Panel



Joint Configurations Have Effect on Bolt Loading

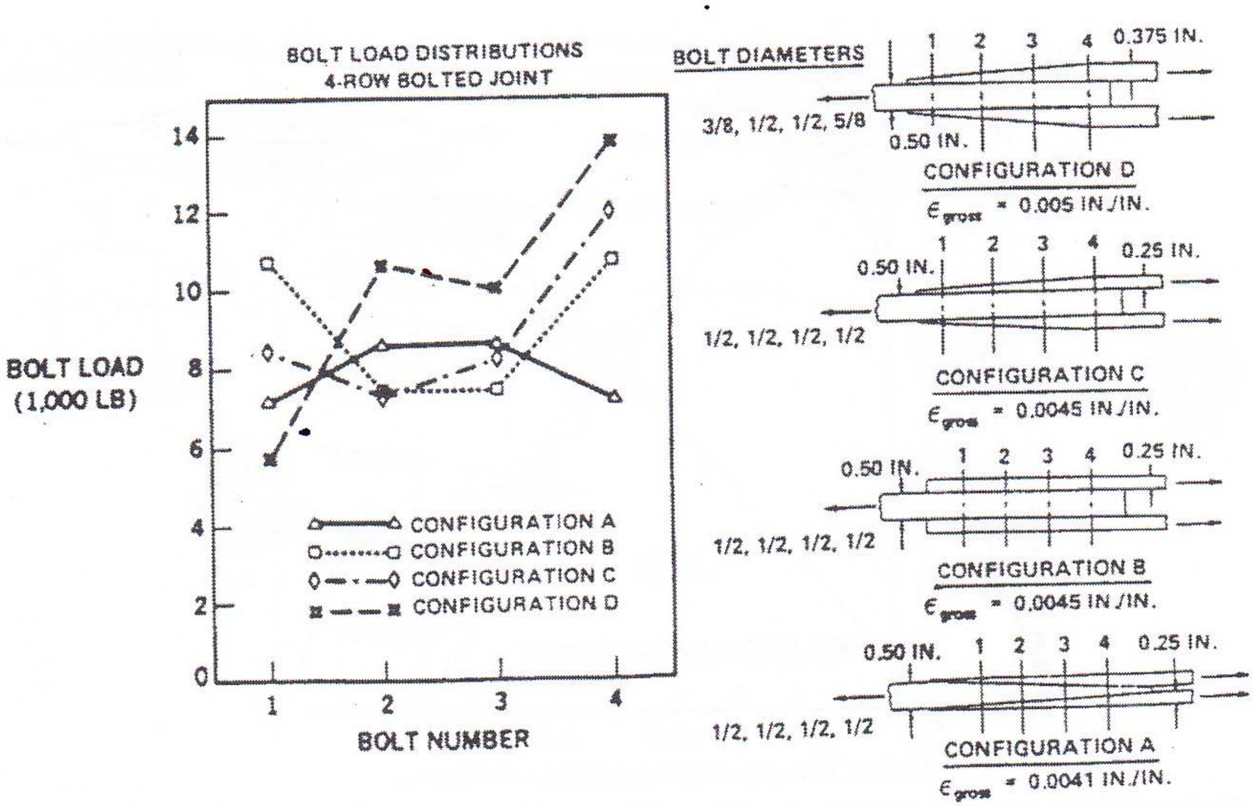


FIGURE 6.32. Effect of Joint Configuration on Bolt Load Distribution.

Preliminary Estimation Fastener Load Sharing, Basic Parameters

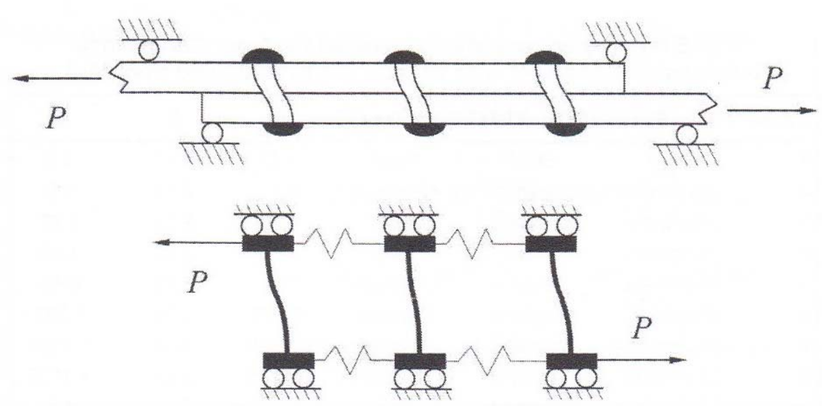
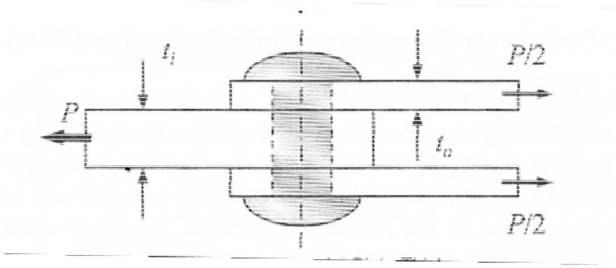


Plate Stiffness

$$K_p = \frac{A \cdot E}{L}$$

Fastener Bending Stiffness

$$K_f = \frac{1}{C}$$



$C \equiv$ Fastener Effective Compliance

$$= \frac{8}{t_{av} \cdot E_f} \left\{ A \cdot \left(\frac{t_{av}}{d} \right)^2 \cdot \left[B + \left(\frac{t_{av}}{d} \right)^2 \right] + H \right\}$$

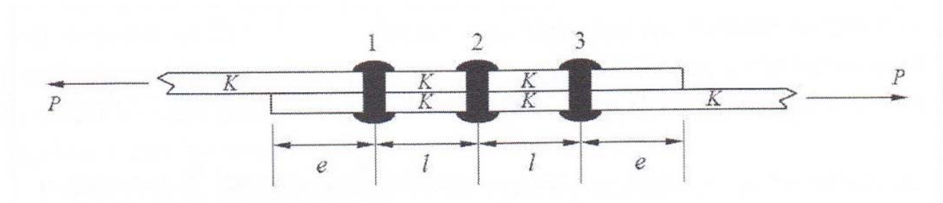
$$t_{av} = \frac{2 \cdot t_0 + t_i}{2}$$

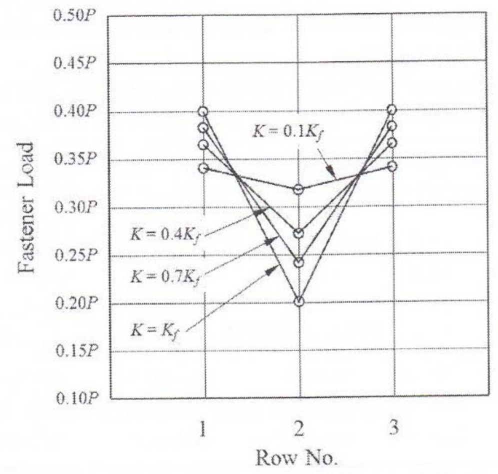
$E_f \equiv$ Fastener Modulus of Elasticity

d $\equiv$ Fastener Diameter

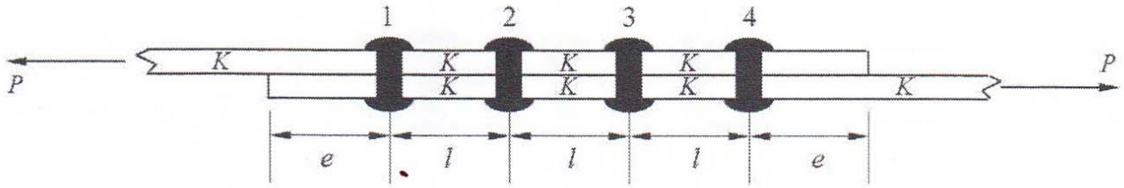
Parameter Summary

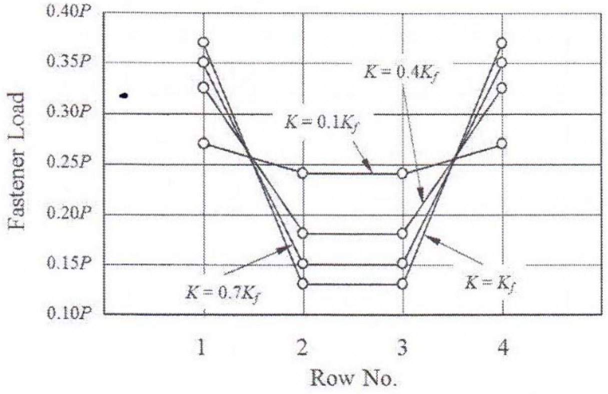
Case	Inner Plate	Outer Plate	Fastener	A	B	H
1	St	St	St	0.13	2.12	1.0
2	Al	Al	Al	0.13	2.12	1.0
3	Al	Al	St	0.13	2.12	1.87
4	Al	St	St	0.13	2.12	1.43
5	Al	St	Al	0.13	2.12	0.84
6	Al	Al	Ti	0.133	2.06	1.242
7	Al	Ti	Ti	0.1325	2.06	1.1125
8	CFRP (QI)	Ti	Ti	0.1325	2.06	1.1125
9	GFRP (QI)	Al	Al	0.13	2.12	1.0



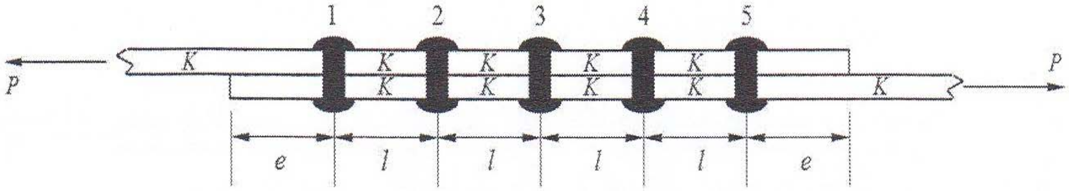


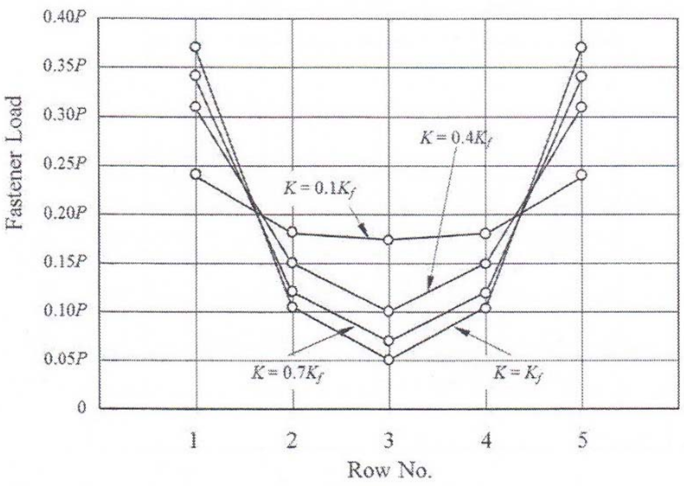
Number of Rows	Inner-to-Outer Plate Stiffness Ratio	Fastener Row Load Shear Expression RLF-Relative Load Fraction
3	$K_{inner}=K_{outer}$	$RFL_{R1}=RFL_{R3}=-0.0167(K_t/K_p)^2+0.0803(K_t/K_p)+0.3362$
		$RFL_{R2}=0.0333(K_t/K_p)^2+0.1607(K_t/K_p)+0.3275$



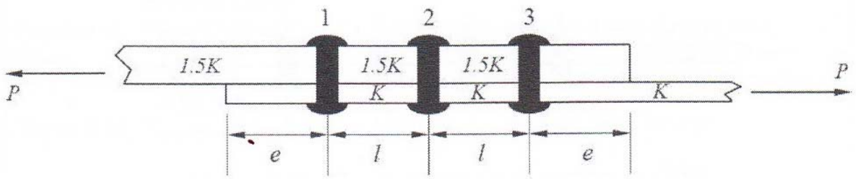


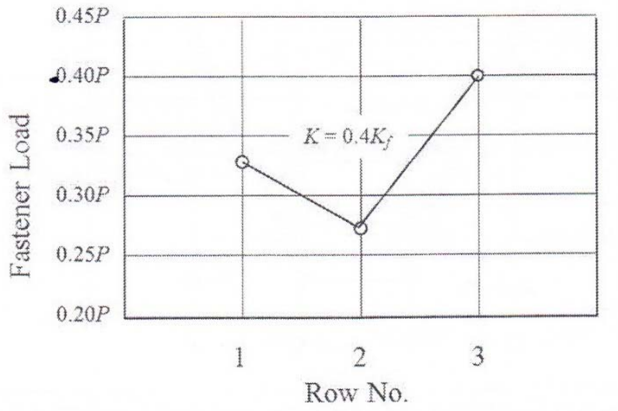
Number of Rows	Inner-to-Outer Plate Stiffness Ratio	Fastener Row Load Shear Expression RLF-Relative Load Fraction
4	$K_{inner}=K_{outer}$	$RFL_{R1}=RFL_{R4}=-0.0556(K_t/K_p)^2+0.1678(K_t/K_p)+0.258$
		$RFL_{R2}=RFL_{R3}=0.0556(K_t/K_p)^2-0.1678(K_t/K_p)+0.2471$



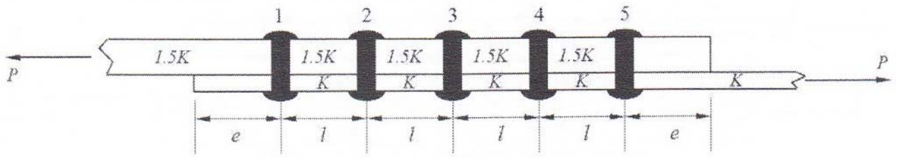


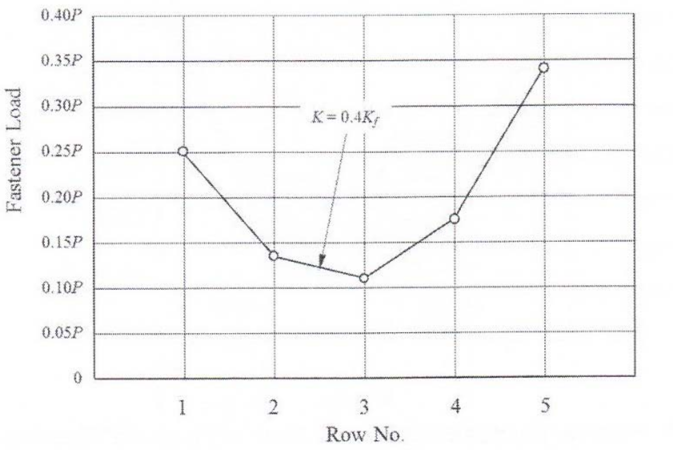
Number of Rows	Inner-to-Outer Plate Stiffness Ratio	Fastener Row Load Shear Expression RLF-Relative Load Fraction
5	$K_{inner}=K_{outer}$	$RFL_{R1}=RFL_{R5}=-0.1389(K_t/K_p)^2+0.2994(K_t/K_p)+0.202$
		$RFL_{R2}=RFL_{R4}=0.0694(K_t/K_p)^2-0.1581(K_t/K_p)+0.1994$
		$RFL_{R3}=0.1389(K_t/K_p)^2-0.2828(K_t/K_p)+0.1954$



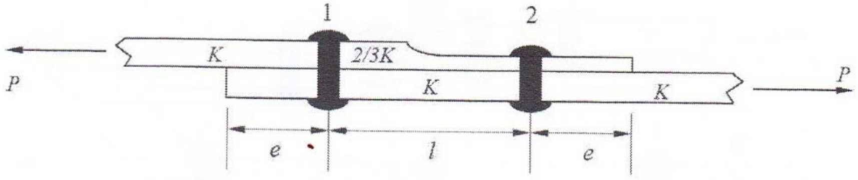


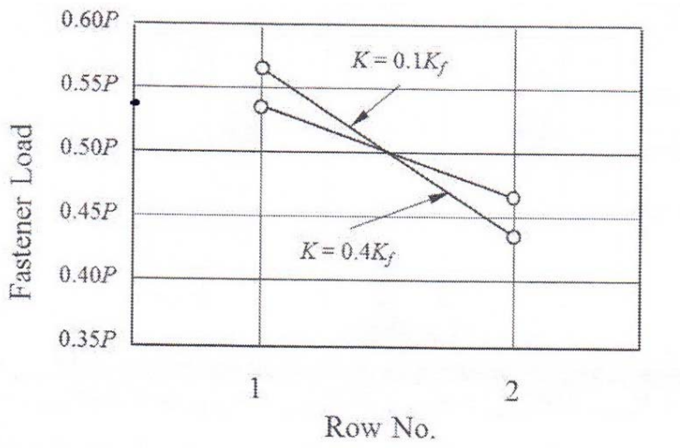
Number of Rows	Inner-to-Outer Plate Stiffness Ratio	Fastener Row Load Shear Expression RLF-Relative Load Fraction
3	K_{inner}=1.5 K_{outer}	$RFL_{R1}=-0.0278(K_t/K_p)^2+0.0194(K_t/K_p)+0.3128$
		$RFL_{R2}=0.0138(K_t/K_p)^2-0.1064(K_t/K_p)+0.3205$
		$RFL_{R3}=0.0139(K_t/K_p)^2-0.0869(K_t/K_p)+0.3776$



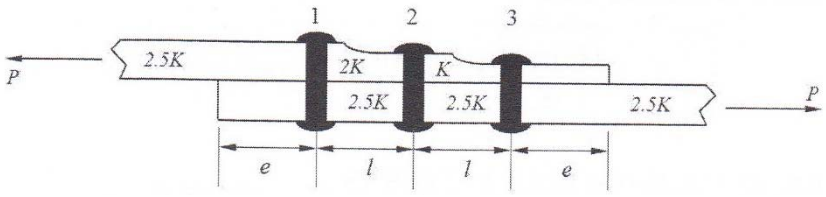


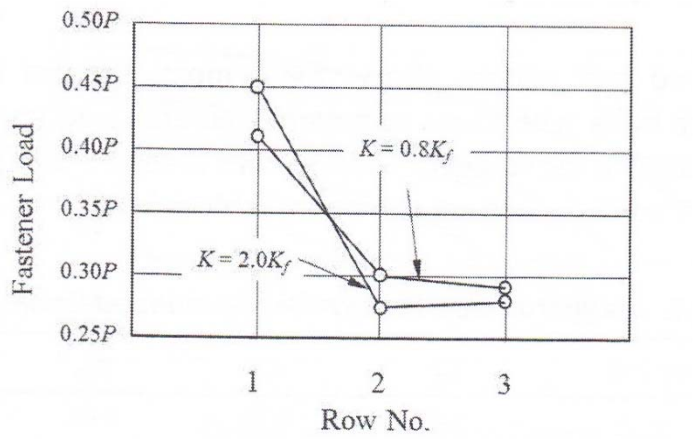
Number of Rows	Inner-to-Outer Plate Stiffness Ratio	Fastener Row Load Shear Expression RLF-Relative Load Fraction
5	K_{inner}=1.5 K_{outer}	$RFL_{R1}=-0.125(K_t/K_p)^2+0.2692(K_t/K_p)+0.1551$
		$RFL_{R2}=0.0417(K_t/K_p)^2-0.1142(K_t/K_p)+0.1618$
		$RFL_{R3}=0.1167(K_t/K_p)^2-0.2677(K_t/K_p)+0.2008$
		$RFL_{R4}=0.0972(K_t/K_p)^2-0.1953(K_t/K_p)+0.2383$
		$RFL_{R5}=0.1306(K_t/K_p)^2-0.3079(K_t/K_p)+0.2441$



