Mechanism Synthesis Rules

- Grübler’s Equation Modification Summary
- Linkage Transformation Rules Summary
- Grashof’s Law
- Inversion
Modifying Grübler’s Equation

\[ M = 3 \cdot (L - 1) - 2 \cdot J \]

\[ M \equiv \text{Mobility of Mechanism} \]
\[ L \equiv \text{Number of Links} \]
\[ J \equiv \text{Number of Joints} \]

\[ M_{b+} = 3 \cdot (L_{b+} - 1) - 2 \cdot J_{b+} \]

\[ M_{b+} \equiv \text{Mobility after adding Binary Links} \]
\[ L_{b+} \equiv \text{Total Number of Links} \]
\[ = L + b \]
\[ b \equiv \text{Number of Binary Links Added} \]
\[ J_{b+} \equiv \text{Total Number of Joints} \]
\[ = J + j \]
\[ j \equiv \text{Number of Joints Added} \]
LINKAGE TRANSFORMATION RULES

RULE 1: Revolute joints in any loop can be replaced by prismatic joints with no change in DOF of the mechanism, provided that at least two revolute joints remain in the loop.

RULE 2: Any full joint can be replaced by a half joint, but this will INCREASE the DOF by one.
- A half joint adds one imaginary link and one joint to the system

RULE 3: Addition of a link will Reduce the DOF by one, Removal of a link will Increase the DOF by one
- The DoF distribution Principle must be maintained
- This rule adds (subtracts) one link and two joint to (from) the system

\[
M_{ba} \equiv \text{Mobility w/ Added Binary Links} = 3 \cdot (L_{ba} - 1) - 2 \cdot J_{ba} \quad \Rightarrow \quad M_{ba} = M - n_{ba}
\]

\[n_{ba}\] \equiv \text{number of Binary links Added}

(is negative if links are subtracted)

\[L_{ba} \equiv \text{number of links in new mechanism} = L + n_{ba}\]

\[J_{ba} \equiv \text{number of Joints in new mechanism} = J + 2 \cdot n_{ba}\]
RULE 4: The Combination of rules 2 and 3 (Addition) above will keep the original DOF unchanged.

RULE 5: Any ternary or higher-order link can be partially “shrunk” to a lower-order link by coalescing nodes. This will create a multiple joint but will not change the DOF of the mechanism.

RULE 6: Complete shrinkage of a higher-order link is equivalent to its removal. A multiple joint will be created, and the DOF will be reduced.
Grashof’s Law

- In motor driven mechanisms it is important to ensure that the input crank can make a complete revolution.

- For a planar four bar linkage, the sum of the SHORTEST and LONGEST link lengths can not be greater than the sum of the REMAINING TWO link lengths if there is to be continuous relative rotation between two members.
Grashof’s Law

- $l$ – is the length of the longest link
- $s$ – is the length of the shortest link
- $p, q$ – lengths of the other two links

$s + l \leq p + q$

- Does not specify link order
- Does not specify which link is fixed
Examples of Grashof Mechanisms

- Crank Rocker,
  \[ s + l < p + q \]
  \[ 2 + 6 < 4 + 5 \]
  \[ s \text{-Drive} \]

- Double Crank,
  \[ s + l < p + q \]
  \[ 2 + 6 < 4 + 5 \]
  \[ s \text{-Ground} \]
Examples of Grashof Mechanisms

- **Non-Grashof** Rocker-Rocker (Triple Rocker),
  \[ s+l > p+q \]

- **Double Rocker,**
  \[ s+l = p+q \]
  \[ 2+6=4+4 \]
Kinematic Inversion

☐ Until frame link has been chosen
  ▪ Kinematic chain

☐ When different links are chosen as the frame link
  ▪ RELATIVE motion between links is not altered
  ▪ ABSOLUTE motion measured with respect to the frame is drastically changed
Kinematic Inversion
Slider Crank

Internal Combustion Engine Piston
Kinematic Inversion
Slider Crank – Link 2 Fixed

Rotary Engine Pistons
Kinematic Inversion
Slider Crank – Link 3 Fixed

Oscillating Cylinder Engine

Used to
• Convert reciprocating motion into rotary motion or
• Convert rotary motion into reciprocating motion
Kinematic Inversion
Slider Crank – Link 4 Fixed

Bull or Pendulum Engine

Used to in water pumps
Synthesize a simple slider drive mechanism. There is to only be a single drive that will complete define the motion of the mechanism.

What types of mechanisms could be used if the type of output sliding motion is well defined?
Solution 1: $M=+1$, $L=4$
Example 1: M=+1, L=6

Synthesize a simple slider drive mechanism. There is to only be a single drive that will complete define the motion of the mechanism.

What types of mechanisms could be used if the type of output sliding motion is well defined?
Solution 1: M=+1, L=6
Solution 1: M=+1, L=6
Example 1: $M=+1$, $L=8$

Synthesize a simple slider drive mechanism. There is to only be a single drive that will complete define the motion of the mechanism.

What types of mechanisms could be used if the type of output sliding motion is well defined?
Solution 1: M=+1, L=8
Example 2:

Synthesize the simplest possible drive mechanism for oscillating two widely separated agitator shafts.
Solution 2: