

Introduction to Isomers and Linkage Synthesis

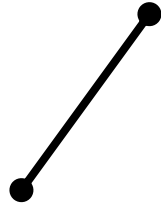
- Linkage Synthesis Overview
- Isomers

Number Synthesis

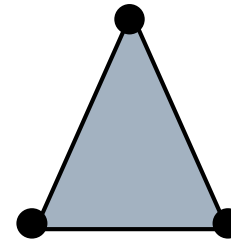
- Determining the number and order of links and joints necessary to produce motion of a particular mobility

- Link order
 - Number of Binary (B), Ternary (T), Quaternary (Q), Pentagonal (P), Hexagonal (H) links
 - $L = B + T + Q + P + H$
≡ total number of links

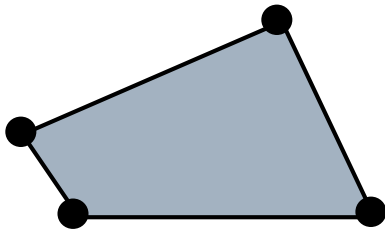
Link Order



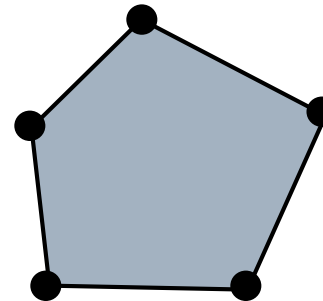
Binary



Ternary



Quaternary



Pentagonal

Isomer Joint Type Nomenclature

- Dashed Line: Imaginary Link
- R: Revolute Joint
 - Pin Joint
 - Hinged connection
- P/T: Lower Pair Sliding Joint
 - Prismatic Joint
 - Relative Translational Motion
- C: Center of Curvature
- B: Base Point

Grübler/Kutzbach Criterion

$$M = 3 \cdot (L - 1) - 2 \cdot j_1 \quad (\text{Grübler})$$

$$M = 3 \cdot (L - 1) - 2 \cdot j_1 - j_2 \quad (\text{Kutzbach})$$

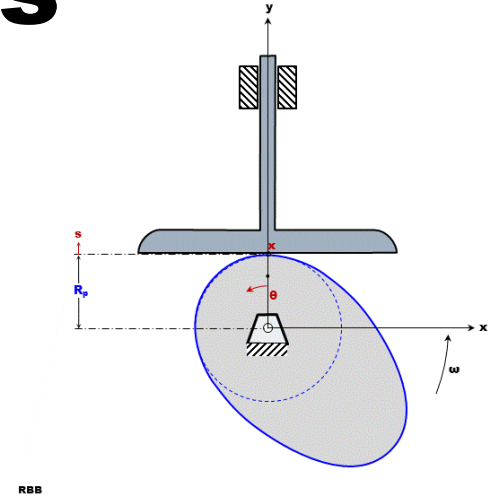
- **M=1**
 - **Mechanism can be driven by a single input direction**
- **M=2**
 - **Two separate input motions are necessary to produce constrained motion for the mechanism**
 - **Differential Mechanism**
- **M=0**
 - **Motion is impossible and the mechanism is a structure**
 - **Exact Constraint**
- **M=-1**
 - **Redundant constraint**
 - **Pre-Load**

Number Synthesis Uses Gröbler's Equation

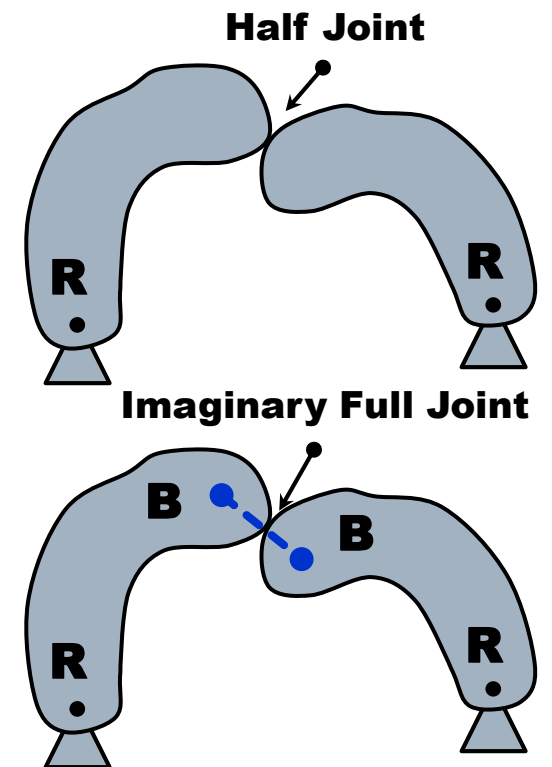
- Gröbler's Criterion: $M=3(L-1)-2J$
 - A half joint is just a full joint with an imaginary link through the base points
 - Base points are the location of the center of curvatures

- For Linkages with Full Joints
 - $L=B+T+Q+P+H+\dots$
 - $L-(M+3)=T+2Q+3P+4H+\dots$

- DoF Must Be Uniformly Distributed

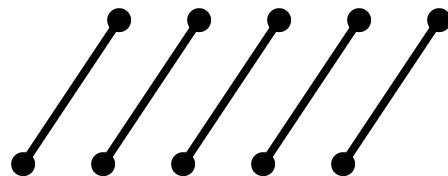


RBB

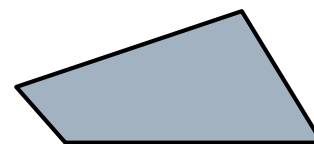


Uniformly Distributed DoF

Basic Links
To Work With

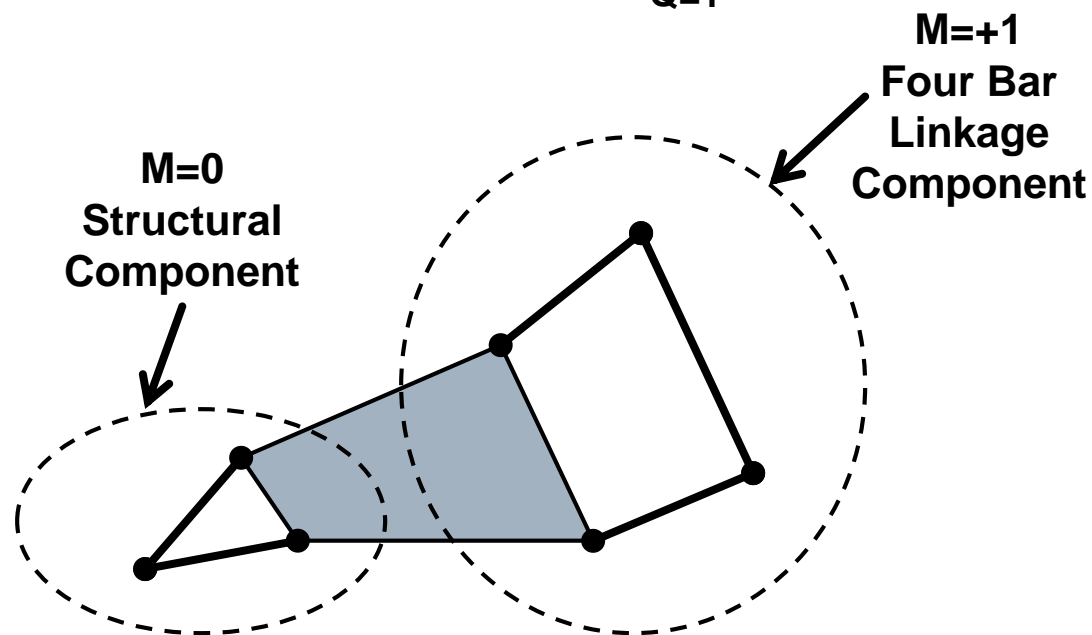


B=5



Q=1

Isomers of this link
combination
contribute nothing
new for M=+1
synthesis use



Implication of Grübler's Equation

$$J = \frac{3}{2} \cdot L - \frac{(M + 3)}{2}$$

- If all joints are full joints
 - An **ODD** number of **DOF** (M) requires an **EVEN** number of links (L)
 - An **EVEN** number of **DOF** (M) requires an **ODD** number of links (L)

Determining All Possible Combinations of Links for a DOF

□ Total Number of Links in a Mechanism

$$L = B + T + Q + P + \dots$$

□ Total Number of Joints in a Mechanism

- A node is a location in a link that can be used for a joint
- 2 nodes are needed to make one joint

$$J = \frac{\text{total nodes}}{2} = \frac{2 \cdot B + 3 \cdot T + 4 \cdot Q + 5 \cdot P + \dots}{2}$$

Equations Relevant For Number Synthesis

- Simultaneous Progressive Solution for up to Pentagonal Links

$$L = B + T + Q + P$$

$$L - (M + 3) = T + 2 \cdot Q + 3 \cdot P$$

- Total Number of Joints

$$J = \frac{3}{2} \cdot L - \frac{(M + 3)}{2}$$

Consider $M=+1$

N must be even: 0, 2

□ $M=+1, L=0 \Rightarrow$ Not Valid

■ $0 = B + T + Q + P$

Can Not have 0 Link Mechanism

$T + 2 \cdot Q + 3 \cdot P = -4$

T, Q, P can not be negative

□ $M=+1, L=2 \Rightarrow$ Not Valid

■ $2 = B + T + Q + P$

$T + 2 \cdot Q + 3 \cdot P = -3$

T, Q, P can not be negative

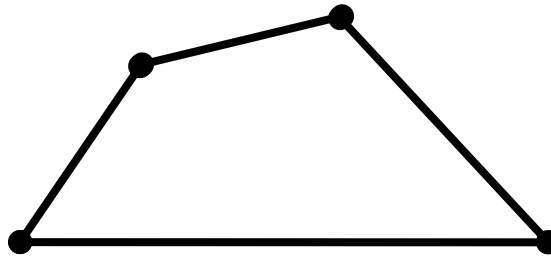
Considering $M=+1$ Continued N must be even: 4

□ $M=+1, L=4$

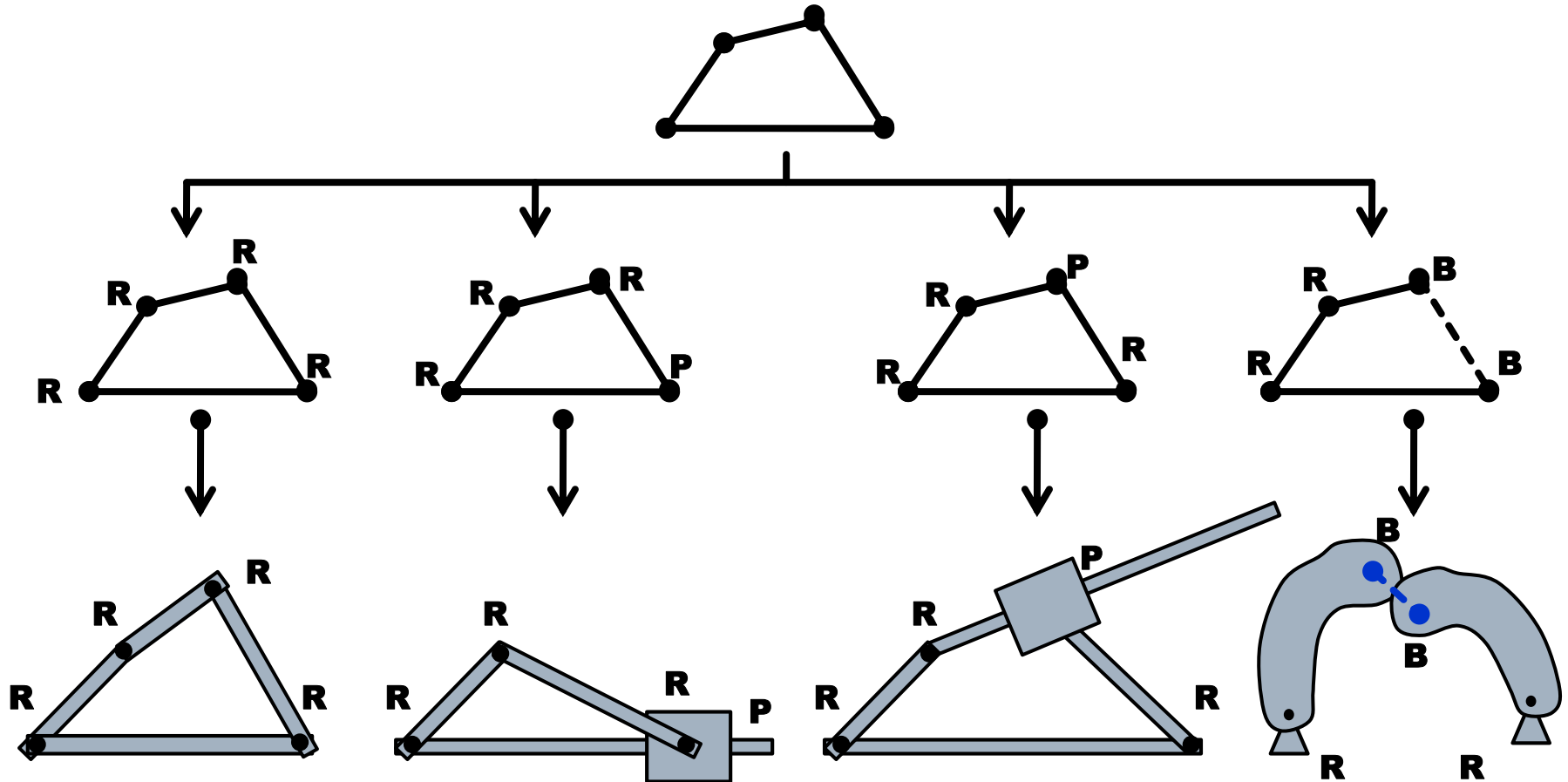
■ $4 = B + T + Q + P \Rightarrow 4 = B$

$T + 2 \cdot Q + 3 \cdot P = 0 \quad T=Q=P=0$

■ $J = \frac{3}{2} \cdot 4 - \frac{(1+3)}{2} = 4$



Synthesis of Mechanical Devices From The Isomer



Considering $M=+1$ Continued L must be even: 6

□ $M=+1, L=6$

■ $L = B + T + Q + P \quad \Rightarrow \quad 6 = B + T + Q + P$

$T + 2 \cdot Q + 3 \cdot P = L - (M + 3) \quad \Rightarrow \quad \boxed{T + 2 \cdot Q + 3 \cdot P = 2}$

■ From the second equation $P=0$

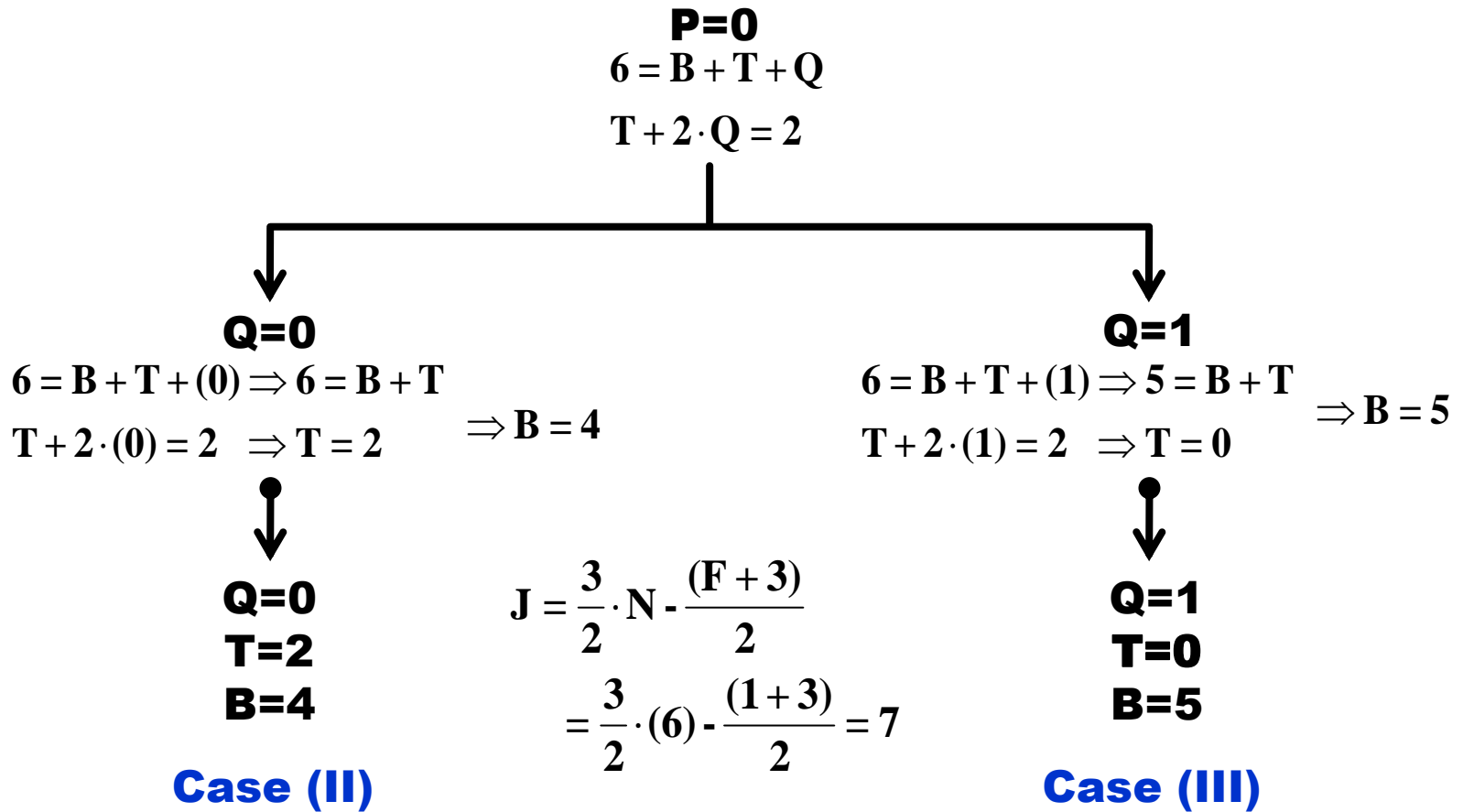
□ T and Q are positive integers

□ The equations reduce to

$$6 = B + T + Q$$

$$T + 2 \cdot Q = 2$$

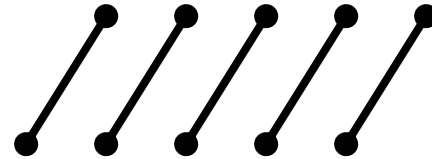
Number Synthesis for M=+1, L=6, P=0



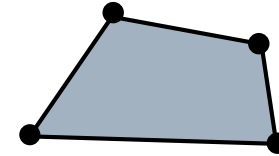
Case III – No Acceptable Isomers

$M=+1$, $L=6$, $P=0$, $Q=1$, $T=0$, $B=5$

Basic Links
To Work With

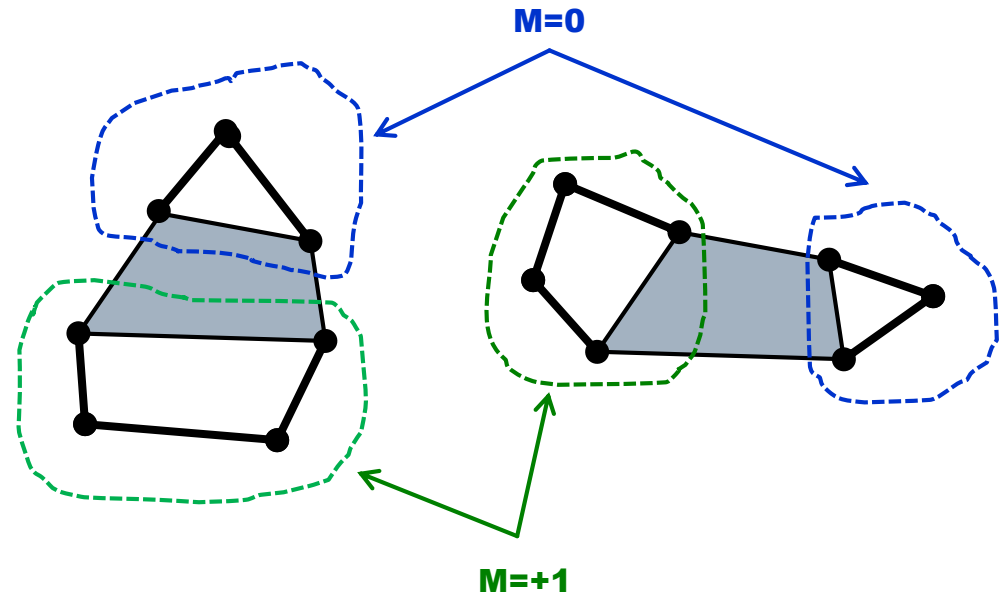


$B=5$



$Q=1$

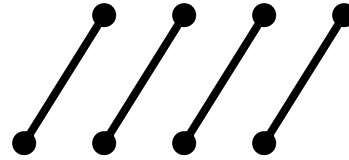
Can Not Connect
Fifth Link and Have
Uniformly Distributed
 $M=+1$



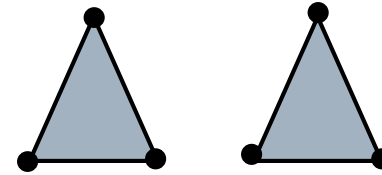
Case II – 2 Acceptable Isomers

M=+1, L=6, P=0, Q=0, T=2, B=4

Basic Links
To Work With

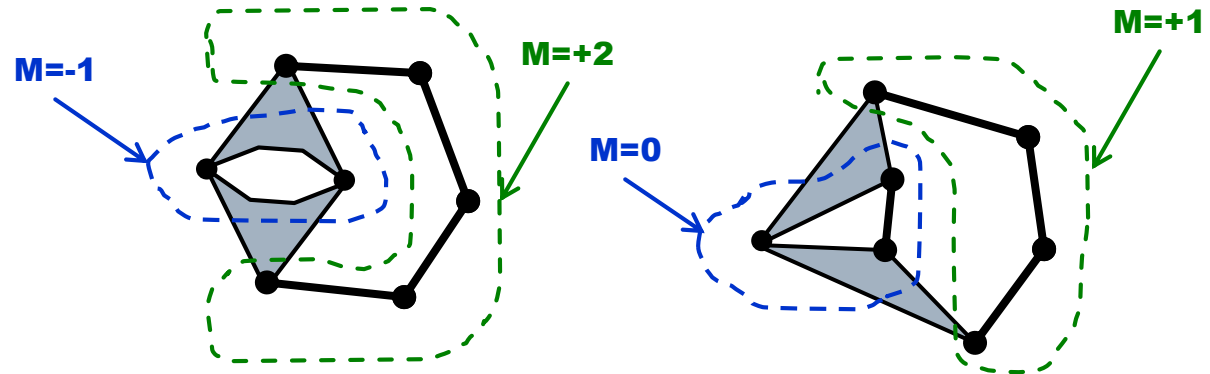


B=4

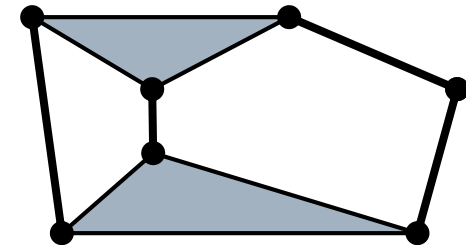
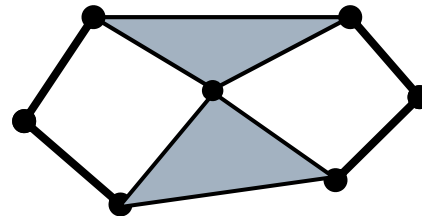


T=2

Unacceptable
Isomers for M=+1



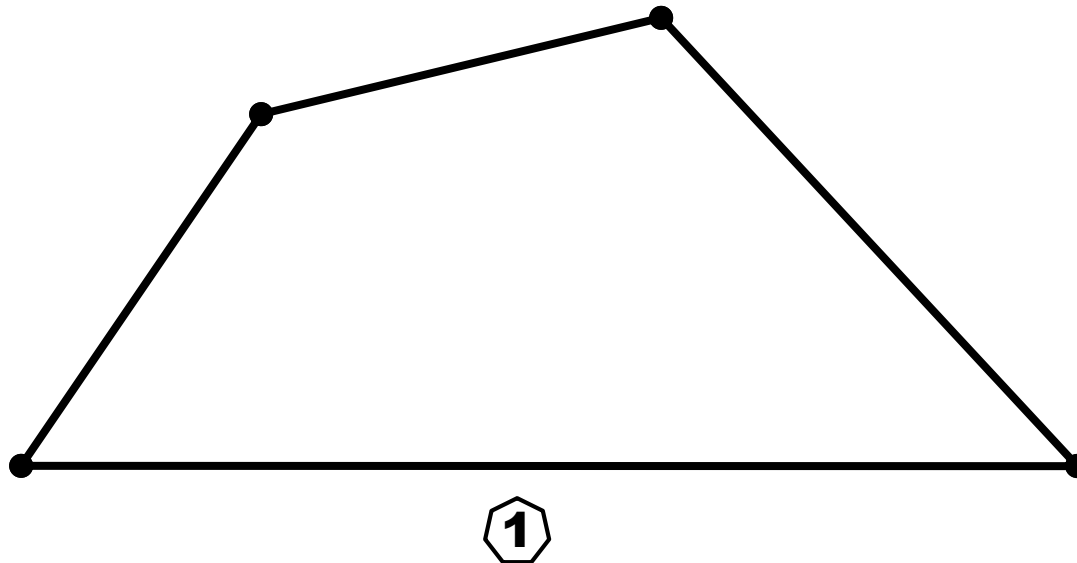
Acceptable
Isomers for M=+1



Link Combinations for Single Pin-Jointed Plane Linkages, $M=+1$

M	L	B	T	Q	P	Designation
+1	4	4	0	0	0	I
	6	4	2	0	0	II
	6	5	0	1	0	III
	8	4	4	0	0	IV
	8	5	2	1	0	V
	8	6	0	2	0	VI
	8	6	1	0	1	VII

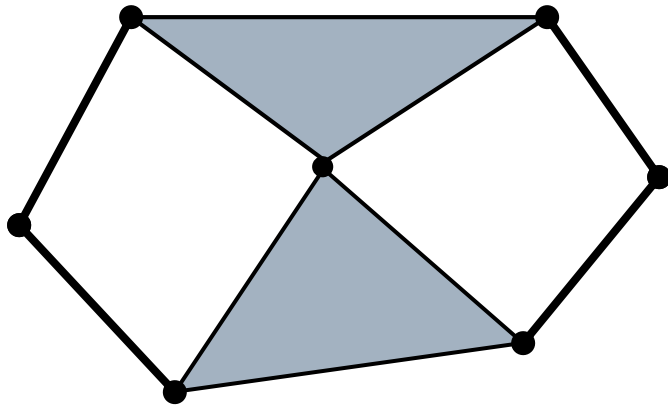
M	L	B	T	Q	P	Designation
+1	4	4	0	0	0	I



M	L	B	T	Q	P	Designation
+1	6	4	2	0	0	II

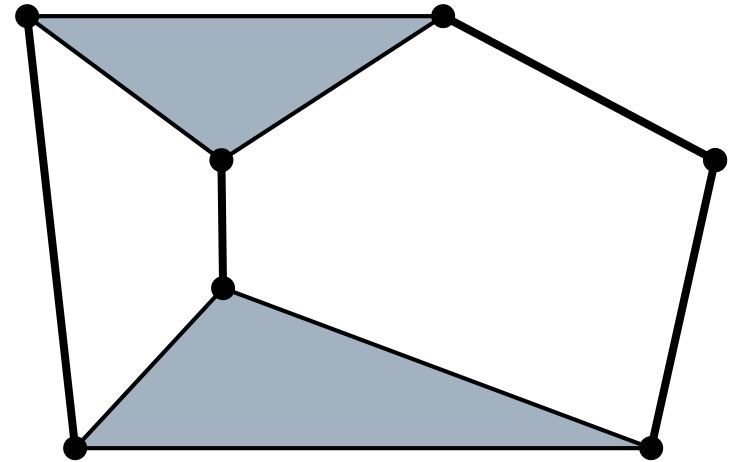
MER312: Dynamics of Mechanisms (RBB)

Watt's Linkage



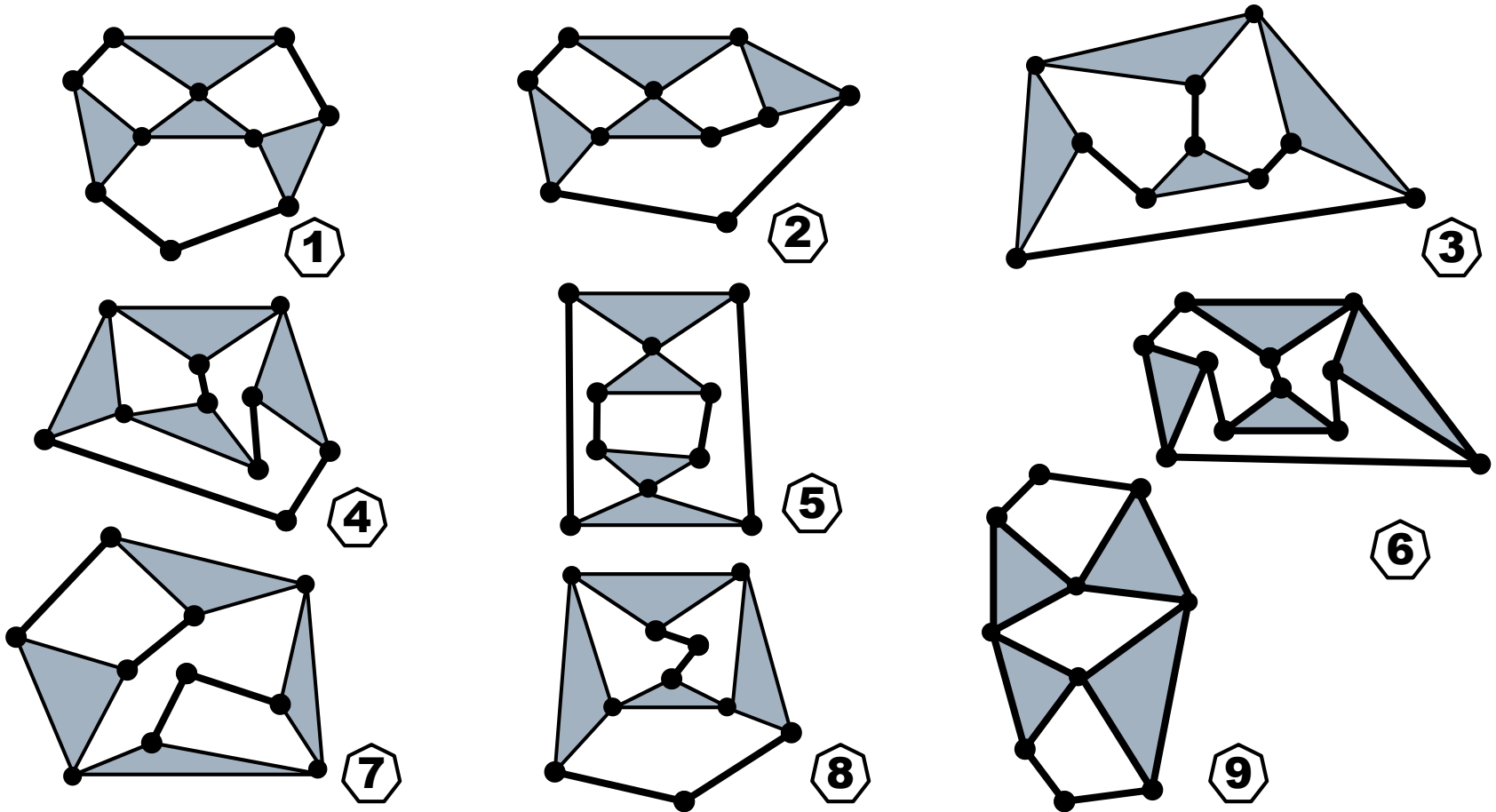
1

Stephenson's Linkage

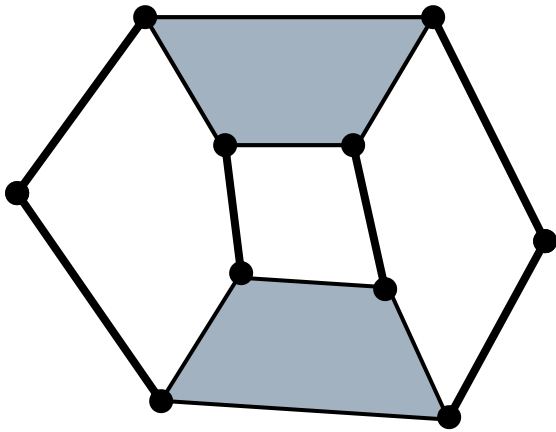


2

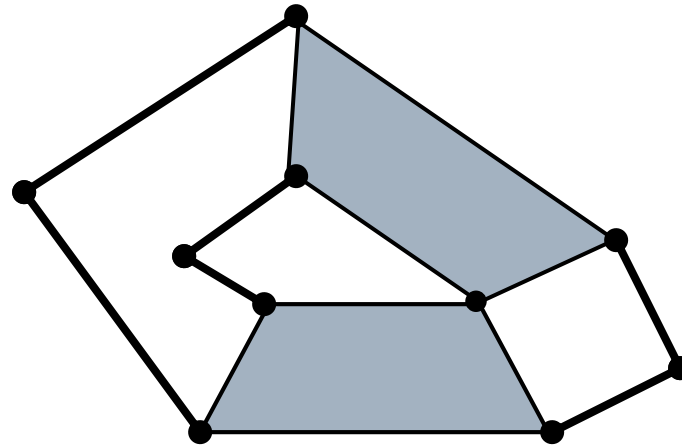
M	L	B	T	Q	P	Designation
+1	8	4	4	0	0	IV



M	L	B	T	Q	P	Designation
+1	8	6	0	2	0	VI



1

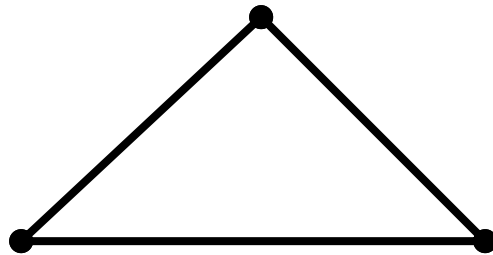


2

Link Combinations for Single Pin-Jointed Plane Linkages, $M=0$

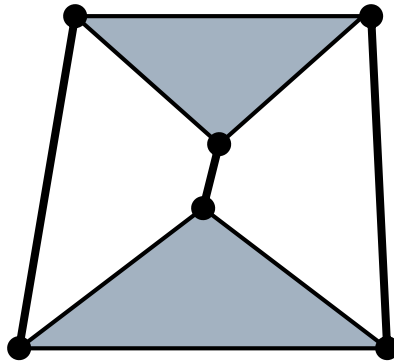
M	L	B	T	Q	P	Designation
0	3	3	0	0	0	VIII
	5	3	2	0	0	IX
	5	4	0	1	0	X
	7	5	1	0	1	XI
	7	3	4	0	0	XII
	7	4	2	1	0	XIII
	7	5	0	2	0	XIV

M	L	B	T	Q	P	Designation
0	3	3	0	0	0	VIII

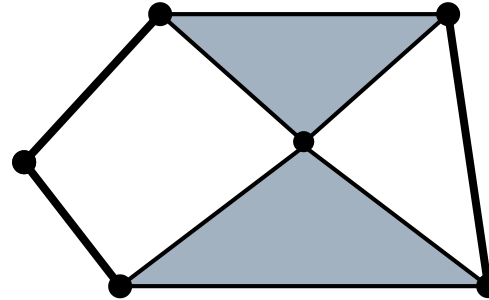


1

M	L	B	T	Q	P	Designation
0	5	3	2	0	0	IX

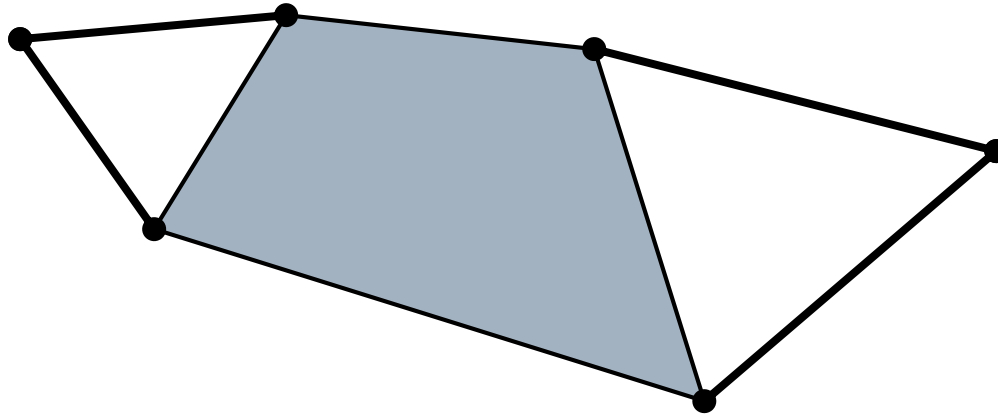


1



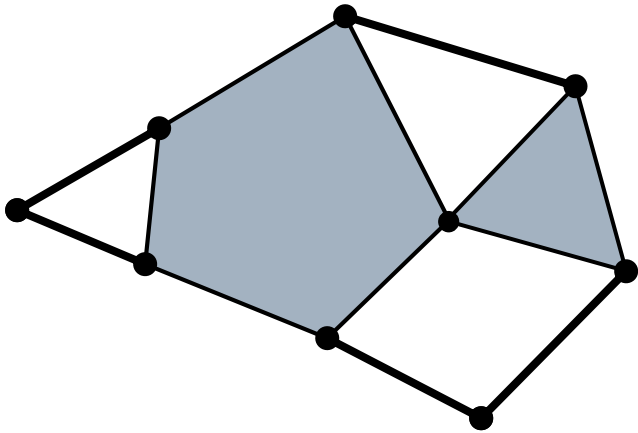
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M	L	B	T	Q	P	Designation
0	5	4	0	1	0	X

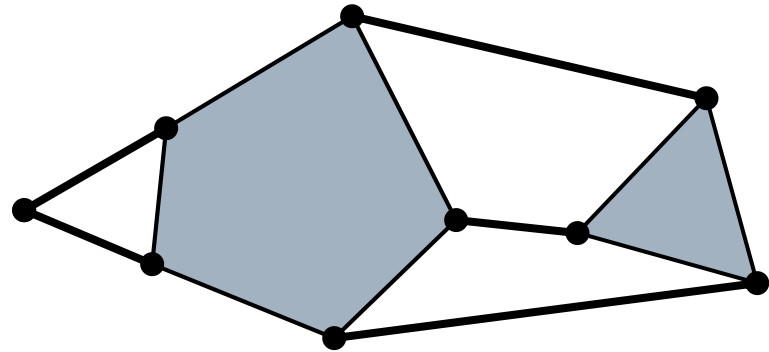


1

M	L	B	T	Q	P	Designation
0	7	5	1	0	1	XI

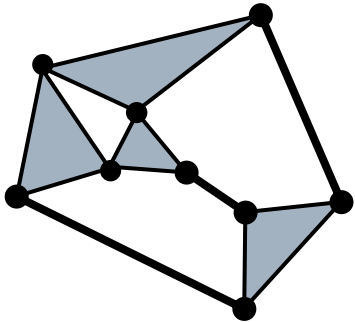


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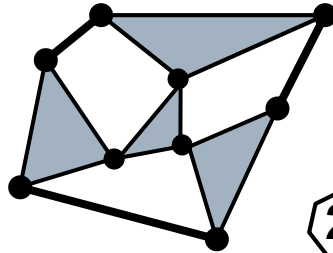


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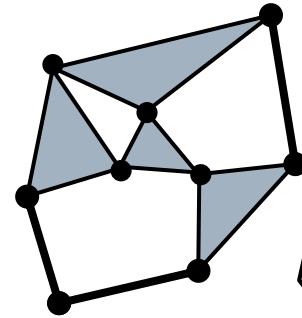
M	L	B	T	Q	P	Designation
0	7	3	4	0	0	XII



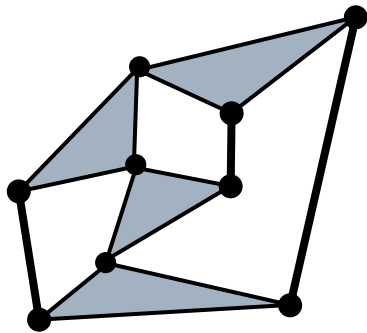
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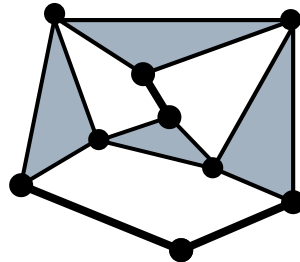
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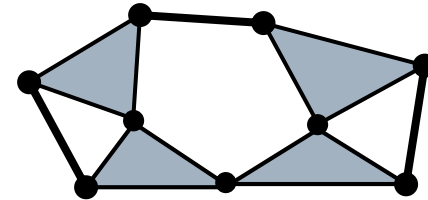
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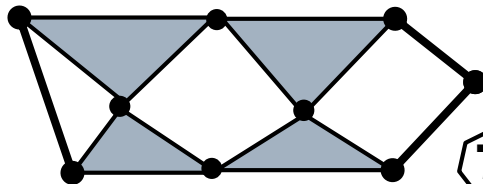
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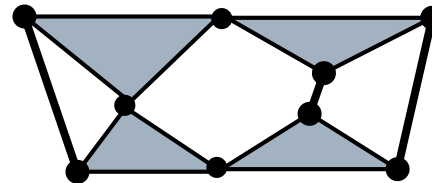
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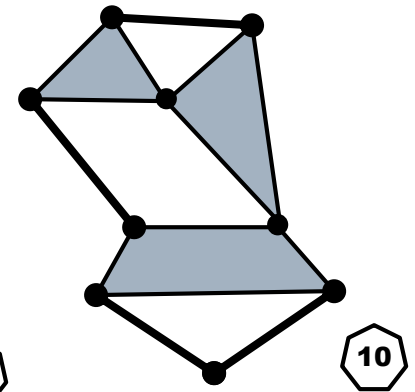
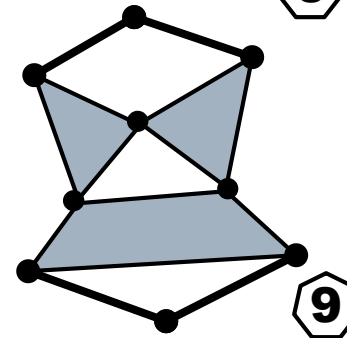
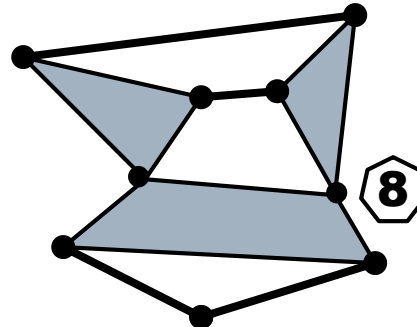
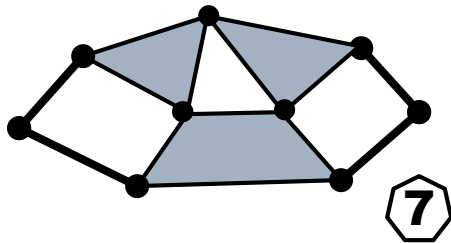
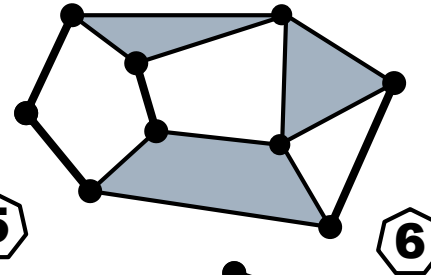
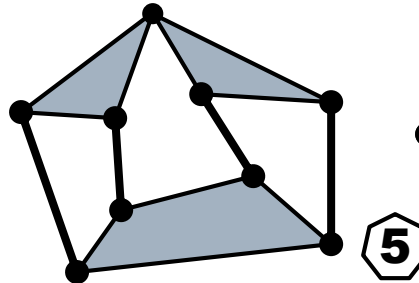
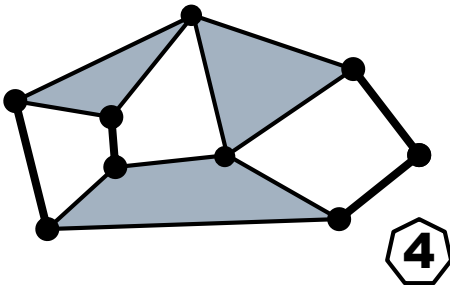
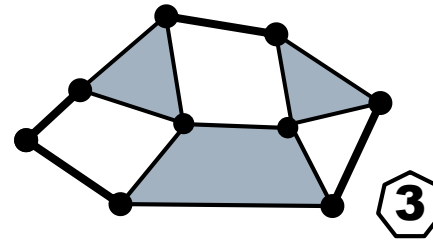
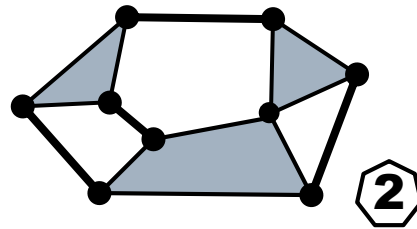
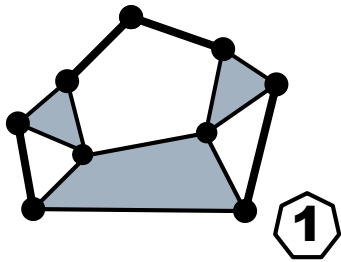


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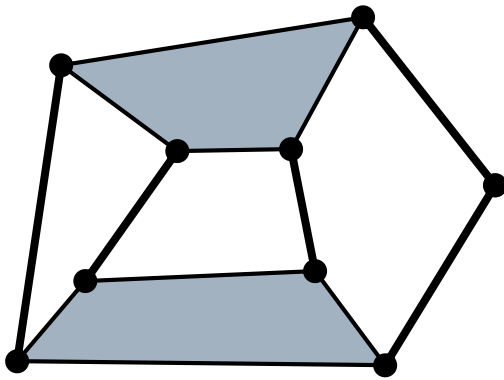


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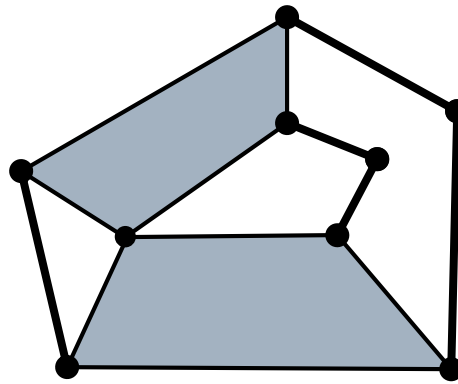
M	L	B	T	Q	P	Designation
0	7	4	2	1	0	XIII



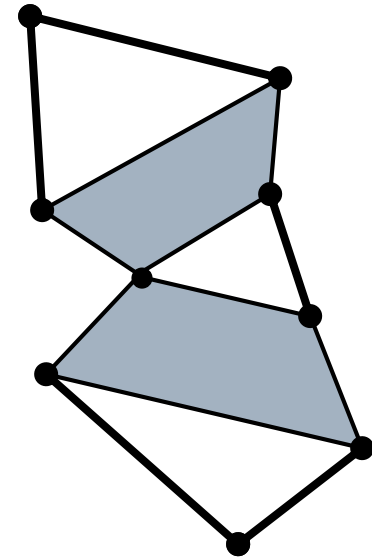
M	L	B	T	Q	P	Designation
0	7	5	0	2	0	XIV



1



2



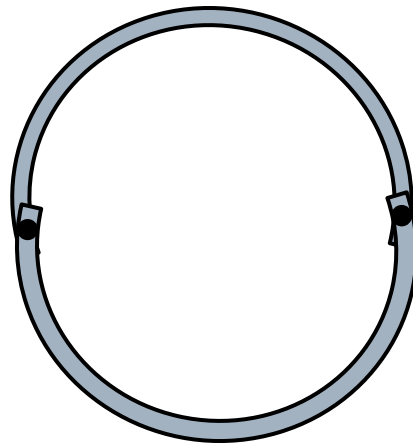
3

Link Combinations for Single Pin-Jointed Plane Linkages, $M=-1$

M	L	B	T	Q	P	Designation
-1	2	2	0	0	0	XV
	4	2	2	0	0	XVI
	4	3	0	1	0	XVII
	6	4	1	0	1	XVIII
	6	4	0	2	0	XIX
	6	3	2	1	0	XX
	6	2	4	0	0	XXI

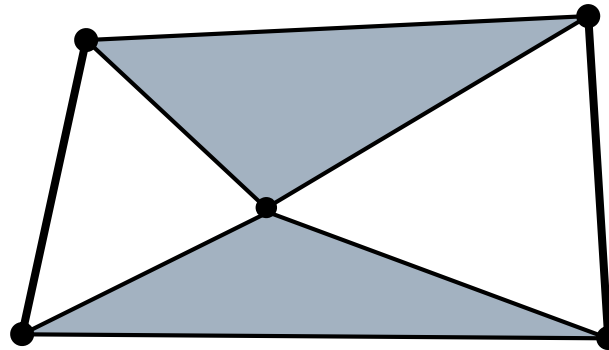
M	L	B	T	Q	P	Designation
-1	8	6	0	0	2	XXII
	8	5	1	1	1	XXIII
	8	4	3	0	1	XXIV
	8	5	0	3	0	XXV
	8	4	2	2	0	XXVI
	8	3	4	1	0	XXVII
	8	2	6	0	0	XXVIII

M	L	B	T	Q	P	Designation
-1	2	2	0	0	0	XV



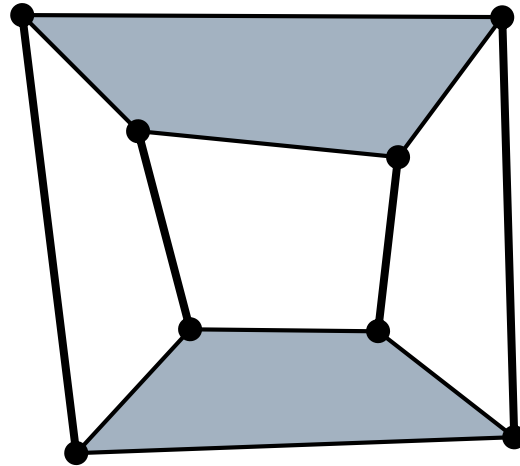
1

M	L	B	T	Q	P	Designation
-1	4	2	2	0	0	XVI



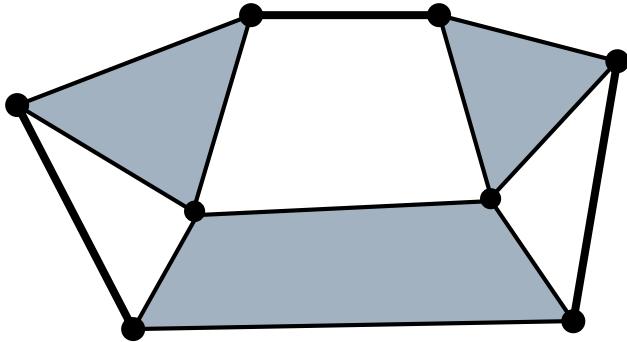
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M	L	B	T	Q	P	Designation
-1	6	4	0	2	0	XIX

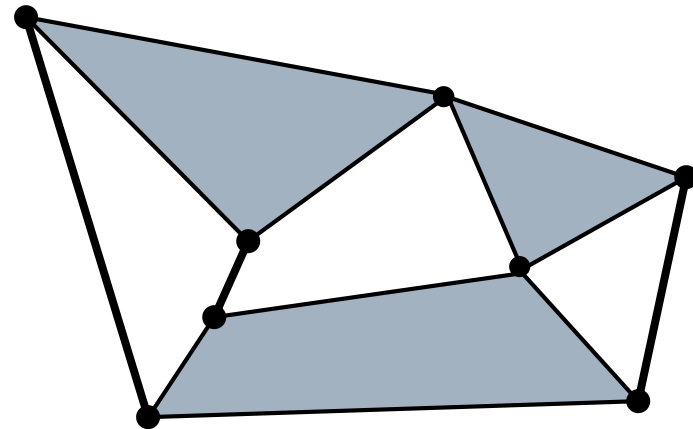


1

M	L	B	T	Q	P	Designation
-1	6	3	2	1	0	XX

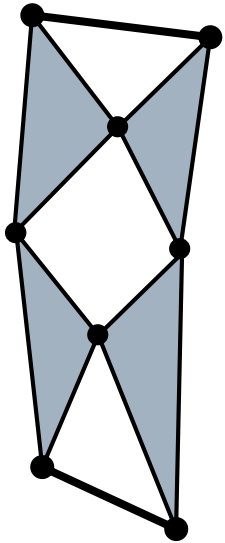


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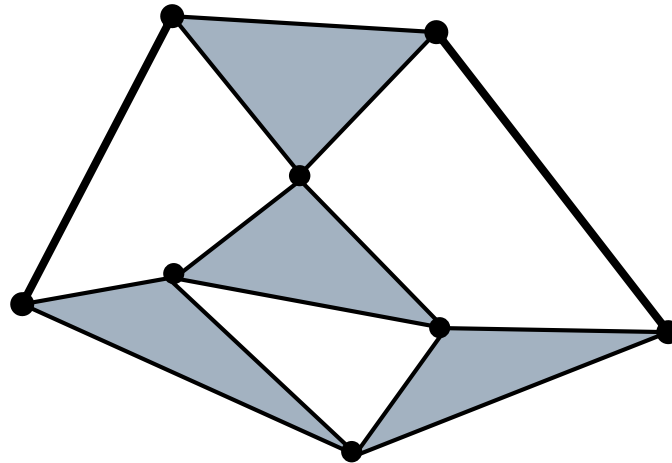


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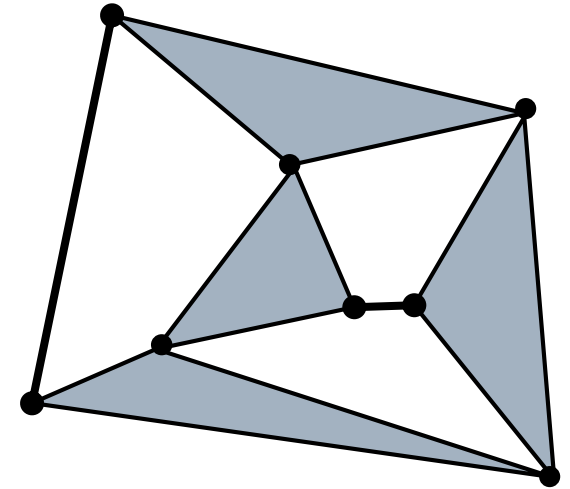
M	L	B	T	Q	P	Designation
-1	6	2	4	0	0	XXI



1

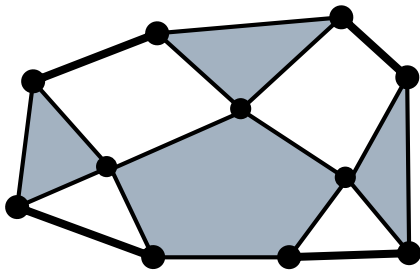


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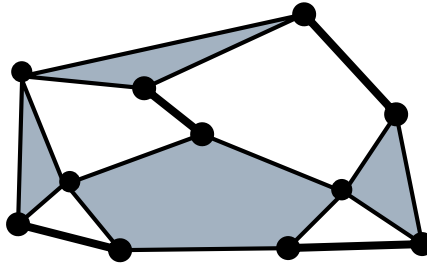


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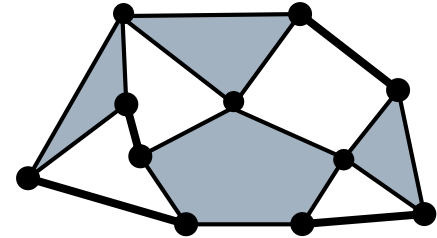
M	L	B	T	Q	P	Designation
-1	8	4	3	0	1	XXIV



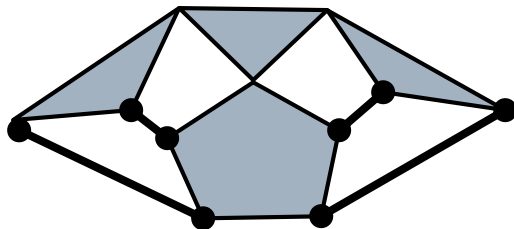
1



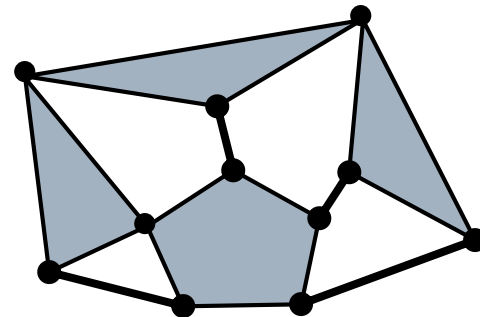
2



3

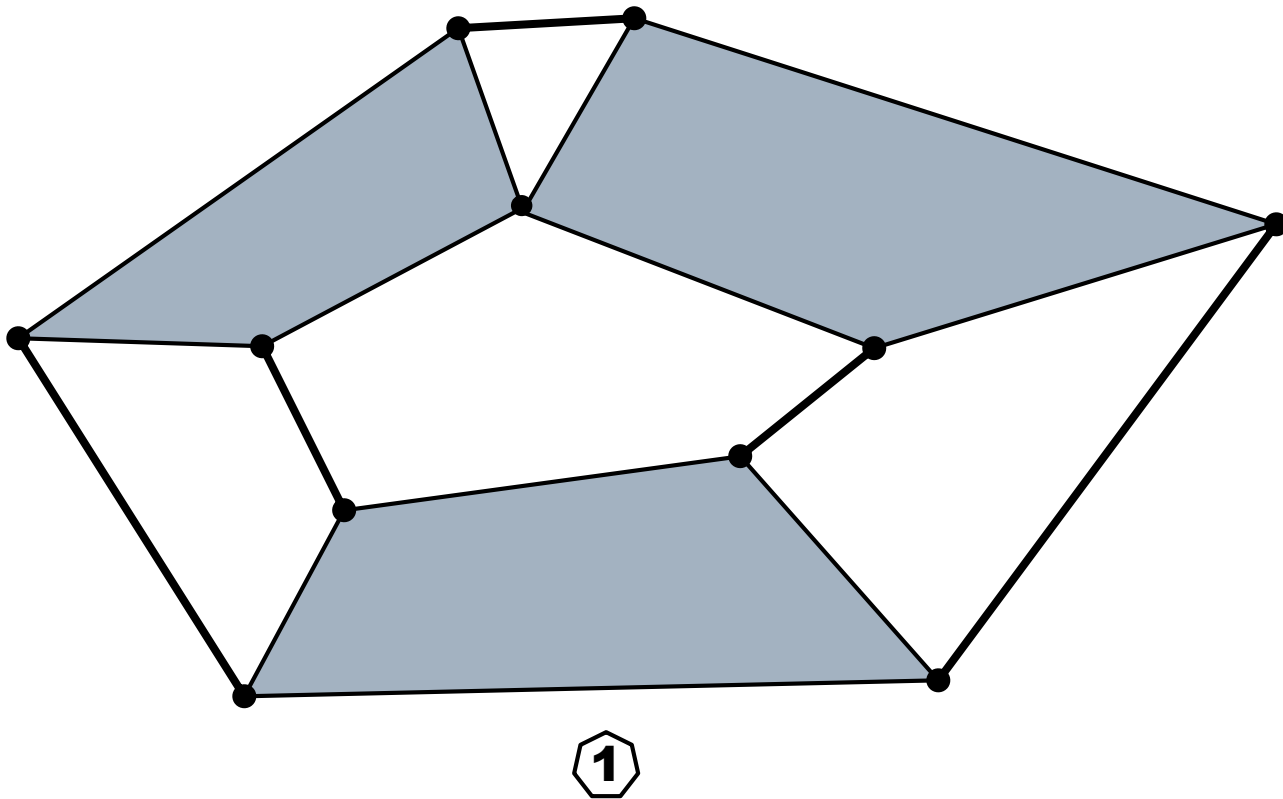


4



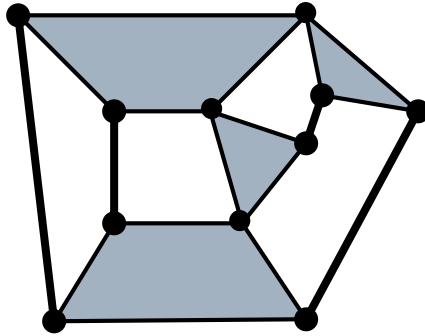
5

M	L	B	T	Q	P	Designation
-1	8	5	0	3	0	XXV

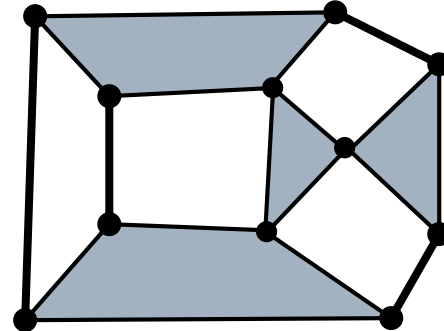


M	L	B	T	Q	P	Designation
-1	8	4	2	2	0	XXVI

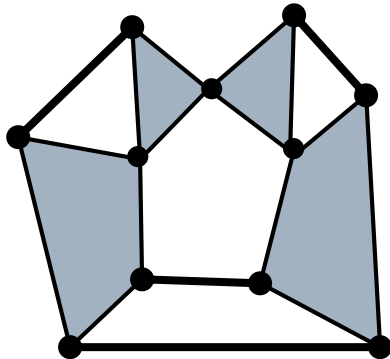
(Plus 11 more not shown)



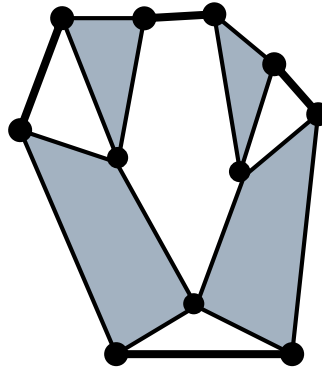
1



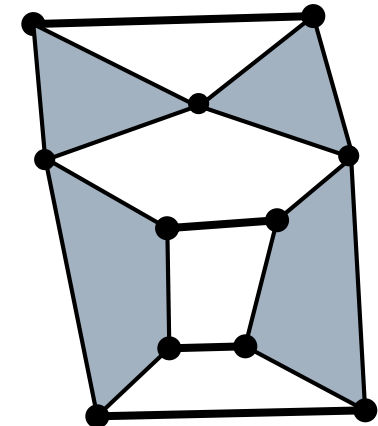
2



3



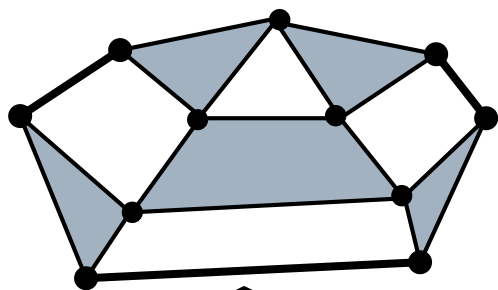
4



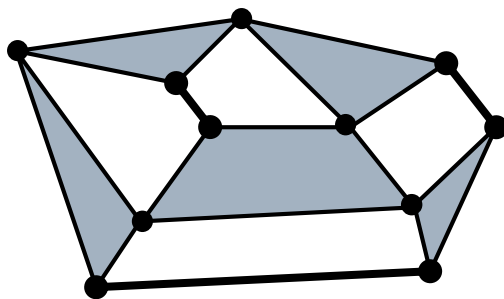
5

M	L	B	T	Q	P	Designation
-1	8	3	4	1	0	XXVII

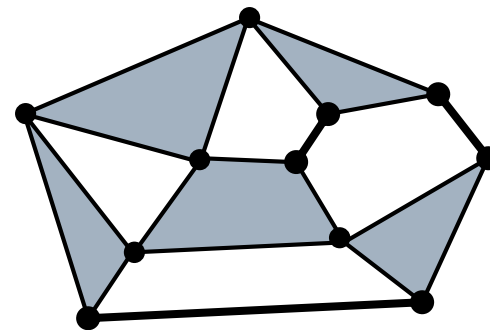
(Plus 16 more not shown)