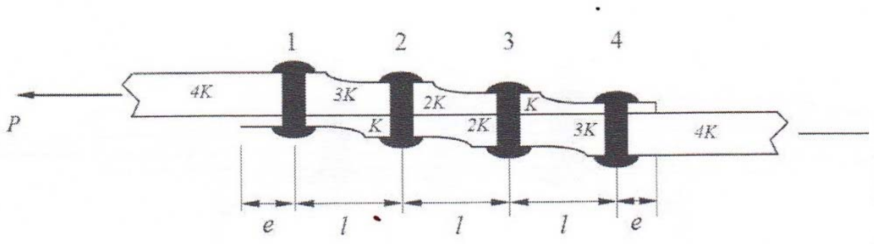


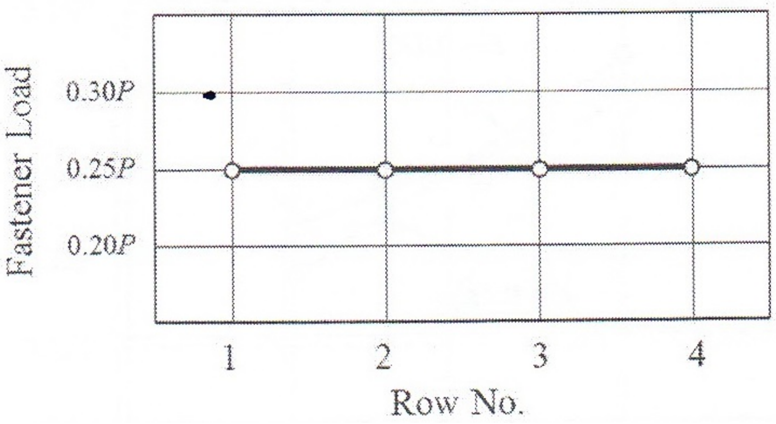
Number of Rows	Inner-to-Outer Plate Stiffness Ratio	Fastener Row Load Shear Expression RLF-Relative Load Fraction
2	$K_{inner}=K_{outer}$	$RFL_{R1}=0.0333(K_t/K_p)+0.5267$
	$K_{inner}=2/3\ K_{outer}$	$RFL_{R2}=-0.0333(K_t/K_p)+0.4733$





Number of Rows	Inner-to-Outer Plate Stiffness Ratio	Fastener Row Load Shear Expression RLF-Relative Load Fraction
3	$K_{inner}=K_{outer}$	$RFL_{R1}=-0.0148(K_t/K_p)^2+0.0852(K_t/K_p)+0.3430$
	$K_{inner}=4/5\ K_{outer}$	$RFL_{R2}=0.0116(K_t/K_p)^2-0.0638(K_t/K_p)+0.3471$
	$K_{inner}=2/5\ K_{outer}$	$RFL_{R3}=0.0032(K_t/K_p)^2-0.0213(K_t/K_p)+0.3039$

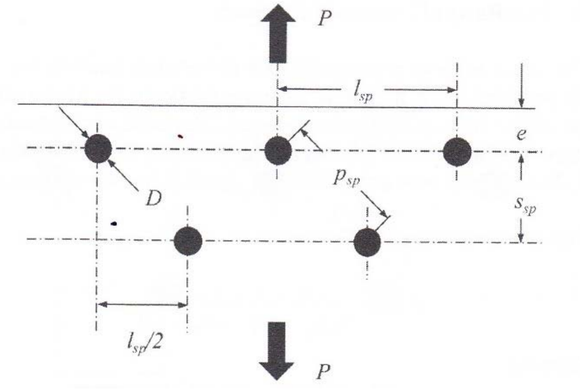




Number of Rows	Inner-to-Outer Plate Stiffness Ratio	Fastener Row Load Shear Expression RLF-Relative Load Fraction
5	4.5 $K_{inner}=K_{outer}$	$RFL_{R1}=RFL_{R2}=RFL_{R3}=RFL_{R4}=RFL_{R5}=0.2$
	4 $K_{inner}=K_{outer}$	
	1.5 $K_{inner}=K_{outer}$	
	2/3 $K_{inner}=K_{outer}$	
	$2/9K_{inner}=K_{outer}$	

Bolt Stager Patter Parameters

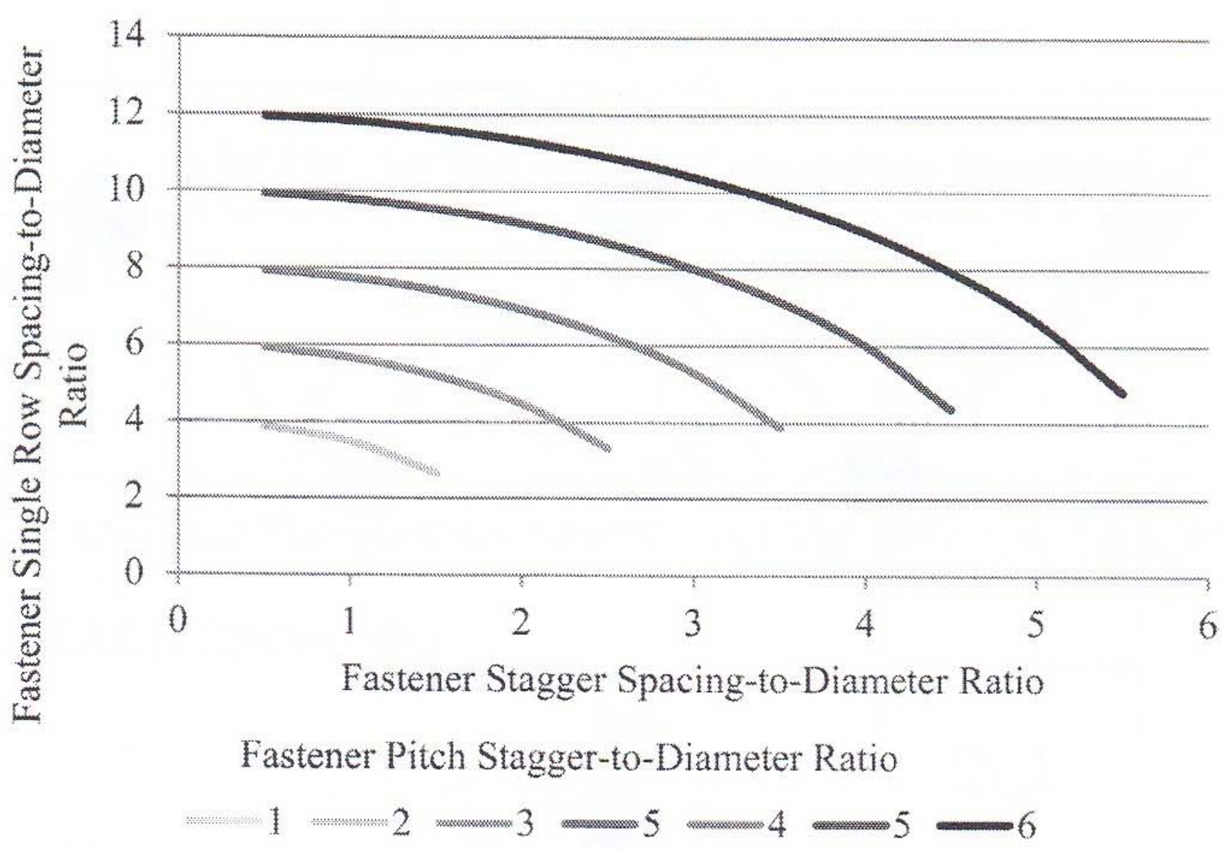
Fastener Diameter (in)	l_{sp} Single Row Spacing	s_{sp} Fastener Stager Spacing	p_{sp} Stager Distance	e
5/32	1.0	0.39	0.63	0.34
3/16	1.18	0.47	0.75	0.41
1/4	1.56	0.63	1.0	0.53
5/16	1.8	0.78	1.19	0.66
3/8	2.35	0.94	1.5	0.78



$$l_{sp} = 2\cdot\sqrt{\left[p_{sp}^2 - s_{sp}^2\right]}$$

$$\frac{l_{sp}}{D} = 2\cdot\sqrt{\left[\left(\frac{p_{sp}}{D}\right)^2 - \left(\frac{s_{sp}}{D}\right)^2\right]}$$

Stager Distance for a Single Row of Fasteners



Two Rows of Staggered Fasteners

Net Tension

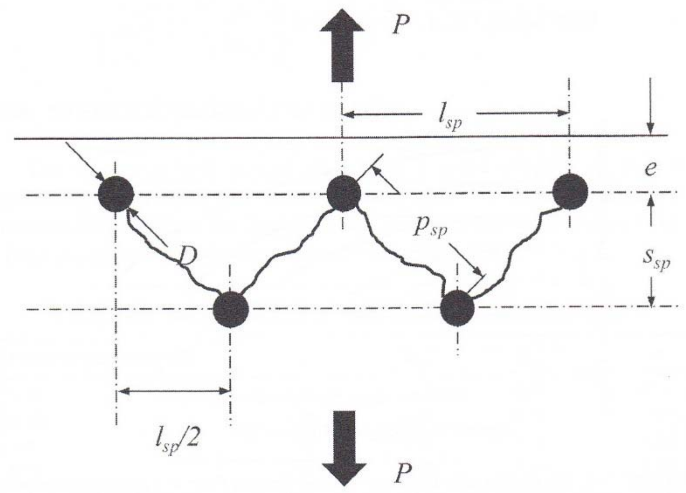
$$\sigma_{NT} = \frac{P}{2 \cdot t \cdot \left(p_{sp} - D\right)} = \frac{P}{t \cdot \left(l_{sp} - D\right)}$$