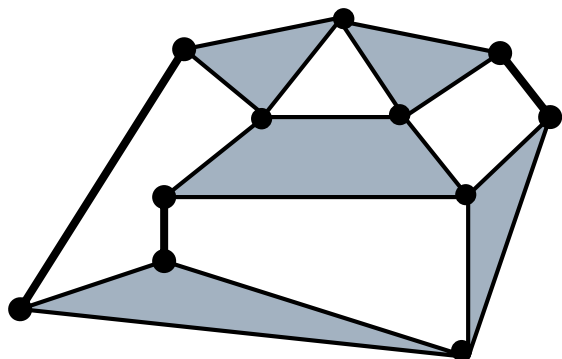
1



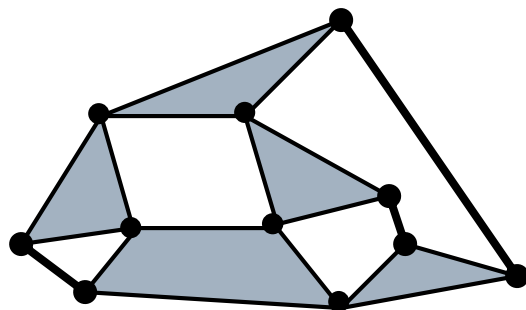
2



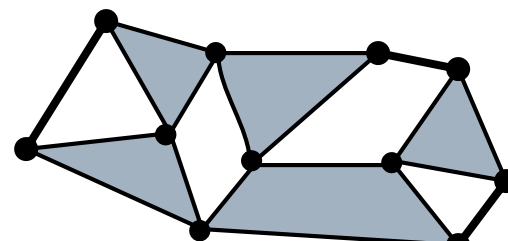
3



4



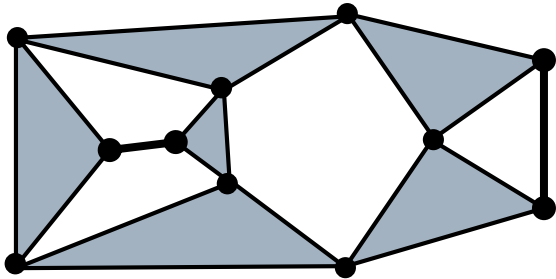
5



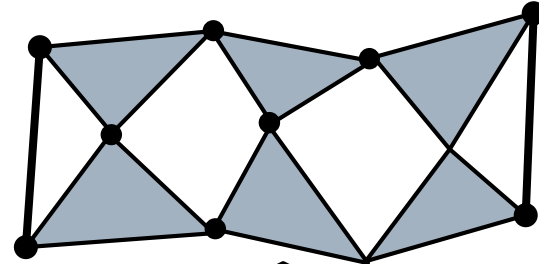
6

M	L	B	T	Q	P	Designation
-1	8	2	6	0	0	XXVIII

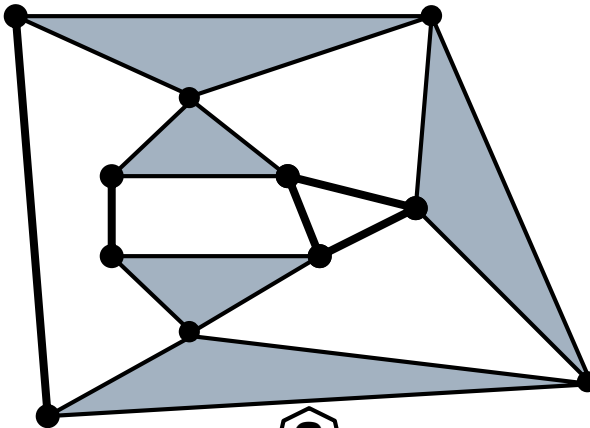
(Plus 11 more not shown)



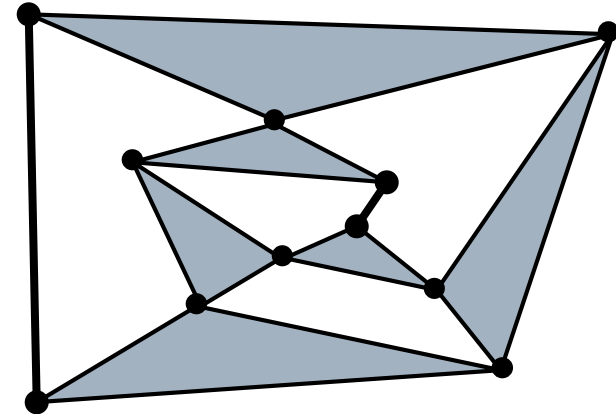
1



2



3

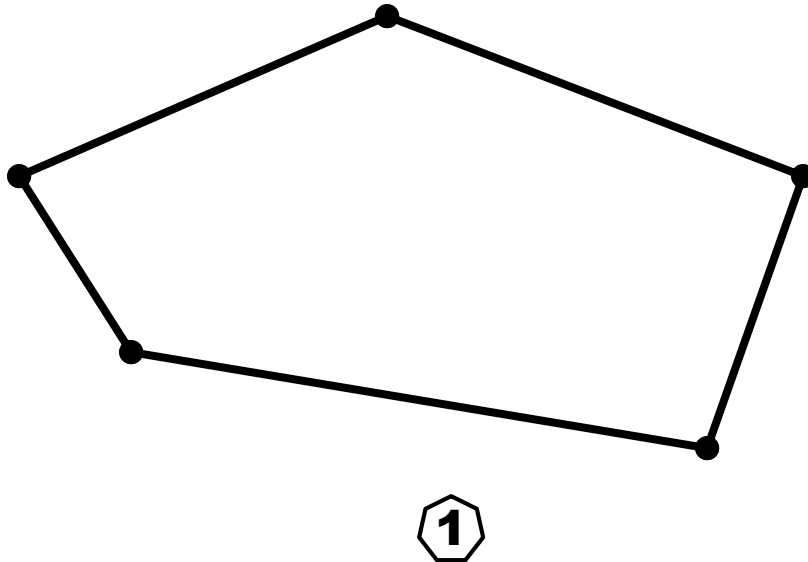


4

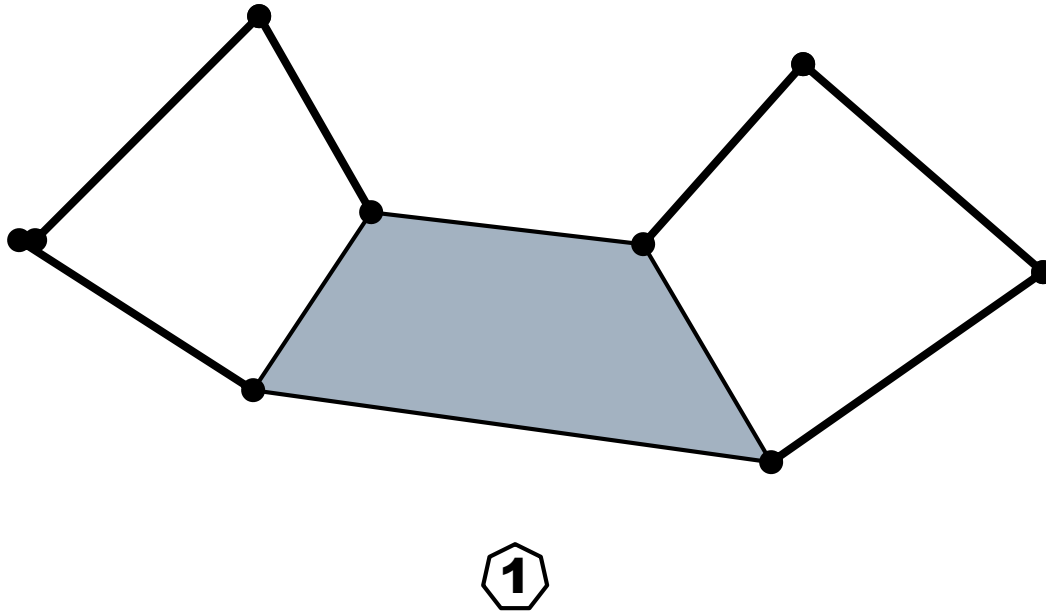
Link Combinations for Single Pin-Jointed Plane Linkages, $M=+2$

M	L	B	T	Q	P	Designation
2	5	5	0	0	0	XXIX
	7	6	0	1	0	XXX
	7	5	2	0	0	XXXI
	9	7	1	0	1	XXXII
	9	7	0	2	0	XXXIII
	9	6	2	1	0	XXXIV
	9	5	4	0	0	XXXV

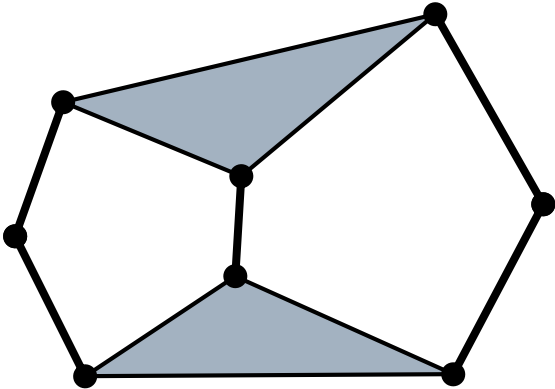
M	L	B	T	Q	P	Designation
2	5	5	0	0	0	XXIX



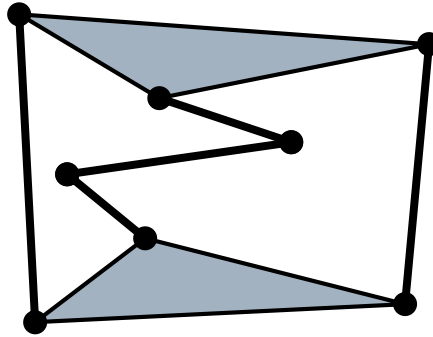
M	L	B	T	Q	P	Designation
2	7	6	0	1	0	XXX



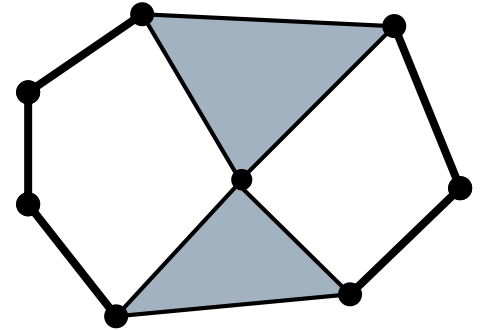
M	L	B	T	Q	P	Designation
2	7	5	2	0	0	XXXI