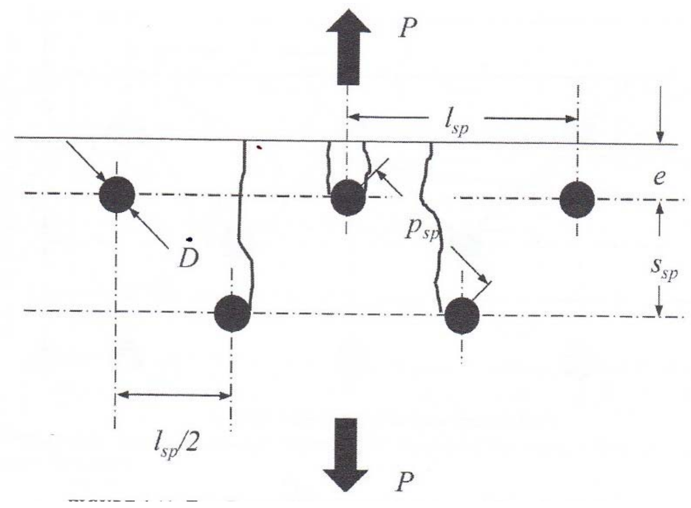
Bearing

$$\sigma_{BR} = \frac{F}{2 \cdot t \cdot D}$$

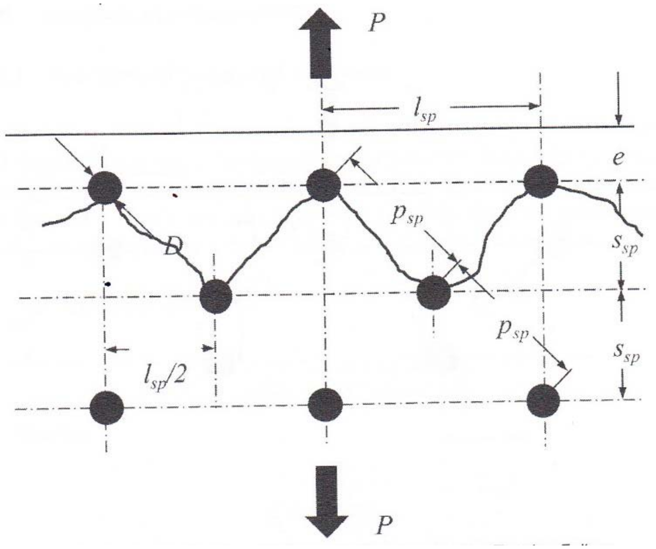
Shear Out

$$\tau_{SO} = \frac{F}{2 \cdot \left(2 \cdot e + s_{sp}\right)}$$





Three Rows of Staggered Fasteners

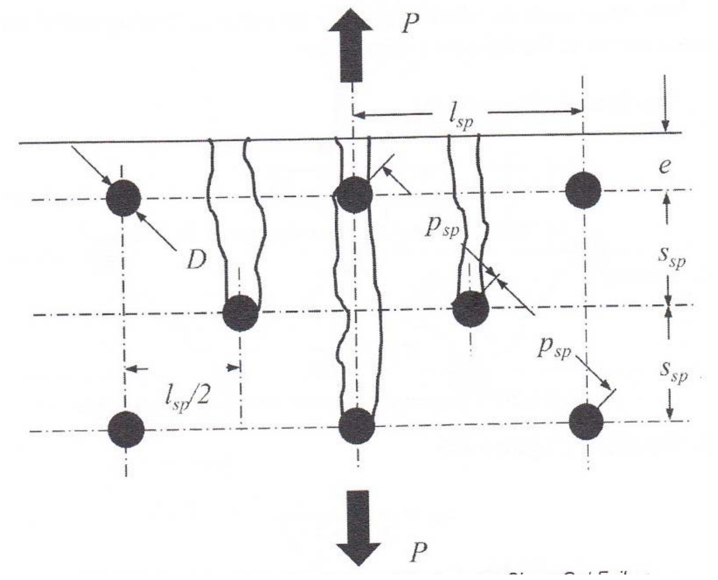


Net Tension

$$\sigma_{NT} = \frac{P}{2 \cdot t \cdot \left(p_{sp} - D\right)}$$

Bearing

$$\sigma_{BR} = \frac{F}{3 \cdot t \cdot D}$$



Shear Out

$$\tau_{SO} = \frac{F}{2 \cdot \left(2 \cdot e + 3 \cdot s_{sp}\right)}$$

Questions and Discussion