1

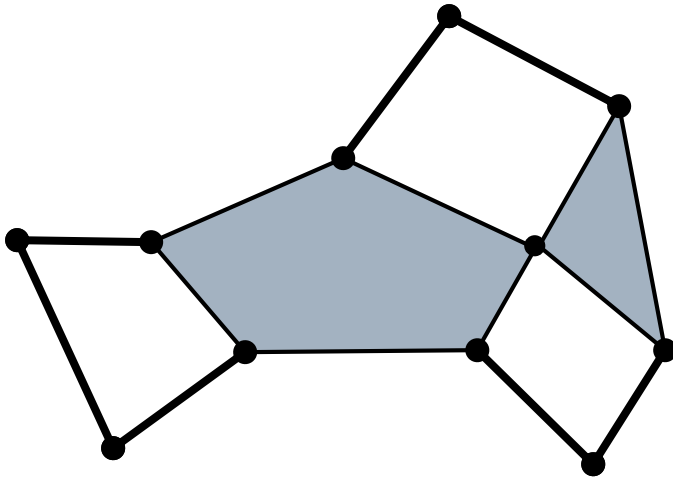


2

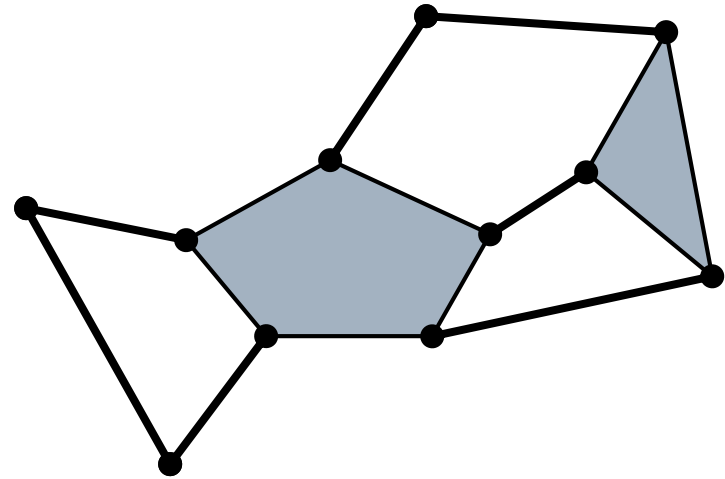


3

M	L	B	T	Q	P	Designation
2	9	7	1	0	1	XXXII

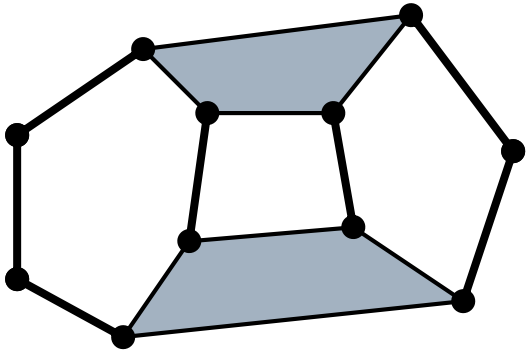


1

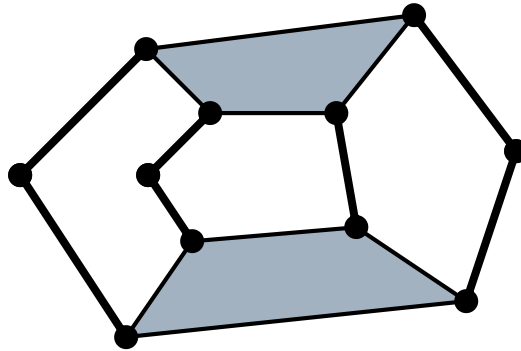


2

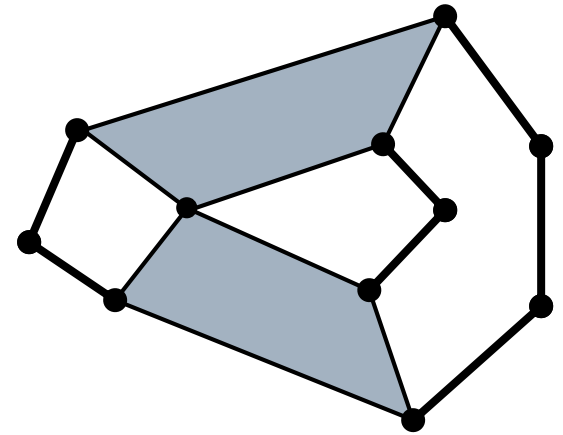
M	L	B	T	Q	P	Designation
2	9	7	0	2	0	XXXIII



1



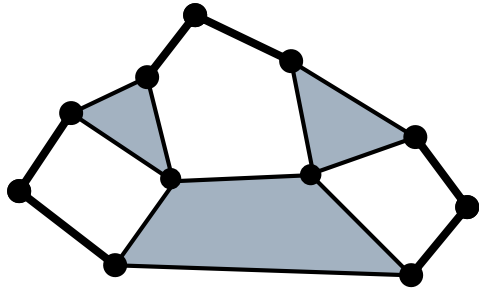
2



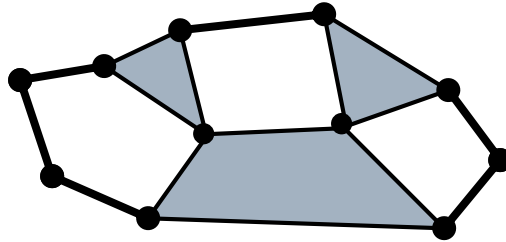
3

M	L	B	T	Q	P	Designation
2	9	6	2	1	0	XXXIV

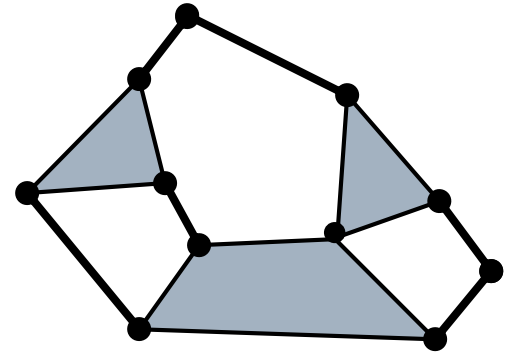
(Plus 10 more not shown)



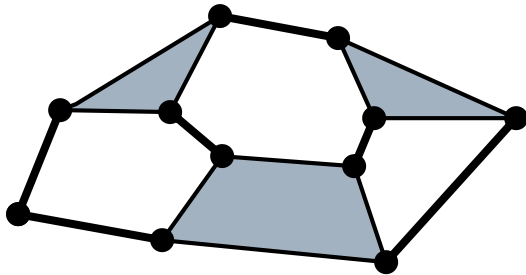
1



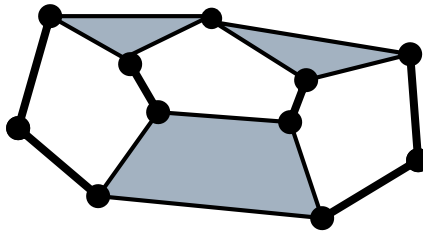
2



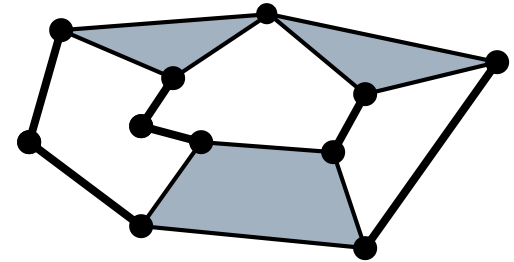
3



4



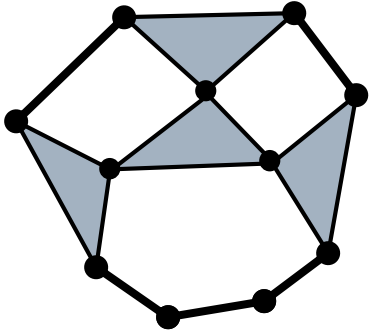
5



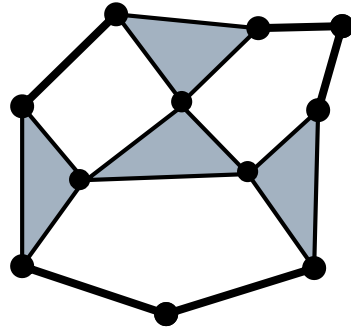
6

M	L	B	T	Q	P	Designation
2	9	5	4	0	0	XXXV

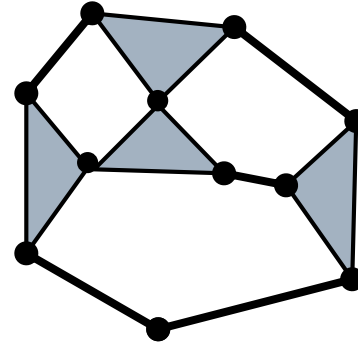
(Plus 11 more not shown)



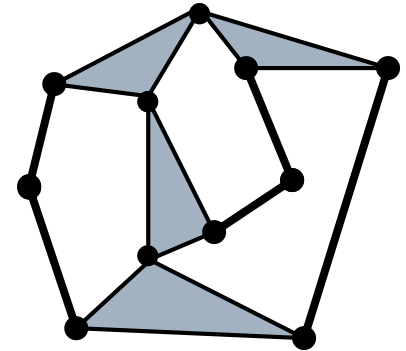
1



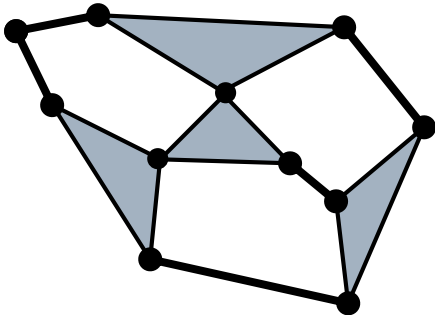
2



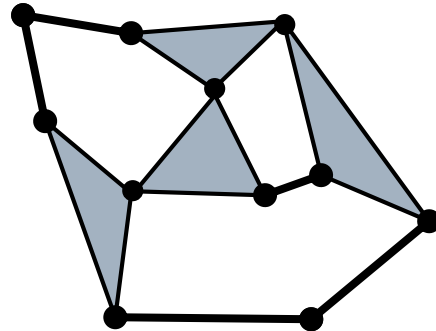
3



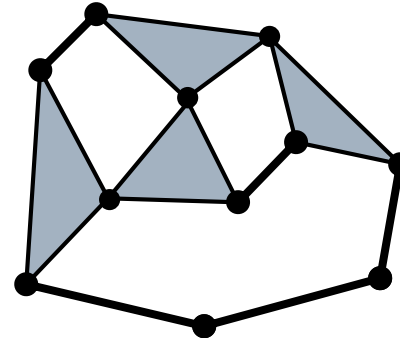
4



5



6



7